Basis for Computing, Programming and Mathematical Modeling

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Programming

- writing instructions, that will make the computer follow and run a program based on those instructions

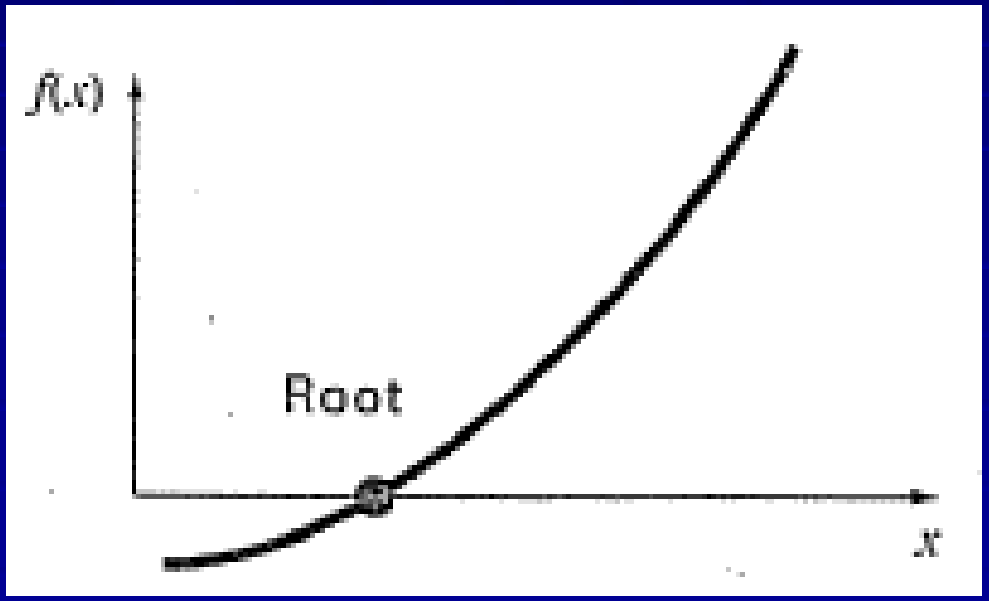
- Engineers use programming to solve numerical problems that come up as a result of mathematical modeling

Numerical Analysis

1. Roots of equations
2. Systems of linear algebraic equations
3. Optimization
4. Curve fitting
5. Integration
6. Ordinary differential equations
7. Partial differential equations

1. Roots of Equations

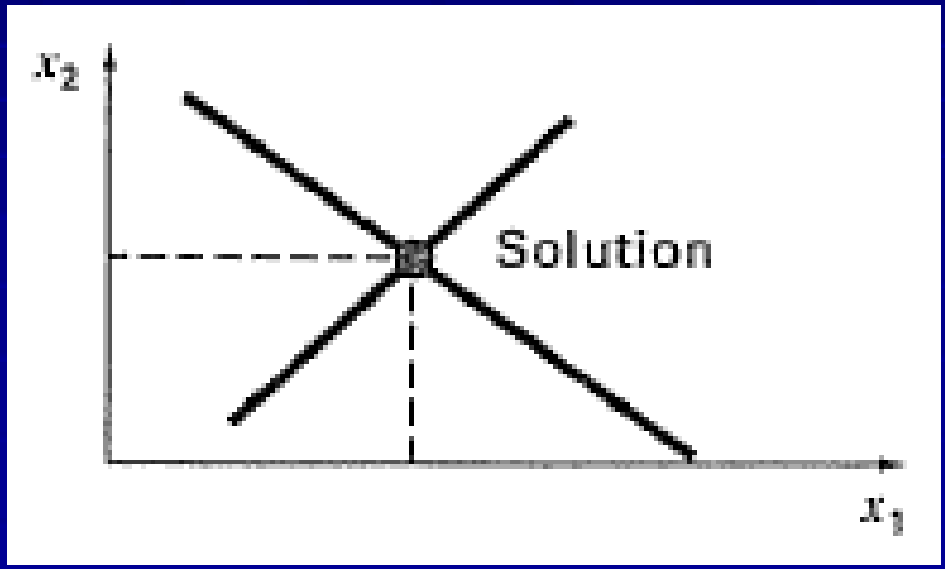
- Solve f(x) = 0 for x



2. Linear Algebraic Equations

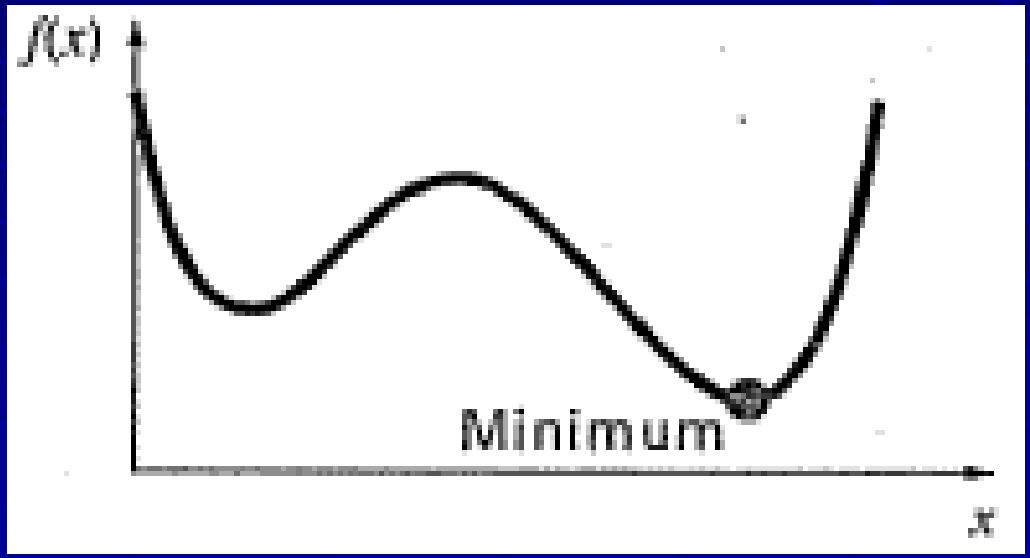
- Given a_{11}, a_{12}, a_{21}, a_{22} and y_1, y_2 as constant parameters, solve for x's

$$a_{11}x_1 + a_{12}x_2 = y_1$$
$$a_{21}x_1 + a_{22}x_2 = y_2$$



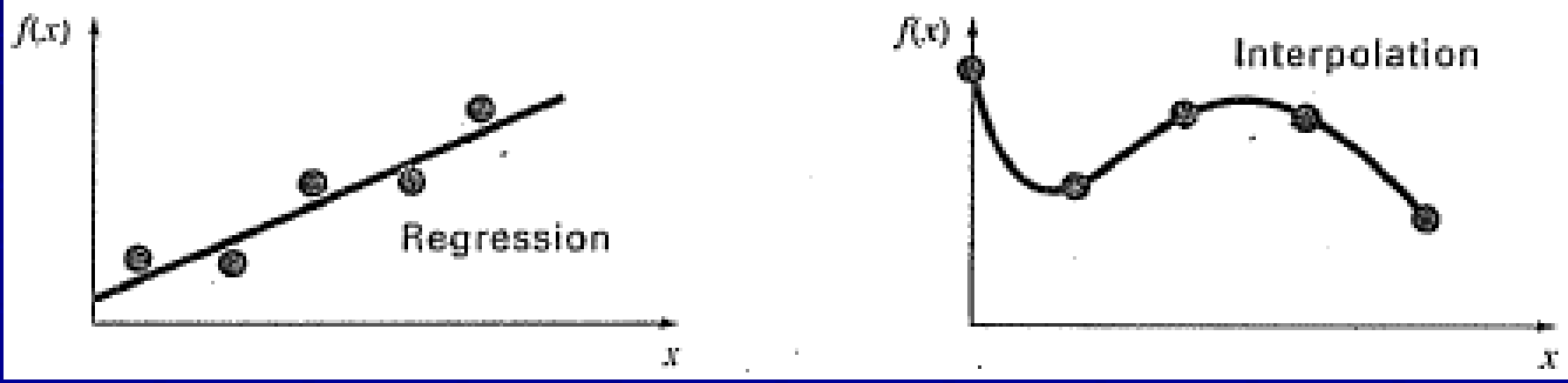
3. Optimization

- Determine x that gives minimum (or maximum) f(x)



4. Curve Fitting

- Fit a function (or functions) to a set of data



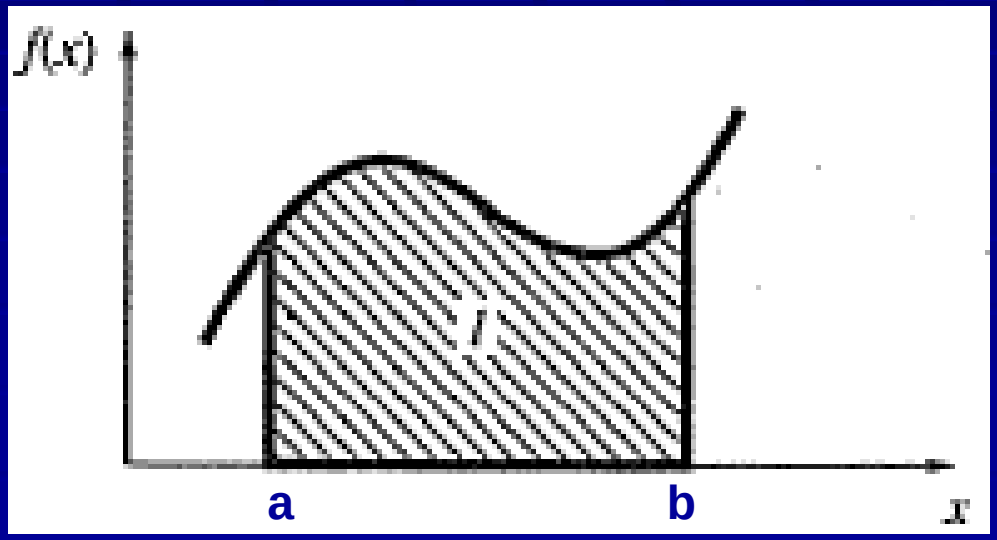
5. Integration

- Find the area *I* under the curve

$$I = \int_a^b f(x)dx$$

$$\frac{dy}{dt} \simeq \frac{\Delta y}{\Delta t} = f(t, y)$$

$$y_{i+1} \simeq y_i + f(t_i, y_i)\Delta t$$



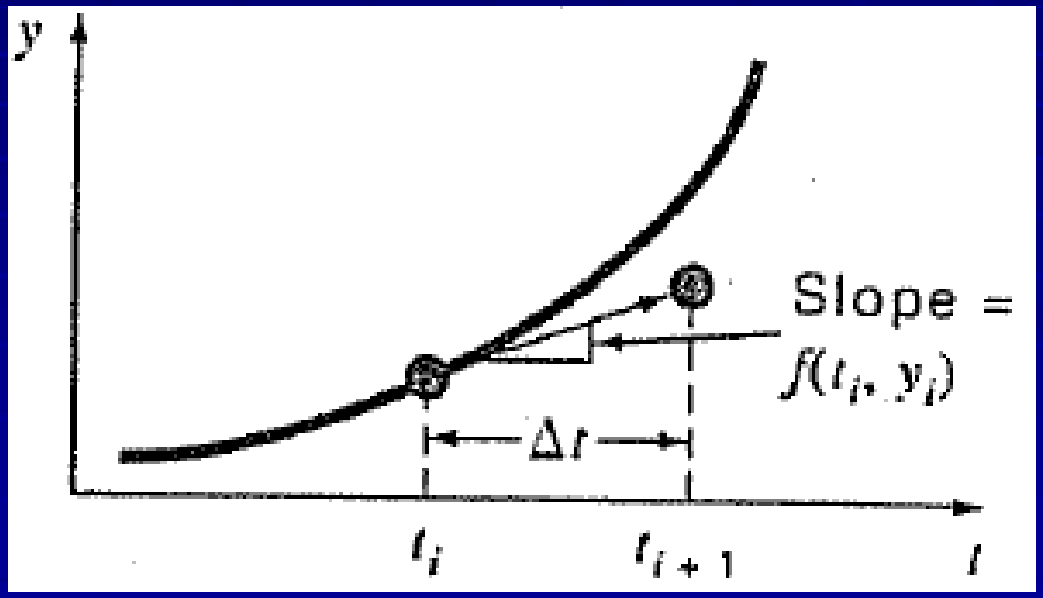
6. Ordinary Differential Equations

Given

$$\frac{dy}{dt} \simeq \frac{\Delta y}{\Delta t} = f(t, y)$$

$$\frac{\partial^2 u}{dx^2} + \frac{\partial^2 u}{dy^2} = f(x, y)$$

Solve for *y* as a function of *t*

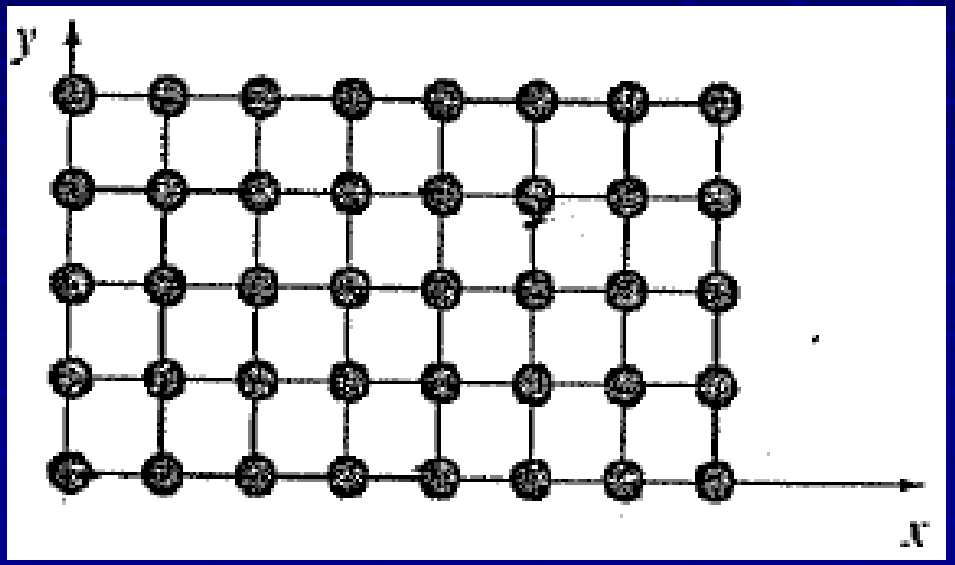


$$y_{i+1} \simeq y_i + f(t_i, y_i)\Delta t$$

7. Partial Differential Equations

Given

$$\frac{\partial^2 u}{dx^2} + \frac{\partial^2 u}{dy^2} = f(x, y)$$



Solve for *u* as a function of *x* and *y*

Mathematical Modeling and Numerical Analysis

Mathematical model→ uses mathematical language to describe a system

Application fields:

- Natural sciences and engineering disciplines
  - Physics, biology, earth science, meteorology, electrical engineering, chemical engineering, mechanical engineering, …
- Social sciences
  - Economics, psychology, sociology, political science, …

Mathematical Modeling and Numerical Analysis

- Eykhoff"s definition of Mathematical model (1974) →

'A representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form'.

- Mathematical models can take many forms: (including but not limited to)
  - dynamical systems,
  - statistical models,
  - differential equations,
  - game theoretic models.

  These models and other types can overlap

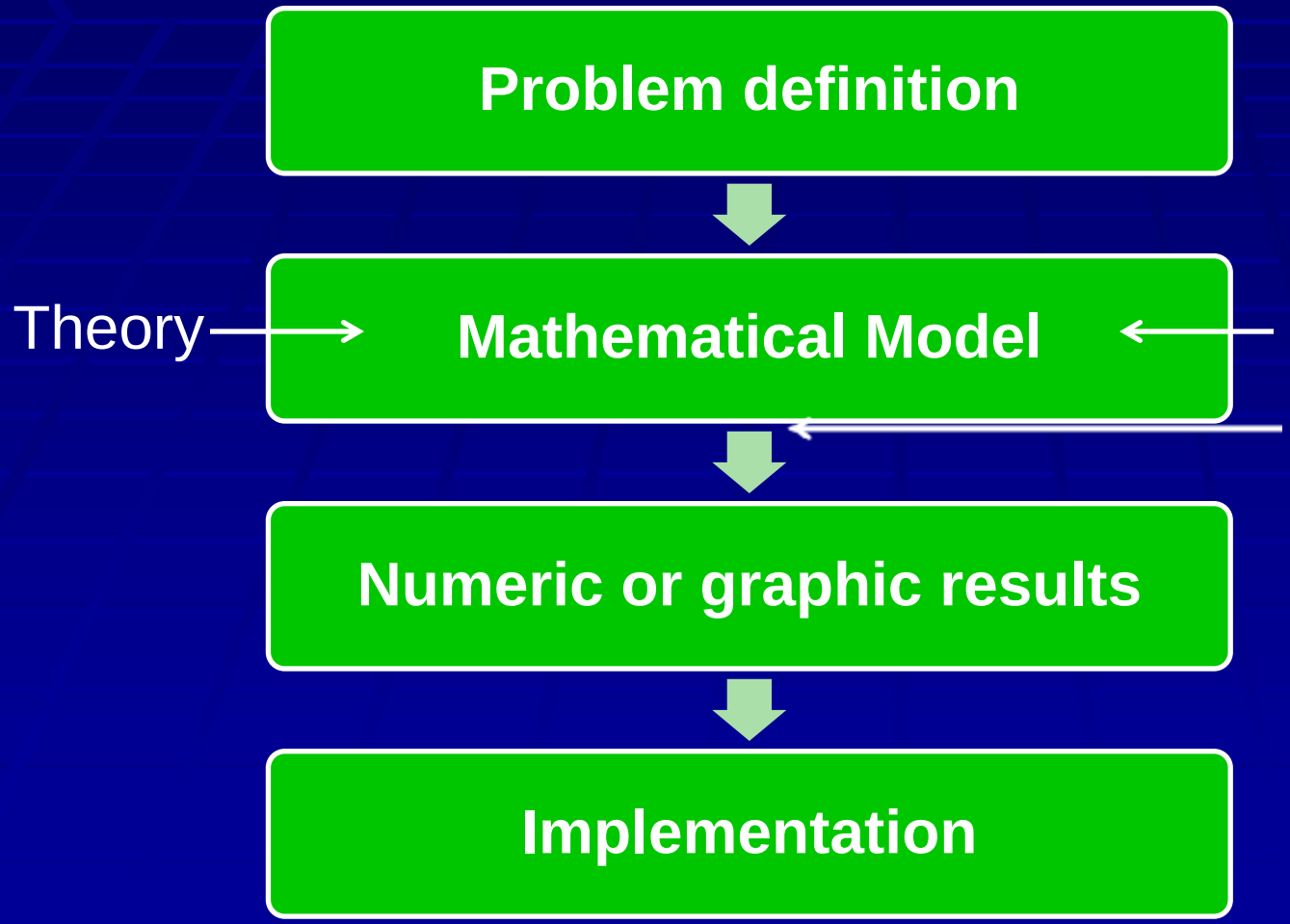
Ref: www.sciencedaily.com/articles/m/mathematical_model.htm

Mathematical Modeling and Numerical Analysis

Diagnosis of a physical problem:

- Define the physical problem
- Formulate it mathematically
- Solve the mathematical formulae
  - Analytical methods (Exact solution)
  - Numerical methods (Approximate solution)
- Interpret the results

Mathematical Modeling



Reading Suggestion

- Read the following article
- Does it give you an idea about what mathematical modeling is

http://pages.cpsc.ucalgary.ca/~gaines/reports/PSYCH/IJISG91/index.html

Modeling Practical Reasoning

Brian R Gaines
Knowledge Science Institute,
University of Calgary
Calgary, Alberta, Canada T2N 1N4
gaines@cpsc.ucalgary.ca

A simple mathematical model

- Formulation or equation that expresses the essential features of a physical system or process in mathematical terms
- Dependent variable = f (
  - Independent variables,
  - Constant parameters,
  - Forcing functions or external influences acting on the system)

Homework

- A parachutist of mass 70 kg jumps out of a helicopter standing still in air. Write the equations to solve for the falling velocity of the man before he opens his chute. The drag coefficient in air is 12.5 kg/s.
- Hint: Balance the upward and downward forces.

Mathematical Modeling and Numerical Analysis

Basic groups of variables:

1. decision variables,
2. input variables,
3. state variables,
4. exogenous variables (fundamental in path analysis and structural equation modeling; in causal modeling these are the variables with no causal links (arrows) leading to them from other variables in the model)
5. random variables,
6. output variables

Ref: www.sciencedaily.com/articles/m/mathematical_model.htm

Random Variable

- is a variable that takes different real values as a result of the outcomes of a random event or experiment
- is a real valued function defined over the elements of a sample space

There can be more than one random variable associated with an experiment.

Ex: if a coin is tossed ten times, one random variable associated with this experiment could be the number times the head shows up, a second random variable could be the number times the tail shows up and a third random variable could be the difference between number of times the head shows up and the number of times the tail shows up.

http://www.statistics.com/resources/glossary/r/randvar.php

Mathematical Modeling and Numerical Analysis

- Mathematical modelling problems are often classified into black box or white box models, according to how much a priori information is available of the system
- Black-box model is a system of which there is no a priori information available
- White-box (glass box or clear box) model is a system where all necessary information is available
- Practically all systems are somewhere between the black-box and white-box models, so this concept only works as an intuitive guide for approach
- It is preferable to use as much a priori information as possible to make the model more accurate

Ref: www.sciencedaily.com/articles/m/mathematical_model.htm

Computer Simulation

- Useful part of mathematical modeling of natural systems in:
  - physics, chemistry and biology,
  - human systems in economics, psychology, social science
  - the process of engineering new technology
- Used to gain insight into the operation of these systems
- Mathematical model
  - Attempts to find analytical solutions to problems
  - Is a set of equations that has physical meaning
  - Uses a set of parameters and initial conditions
  - Enables the prediction of the behavior of the system
- Computer simulations build on, and are a useful adjunct to purely mathematical models in science, technology and entertainment.

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Debugging ?

- Term from early computer programming
- Means: Sseeking out problems in the program and correcting them

- If your computer is asking for debugging, it is simply asking you permission to continue what it thinks will make the site work.

  Choose “yes” and see what it does.

  It probably won't harm anything, and it could help.

Debugging ?

Rear Admiral Grace Murray Hopper

The First "Computer Bug“

Moth found trapped between points at Relay # 70, Panel F, of the Mark II Aiken Relay Calculator while it was being tested at Harvard University, 9 September 1947. The operators affixed the moth to the computer log, with the entry: "First actual case of bug being found". They put out the word that they had "debugged" the machine, thus introducing the term "debugging a computer program".

In 1988, the log, with the moth still taped by the entry, was in the Naval Surface Warfare Center Computer Museum at Dahlgren, Virginia.

Source: http://www.history.navy.mil/photos/pers-us/uspers-h/g-hoppr.htm

Debugging ?

Photo # NH 96566-KN (Color) First Computer "Bug", 1947

9/9

0800 Arctan started
1000 " stopped - arctan ✓ { 1.2700 9.037 847 025
9.037 846 795 correct
13" vc (032) MP - MC ~~1.9821447000~~ ~~2.13047641S (-3)~~ 4.615925059(-2)
(033) PRO 2 2.13047641S
corrected 2.13067641S
Relays 6-2 in 033 failed special speed test
In relay " 11.00 test.
Relays changed
1100 Started Cosine Tape (Sine check)
1525 Started Multy Adder Test.
1545  Relay #70 Panel F
(moth) in relay.
First actual case of bug being found.
1700 1630 Arctangent started.
closed down.

Source: http://www.history.navy.mil/photos/pers-us/uspers-h/g-hoppr.htm

Debugging

- However the term "bug" in the meaning of technical error dates back at least to 1878 and Thomas Edison and "debugging" seems to have been used as a term in aeronautics before entering the world of computers. Indeed, in an interview Grace Hopper remarked that she was not coining the term. The moth fit the already existing terminology, so it was saved.

Watch this web course!

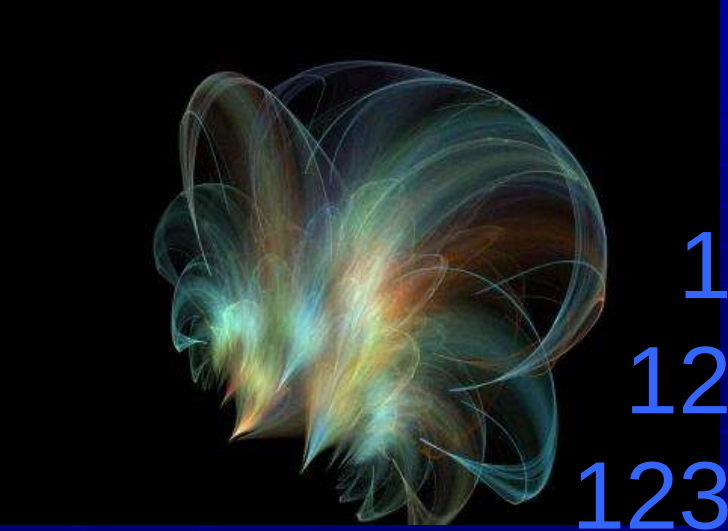
- Listen to the course on “History of Computing” at

http://www.academicearth.org/lectures/the-history-of-computing

The Beauty of Mathematics

Here is an interesting way to look at the beauty of mathematics

Wonderful World



$1 \times 8 + 1 = 9$

$12 \times 8 + 2 = 98$

$123 \times 8 + 3 = 987$

$1234 \times 8 + 4 = 9876$

$12345 \times 8 + 5 = 98765$

$123456 \times 8 + 6 = 987654$

$1234567 \times 8 + 7 = 9876543$

$12345678 \times 8 + 8 = 98765432$

$123456789 \times 8 + 9 = 987654321$

$1 \times 9 + 2 = 11$

$12 \times 9 + 3 = 111$

$123 \times 9 + 4 = 1111$

$1234 \times 9 + 5 = 11111$

$12345 \times 9 + 6 = 111111$

$123456 \times 9 + 7 = 1111111$

$1234567 \times 9 + 8 = 11111111$

$12345678 \times 9 + 9 = 111111111$

$123456789 \times 9 + 10 = 1111111111$

$9 \times 9 + 7 = 88$

$98 \times 9 + 6 = 888$

$987 \times 9 + 5 = 8888$

$9876 \times 9 + 4 = 88888$

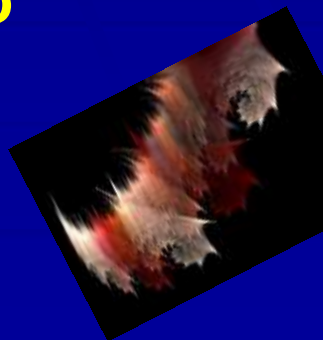
$98765 \times 9 + 3 = 888888$

$987654 \times 9 + 2 = 8888888$

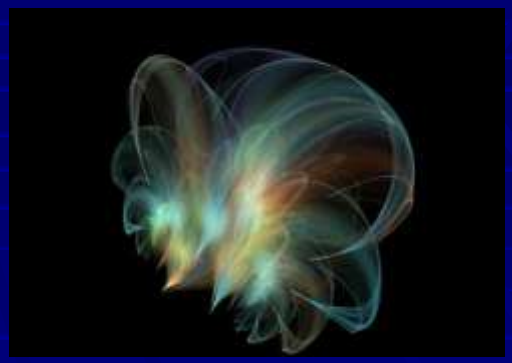
$9876543 \times 9 + 1 = 88888888$

$98765432 \times 9 + 0 = 888888888$

Brilliant, isn’t it?



And look at this symmetry:



1 x 1 = 1

11 x 11 = 121

111 x 111 = 12321

1111 x 1111 = 1234321

11111 x 11111 = 123454321

111111 x 111111 = 12345654321

1111111 x 1111111 = 1234567 654321

11111111 x 11111111 =

123456787654321

111111111 x 111111111 =

12345678987654321

Now, take a look at this…

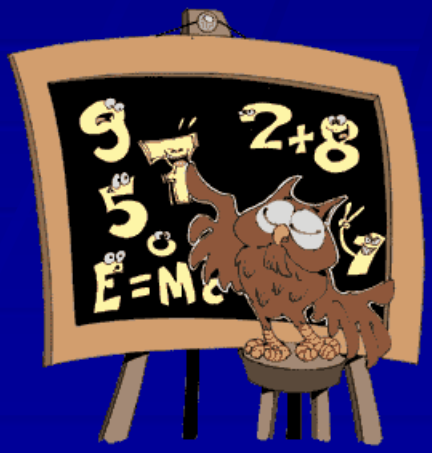
From a strictly mathematical viewpoint:

What Equals 100%?

What does it mean to give 100%?

How about ACHIEVING 100%.

What equals 100% in life?



Here's a little mathematical formula that might help

Answer these questions:

If:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Is represented as:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26.

then

H-A-R-D-W-O-R- K

8+1+18+4+23+15+18+11 = 98%

and

K-N-O-W-L-E-D-G-E

11+14+15+23+12+5+4+7+5 = 96%

But

A-T-T-I-T-U-D-E

1+20+20+9+20+21+4+5 = **100%**

Therefore, one can conclude with mathematical certainty that:

While Hard Work and Knowledge will get you close, Attitude will get you there!

Have a great day!