Solving Simultaneous Nonlinear Equations Substitutive Iteration and Graphical Method

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Quotes of the Day: Keep Moving Forward

"Every worthwhile accomplishment, big or little, has its stages of drudgery and triumph; a beginning, a struggle and a victory." Ghandi

"The price of success is hard work, dedication to the job at hand, and the determination that whether we win or lose, we have applied the best of ourselves to the task at hand."
Vince Lombardi

What are we going to learn today?

- An iterative technique to solve system of multivariate nonlinear equations
- Plotting 2.5D and 3D plots of multivariate functions

Nonlinear Functions of Several Variables

- System with two nonlinear functions and two variables

  $z=f(x_1,x_2)$ and $z=g(x_1,x_2)$

- Problem is more difficult to solve for more variables $f(x_1,x_2,x_3,\dots,x_n)$

Nonlinear Functions of Several Variables

- To find a zero of the system, find the intersection of the curves:

  $f(x_1,x_2)=0$ and $g(x_1,x_2)=0$

ex: $f(x,y)=x^2+y^2-1$ and $g(x,y)=x^2-y$

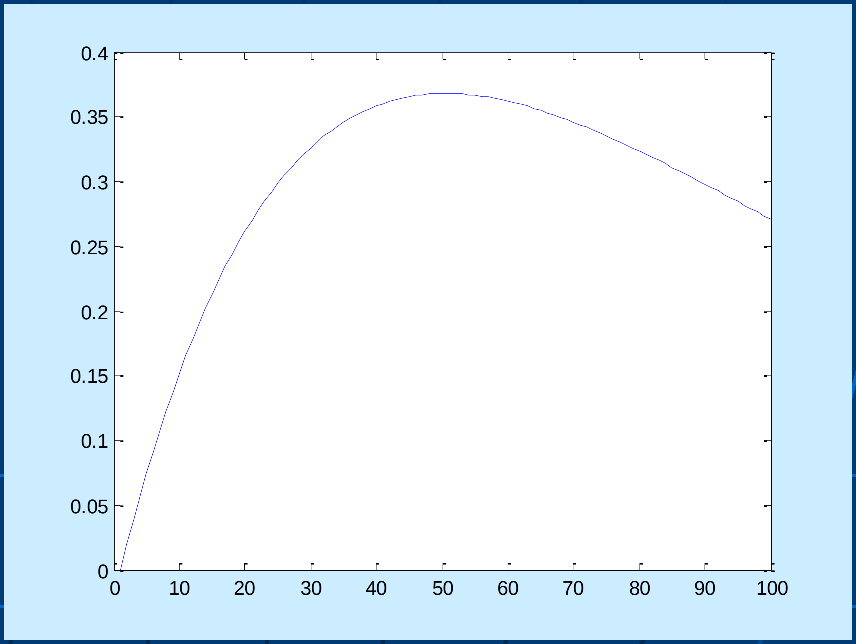
- Use all the information about the problem to find the region where the curves may intersect → SO…???

Nonlinear Functions of Several Variables

- Plot a 2-D or 3-D figure of the functions in MATLAB® !

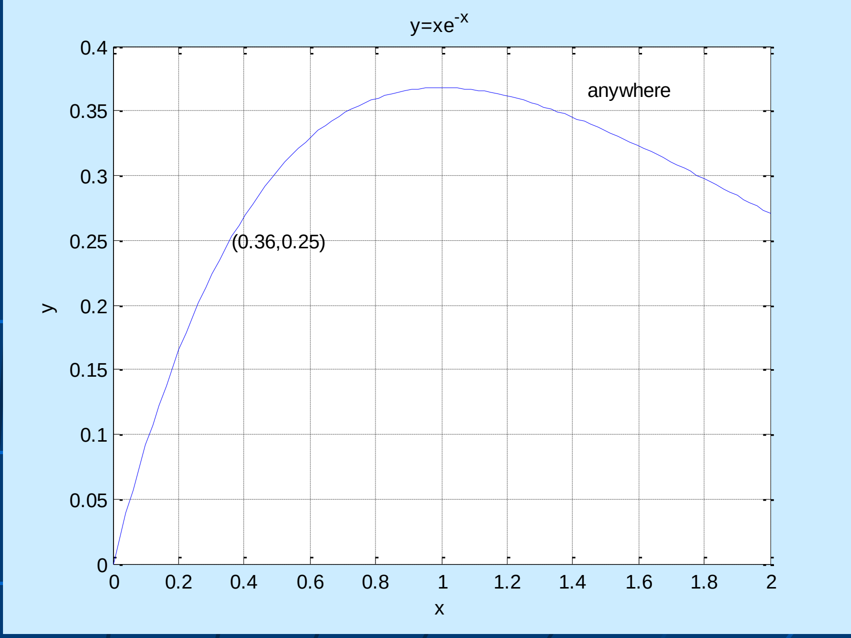
2-D Graphs

```
>> x=linspace(0,2);
% linspace generates a row vector of 100 linearly
% equally spaced points between x1 and x2
>> y=x.*exp(-x);
>> plot(y)
% plots y versus their index
```



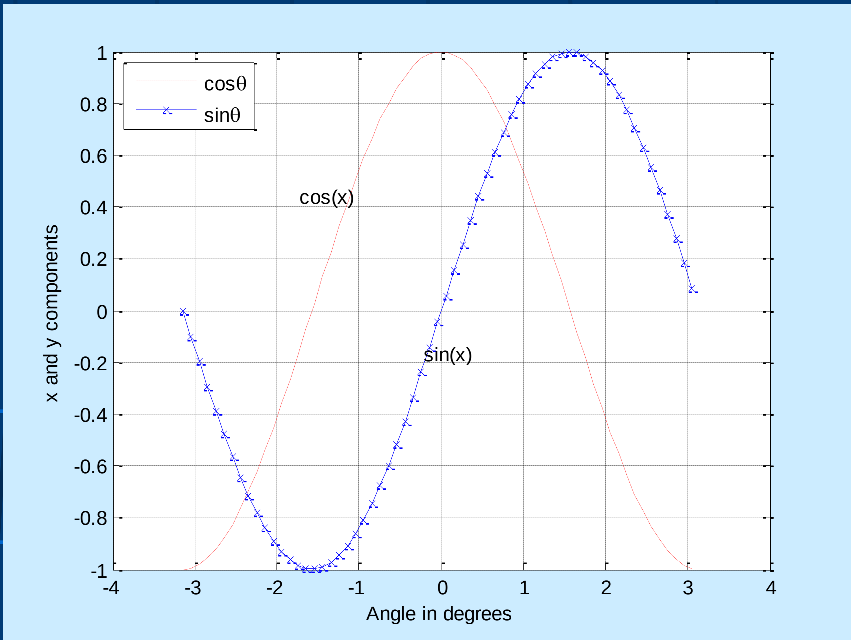
2-D Graphs

```
% ExPlot2DGraph.m
x=linspace(0,2);
% linspace generates a row vector of 100 linearly
% equally spaced points between x1 and x2
y=x.*exp(-x);
plot(x,y)           % plots y agains x
grid on             % adds grid lines to the current axes
xlabel('x')         % adds text below the x-axis
ylabel('y')
% adds text besides the y-axis
title('y=xe^{-x}')
% adds text at the top of graph
gtext('anywhere')
%places text with mouse
text(0.36,0.25,'(0.36,0.25)')
% places text at the specific point
```



2-D Graphs

```
% ExPlot.m: This program draws a graph of sin(x) and cos(x) % where
    0 <= x <= 3.14
angle=-pi:0.1:pi;           % Create array
xcomp=cos(angle);           % Create array
plot(angle,xcomp,'r:');  % Plot using dots(:) with red(r)
hold on                     % Add another plot on the same graph
ycomp=sin(angle);
% Create array
plot(angle,ycomp,'b-x');
% Plot using lines(-) and the symbol x
% at each data point with blue(b)
grid on
xlabel('Angle in degrees');
ylabel('x and y components');
legend('cos{\theta}','sin{\theta}',2)
gtext('cos(x)');
gtext('sin(x)');
% Display mouse-movable text
```



2-D Graphs: Other Commands

- More than one function can be plotted on one graph:

  >>plot(x,X.*exp(-x),'.',x,x*sin(x),'-.')
- More than one graph can be shown in different frames

  >>subplot(2,1,1), plot(x,x.*cos(x))

  >>subplot(2,1,2), plot(x,x.*sin(x)
- Axis limits can be seen or modified

  >>axis

  >>axis([0,1.5,0,1.5])
- Figure window can be cleared

  >>clf

2-D Graphs: Other Commands

- Comet like trajectory of the function

  >> shg, comet(x,y)

% shg brings up the current graphic window

- By using **figure(n)** command, it is possible to use more than one graphic window, n: positive integer

- Another easy way to plot a function:

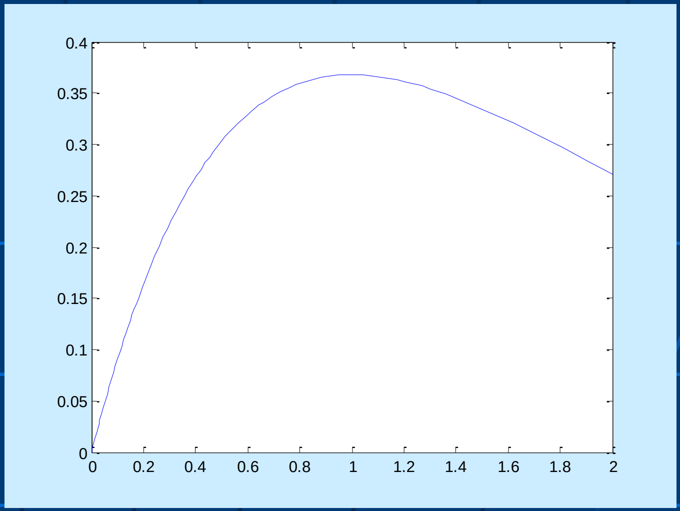
  >>fplot(x*)

2-D Graphs: fplot

Another easy way to plot a function: fplot

- fplot(@humps,[0 1])
- fplot(@(x)[tan(x),sin(x),cos(x)], 2*pi*[-1 1 -1 1])
- fplot(@(x) sin(1./x), [0.01 0.1], 1e-3)
- f = @(x,n)abs(exp(-1j*x*(0:n-1))*ones(n,1)); fplot(@(x)f(x,10),[0 2*pi])

>> fplot('x*exp(-x)',[0,2])

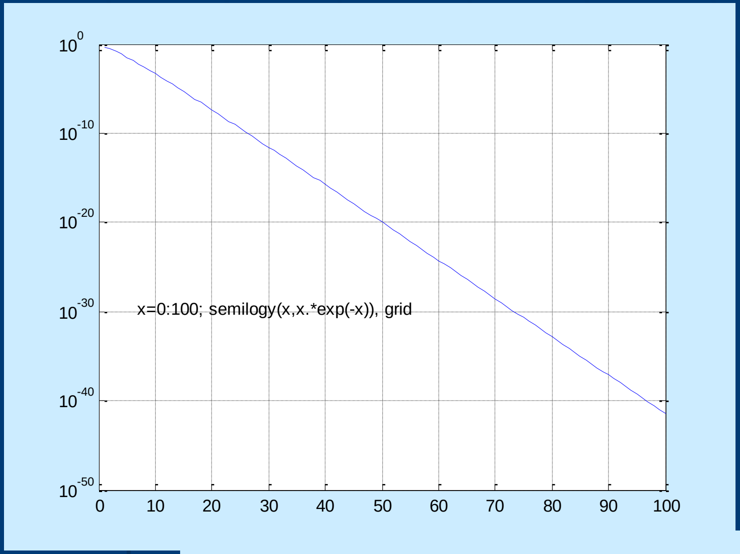


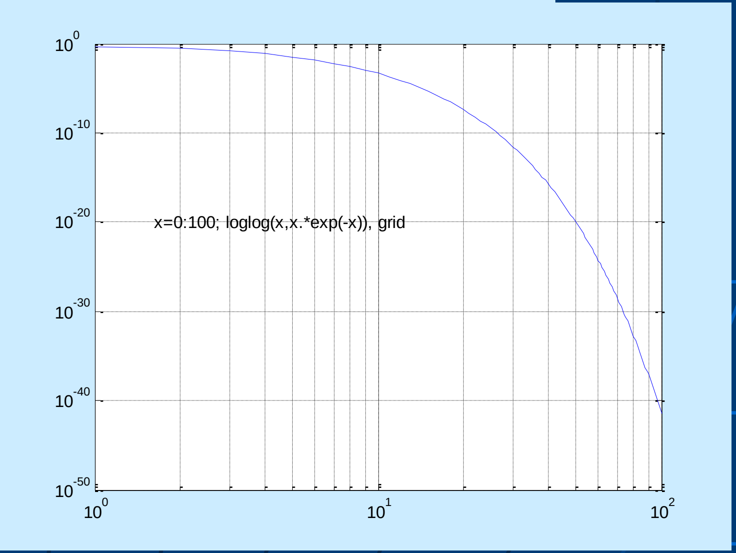
2-D Graphs: Other Commands

semilogx(x,y)
% semilogarithmic plot
% log scale x-axis

semilogy(x,y)
% log scale y-axis

loglog(x,y)
% log scale x- and y-axis





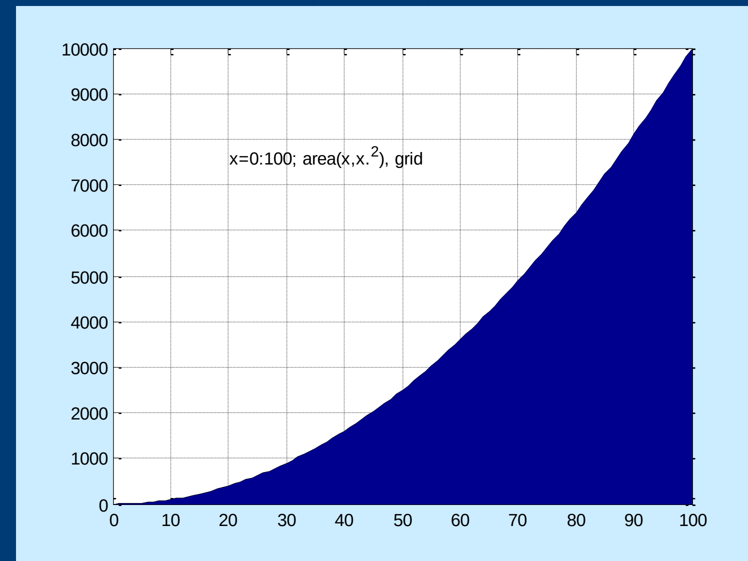
2-D Graphs: Other Commands

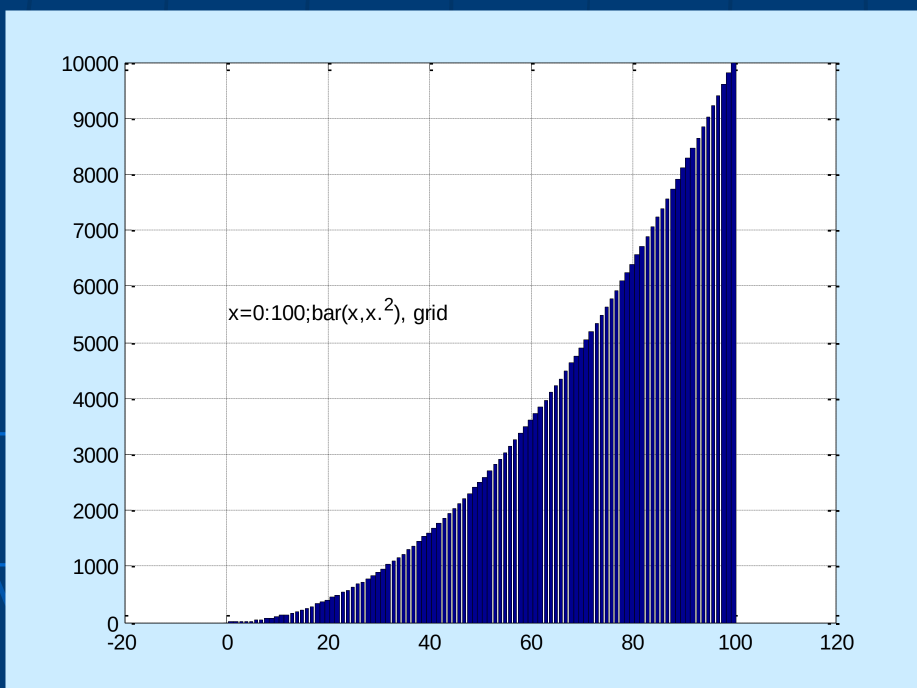
area(x,y) % filled area plot

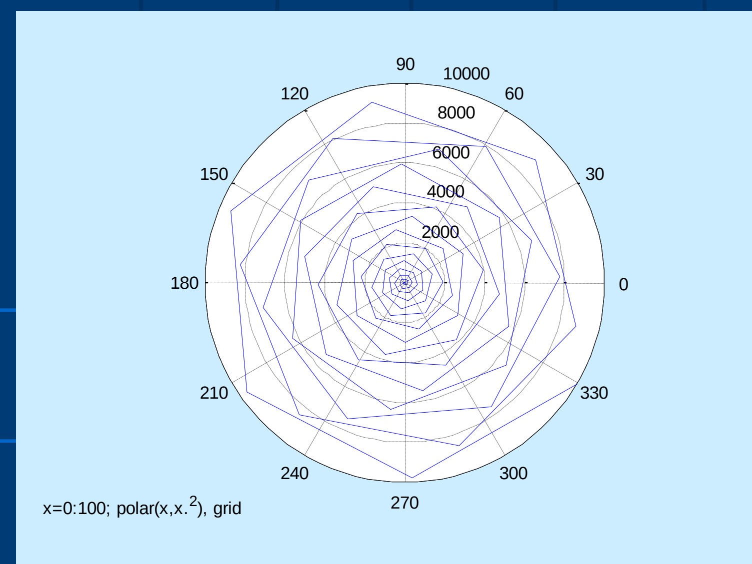
polar(x,y)

% polar coordinate plot

bar(x,y)   % bar graph

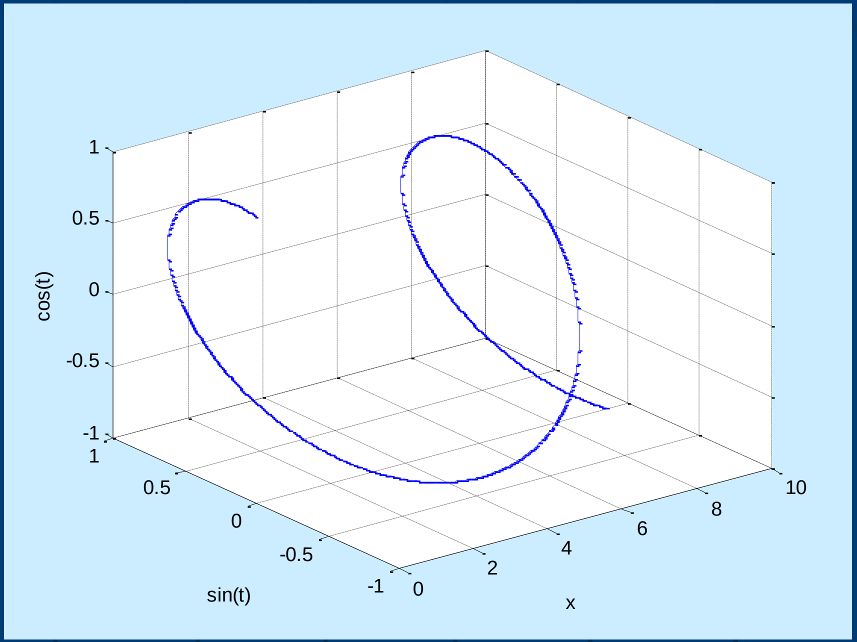






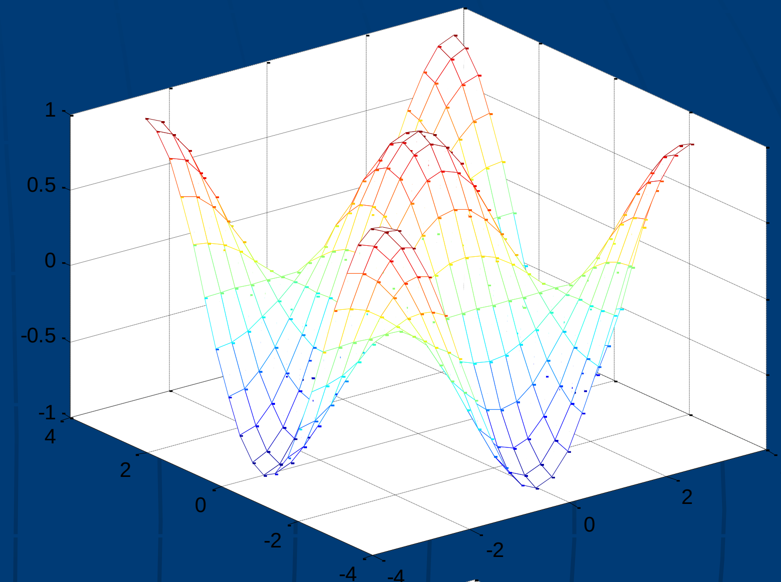
3-D Graphs

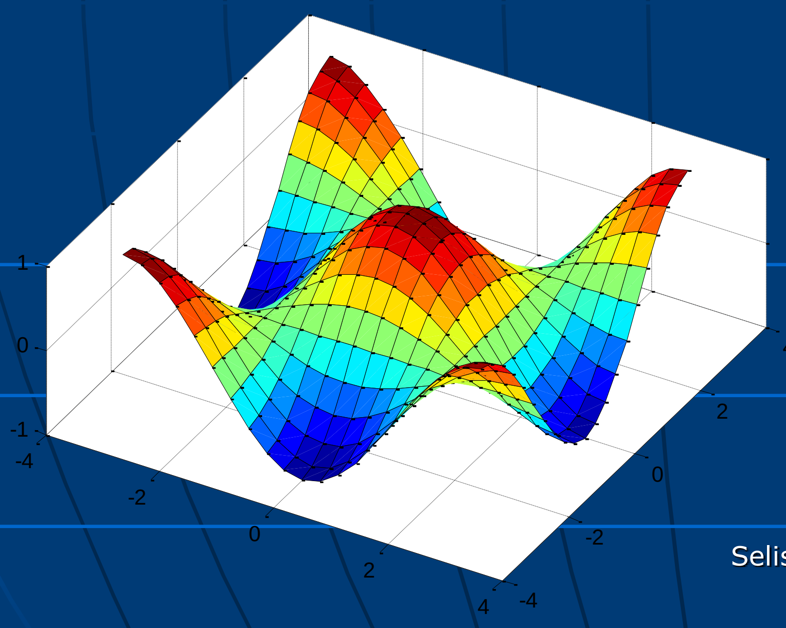
```
% ExPlot3DGraph.m
t=0:0.01:3*pi;
plot3(t,sin(t),cos(t))
xlabel('x'),
ylabel('sin(t)'),
zlabel('cos(t)')
grid
```

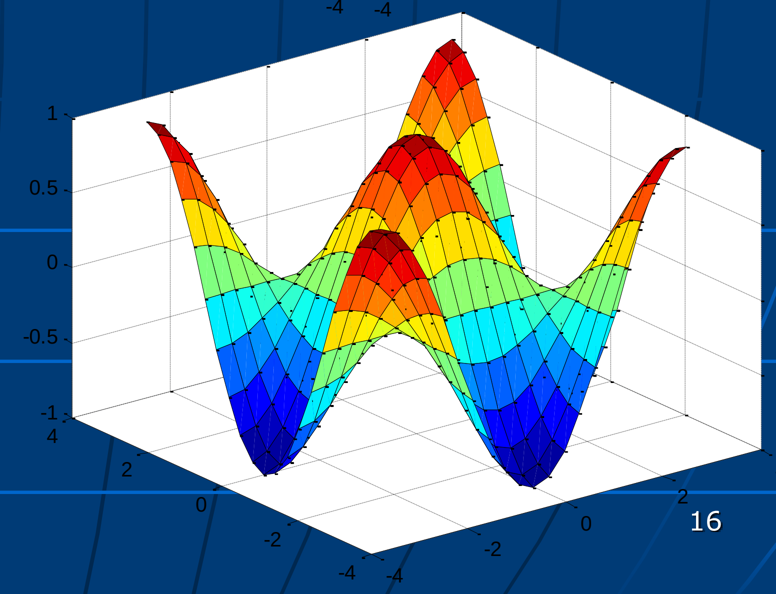


3-D Graphs: Surfaces

```
% ExPlot3DGraphSurface.m
[x,y]=meshgrid(-pi:pi/10:pi, -
   pi:pi/10:pi);
z=cos(x).*cos(y);
figure(1), mesh(x,y,z)
figure(2), surf(x,y,z)
figure(3), surf(x,y,z), view(30,60)
```







3-D Graphs: Surfaces

shading controls the color shading of **surface** and **patch** objects
surface and **patch** objects are created by the functions **surf**, **mesh**, **pcolor, fill**, and **fill3**

Options: shading flat, interp, faceted (default)

Flat shading is piecewise constant; each mesh line segment or surface patch has a constant color determined by the color value at the end point of the segment or the corner of the patch which has the smallest index or indices.

Interpolated shading, which is also known as Gouraud shading, is piecewise bilinear; the color in each segment or patch varies linearly and interpolates the end or corner values.

Faceted shading is flat shading with superimposed black mesh lines. This is often the most effective and is the default.

3-D Graphs: colorbar

COLORBAR Display color bar (color scale)
COLORBAR appends a colorbar to the current axes in the default (right) location
COLORBAR('peer',AX) creates a colorbar associated with axes AX instead of the current axes.
COLORBAR(...,LOCATION) appends a colorbar in the specified location relative to the axes. LOCATION may be any one of the following strings:

'North' inside plot box near top
'South' inside bottom
'East' inside right
'West' inside left
'NorthOutside' outside plot box near top
'SouthOutside' outside bottom
'EastOutside' outside right
'WestOutside' outside left

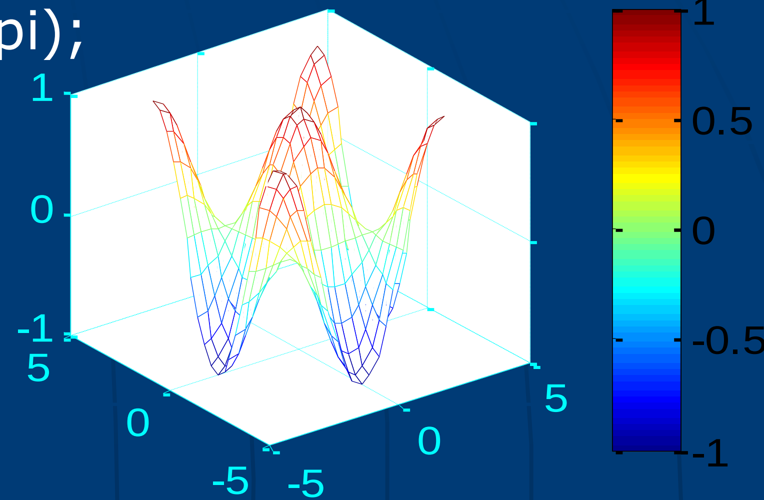
COLORBAR(...,P/V Pairs) specifies additional property name/value pairs for colorbar
H = COLORBAR(...) returns a handle to the colorbar axes

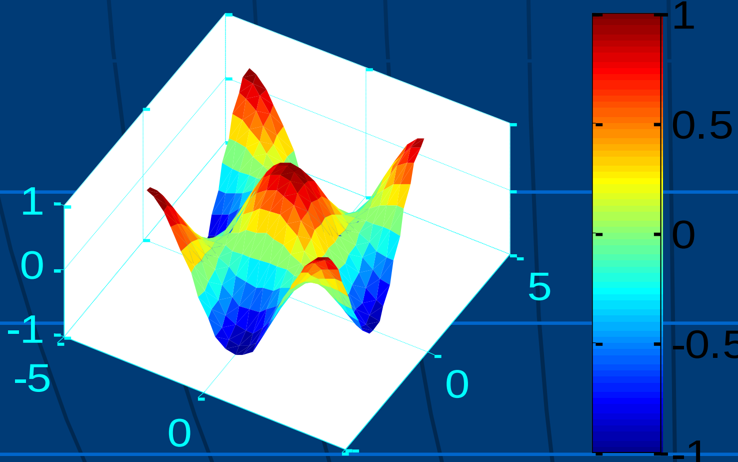
MATLAB® Plotting Command: **surf**

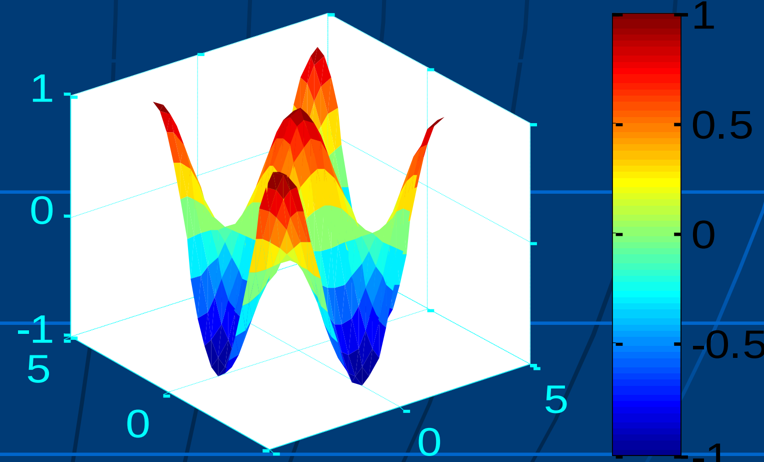
- **Plots 3-D surface**
- **surf**(x,y,z,c) plots the colored parametric surface defined by four matrix arguments.
  - The view point is specified by **view.** The axis labels are determined by the range of X, Y and Z, or by the current setting of **axis**
  - The color scaling is determined by the range of C, or by the current setting of **caxis**.
  - The scaled color values are used as indices into the current **colormap**
  - The shading model is set by **shading**
- **surf**(x,y,z) uses c=z, so color is proportional to surface height

3-D Graphs: Surfaces

```
% ExPlot3DGraphSurface.m
[x,y]=meshgrid(-pi:pi/10:pi,-pi:pi/10:pi);
z=cos(x).*cos(y);
subplot(2,2,2)
mesh(x,y,z), shading interp, colorbar
subplot(2,2,4)
surf(x,y,z), shading interp, colorbar
subplot(2,2,3), surf(x,y,z)
view(30,60), shading interp, colorbar
```







MATLAB® Plotting Command: **meshgrid**

- 2-D arrays, x and y, can be generated from 1-D arrays x1 and y1 as:

  [x,y]=**meshgrid**(x1,y1)

  - x1 and y1 represent x_i and y_j
  - x and y represent $x_{i,j}$ and $y_{i,j}$

Plot the grid using:

mesh(x,y,0*x);

view([0,0,10000]);

xlabel(‘x’); ylabel(‘y’)

MATLAB® Plotting Command: **mesh**

- 2-D function $z_{i,j}=f(x_{i,j},y_{i,j})$ can be plotted using the mesh command

Ex: $x_{i,j} = x_i = -2+0.2(i-1), \quad 1 \le i \le 21$

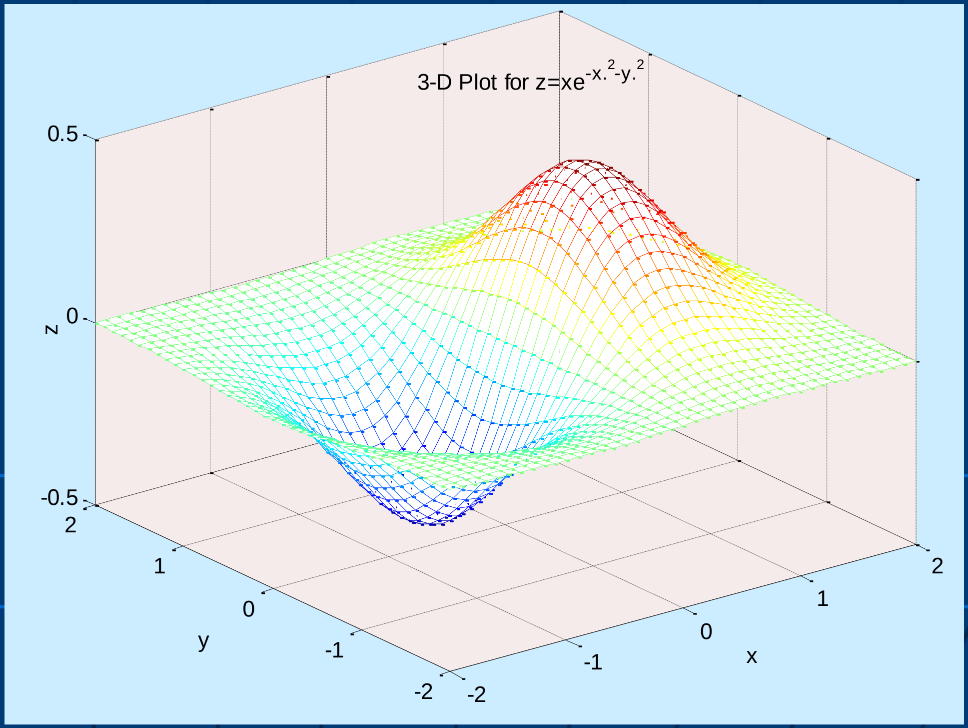
$y_{i,j} = y_j = -2+0.2(j-1), \quad 1 \le j \le 21$

The function is defined by $z_{i,j}=x_{i,j}e$^$(-x_{i,j}^2-y_{i,j}^2)$

Plot the grid using **mesh** function…

MATLAB® Plotting Command: **mesh**

```
z=x.*exp(-x.^2-y.^2);  mesh(x,y,z),
title('3-D Plot for z=xe^{-x.^{2}-y.^{2}}'),
xlabel('x');
ylabel('y');
zlabel('z');
```



2.5-D Graphs: Surface

Used for visualizing a 3-D graph on a 2-D system of coordinates, i.e.,

showing different z-levels on an x-y system of coordinates by its contour lines

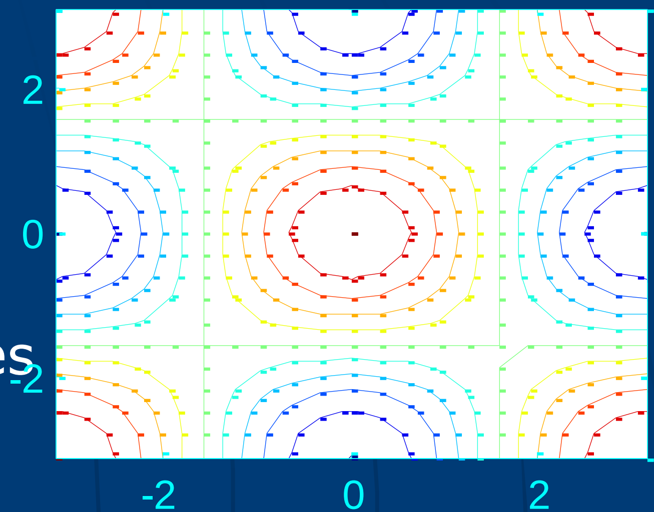
Ex: contour(x,y,z)

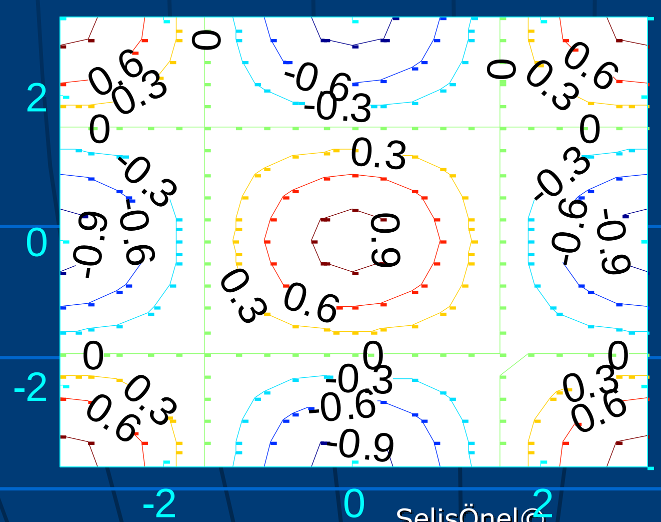
MATLAB® Plotting Command: **contour**

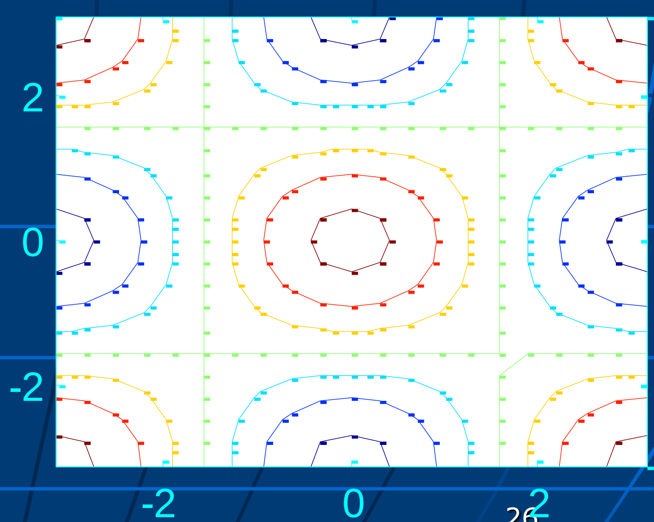
- **contour**(Z) is a contour plot of matrix Z treating the values in Z as heights above a plane. A contour plot are the level curves of Z for some values V. The values V are chosen automatically
- **contour**(X,Y,Z) X and Y specify the (x,y) coordinates of the surface as for **surf**
- **contour**(Z,N) and **contour**(X,Y,Z,N) draw N contour lines, overriding the automatic value
- **contour**(Z,V) and **contour**(X,Y,Z,V) draw **length**(V) contour lines at the values specified in vector V
- Use **contour**(Z,[v v]) or **contour**(X,Y,Z,[v v]) to compute a single contour at the level v.

2.5-D Graphs: Surface

```
figure(5)
subplot(2,2,2), contour(x,y,z)
% Show only the levels required
subplot(2,2,4), contour(x,y,z,[-0.9:.3:.9])
% Show the level of values on the contour lines
subplot(2,2,3),
[c,h]=contour(x,y,z,[-0.9:.3:.9])
clabel(c,h)
```







Other Plot Commands

waterfall(...): Same as mesh(...) except that the column lines of the mesh are not drawn - thus producing a "waterfall" plot. For column-oriented data analysis, use waterfall(z') or waterfall(x',y',z')

ribbon: Draws 2-D lines as ribbons in 3-D

ribbon(x,y) is the same as plot(x,y) except that the columns of

y are plotted as separated ribbons in 3-D. ribbon(y) uses the

default value of x=1:size(y,1).

ribbon(x,y,width) specifies the width of the ribbons to be width. The default value is width = 0.75;

Other Plot Commands

spy : Zero/nonzero values, i.e., it visualizes sparsity pattern.

spy(S) plots the sparsity pattern of the matrix S

spy(S,'LineSpec') uses the color and marker from the line

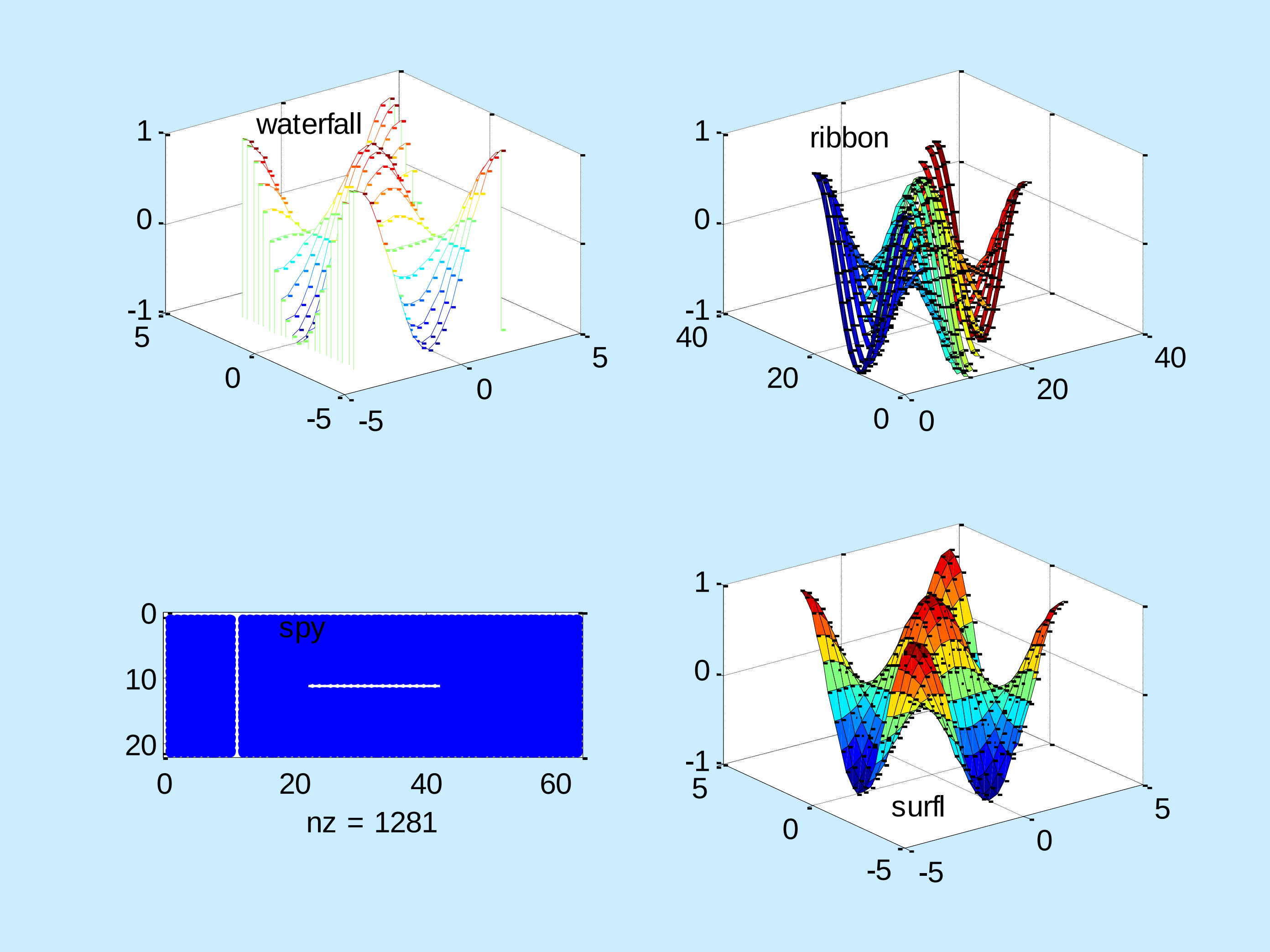
specification string 'LineSpec'

surfl : 3-D shaded surface plot with light effects

same as surf(...) except that it draws the surface with highlights from a light source

Other Plot Commands

```
figure(6)
subplot(2,2,1)
waterfall(x,y,z),
text(-5,-5,-
    1,'waterfall');
subplot(2,2,2)
ribbon(z);
text(0,0,-1,'ribbon');
subplot(2,2,3)
S=[x y z]; spy(S);
gtext('spy');
subplot(2,2,4)
surfl(x,y,z,'light');
text(-5,-5,-1,'surfl');
```



Data Export and Import

A=magic(3); B=magic(4);

% save all variables in MATLAB® workspace

save f1

% to save only certain variables

save f2 A

%These files have .mat extension and can only be retrieved by MATLAB®

% to load use:

load f1

% To save data as text

save f3 B –ascii

Data Export and Import

fprintf : Writes formatted data to file

count = fprintf(fid, format, A, ...)
formats the data in the real part of matrix A (and in any additional matrix arguments) under control of the specified format string, and writes it to the file associated with file identifier fid

fid = fopen(filename, mode)

```
>> x = 0:.1:1; y = [x; exp(x)];
 fid = fopen('exp.txt','wt');
 fprintf(fid,'%6.2f  %12.8f \n',y);
 fclose(fid);
```

```
>>   type exp.txt
  0.00    1.00000000
  0.10    1.10517092
  0.20    1.22140276
  0.30    1.34985881
  0.40    1.49182470
  0.50    1.64872127
  0.60    1.82211880
  0.70    2.01375271
  0.80    2.22554093
  0.90    2.45960311
  1.00    2.71828183
```

Data Export and Import

fread : Reads binary data from file.

A = fread(FID) reads binary data from the specified file and writes it into matrix A. FID is an integer file identifier obtained from fopen.

A = fread(FID,SIZE,PRECISION) reads the file according to the data format specified by the string PRECISION. Valid entries for SIZE are:

N read N elements into a column vector.

inf read to the end of the file.

[M,N] read elements to fill an M-by-N matrix, in column order. N can be inf, but M can't.

Successive Substitution Iteration

- Nonlinear system of equations usually can be written in the same form as linear equations → $Ax=y$
- Then, the coefficient matrix **A** and the inhomogeneous term **y** may be dependent on the solution
- So, an iterative solution for a nonlinear system can be written as:

$A_{n-1}x_n=y_{n-1}$

Successive Substitution Iteration

$A_{n-1}x_n=y_{n-1}$ can be solved using successive substitution iteration, where

A_{n-1} : is computed using the most recent calculation result for x_n

x_n : is the nth iterative solution

y_{n-1} : is an inhomogeneous term assumed to be a function of x_n

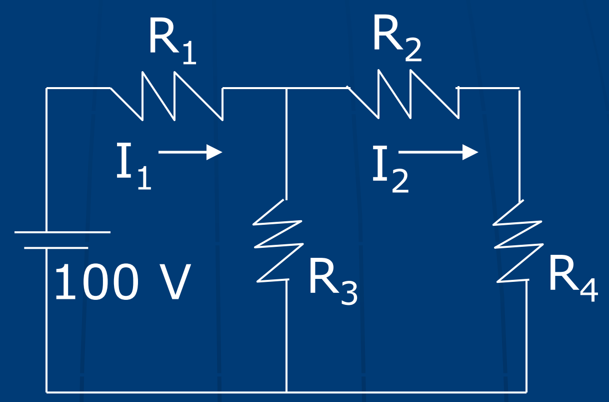
Successive Substitution Iteration

1. Start the iteration with an initial guess for the solution x
2. Determine the coefficient matrix
3. Solve the system as a linear system
4. Get the solution x
5. Rearrange the coefficient matrix
6. Solve the system again
7. In case of instability:
8. Add the (under)relaxation parameter, i.e.

$x_n=\omega inv(A_{n-1})y_{n-1}+(1-\omega)x_{n-1}$ where $0<\omega<1$

Ex: Successive Substitution Iteration

Electric circuit between heating elements can be shown schematically as:



The resistance of the j[th] heating element is a function of temperature:

$$R_j = a_j + b_j T_j + c_j T_j^2$$,where

a_j, b_j, c_j : constants

T_j :temperature of the j[th] element

Temperature of each heating element is determined by:

$$I_j^2 R_j = A_j \sigma (T_j^4 - T_\infty^4) + A_j h (T_j - T_\infty)$$, where

T_∞ : temperature of the surrounding environment

A_j : the surface area of the j[th] element

Ex: Successive Substitution Iteration

Electric currents (I_i) should satisfy:

$$(R_1 + R_3)I_1 - R_3 I_2 = 100$$

$$-R_3 I_1 + (R_2 + R_4 + R_3) I_2 = 0$$

These equations are in fact nonlinear as

$I=f(T)$ from $R_j=a_j+b_jT_j+c_jT_j^2$ and $T=f(R,I)$

If T is low, nonlinear effects vanish and system becomes linear

Otherwise, we need to solve the nonlinear system…

Ex: Successive Substitution Iteration

Solution Algorithm:

First Solve

$$(R_1 + R_3)I_1 - R_3 I_2 = 100$$

$$-R_3 I_1 + (R_2 + R_4 + R_3) I_2 = 0$$

As a simultaneous linear system with initial (cold) values of resistances

Solve

$$I_j^2 R_j = A_j \sigma (T_j^4 - T_\infty^4) + A_j h (T_j - T_\infty)$$

for temperature

Calculate each resistance as a function of T

$$R_j = a_j + b_j T_j + c_j T_j^2$$

Convergence

$|R_k - R_{k-1}| < tol$

Resolve with updated values of resistances

$$(R_1 + R_3)I_1 - R_3 I_2 = 100$$

$$-R_3 I_1 + (R_2 + R_4 + R_3) I_2 = 0$$

END

Quiz

Find the roots of the following nonlinear system of equations using substitutive iteration.

Start with initial guess values of [3/4 2/3]

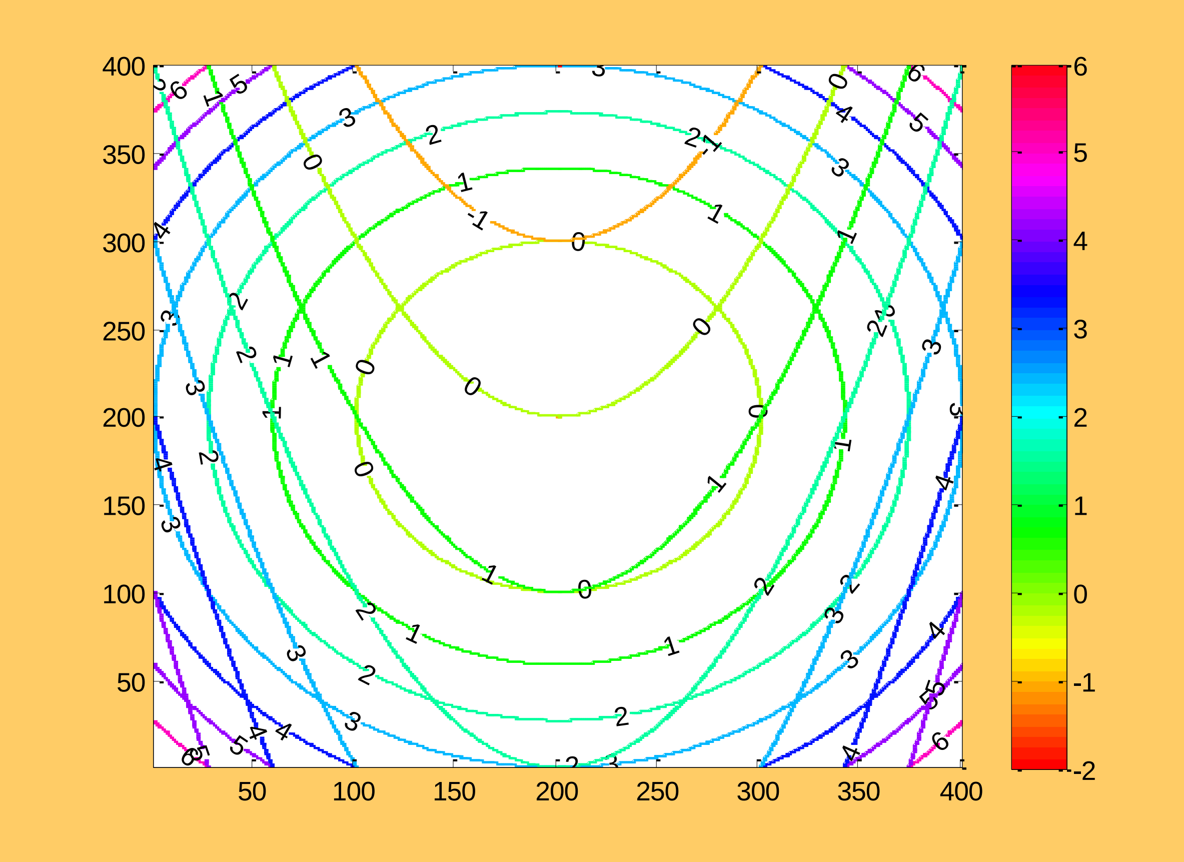
Show three consecutive iterations and check convergence for a tolerance of 0.0001

$f(x,y)=x^2+y^2-1$

$g(x,y)=x^2-y$

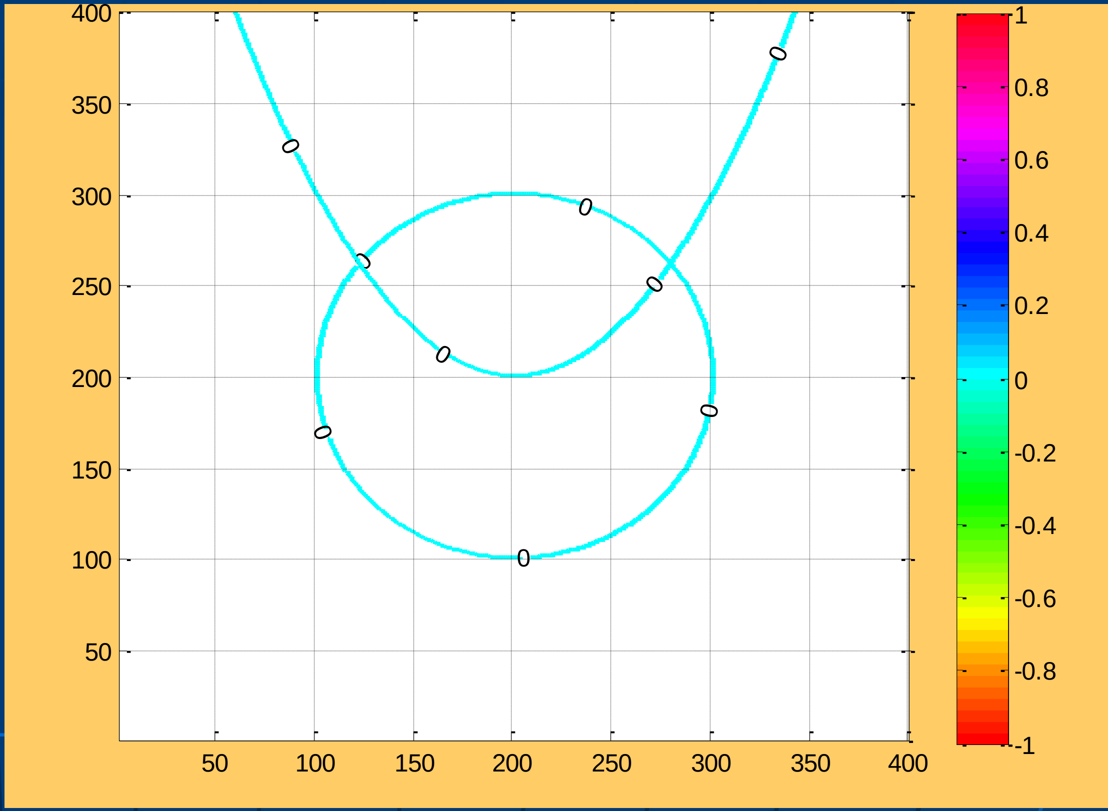
2.5-D Graph in MATLAB®

```
>> x1=-2:0.01:2;
>> x2=-2:0.01:2;
>> [x,y]=meshgrid(x1,x2);
>> f1=x.^2+y.^2-1 ;
>> f2=x.^2-y;
>> [c,h] = contour(f1);
   clabel(c,h),
   colormap hsv, hold on,
>> [c,h] = contour(f2);
   clabel(c,h),
   colorbar, hold off
```



2.5-D Graph in MATLAB®

```
>> x1=-2:0.01:2;
>> x2=-2:0.01:2;
>> [x,y]=meshgrid(x1,x2);
>> f1=x.^2+y.^2-1 ;
>> f2=x.^2-y;
>>  [c,h] = contour(f1,[.0000
   .0000],'linewidth',2);
   clabel(c,h),
   colormap hsv,
   hold on,
>> [c,h] = contour(f2,[.0000
   .0000],'linewidth',2);
   clabel(c,h),
   colorbar,
   hold off,
   grid on,
```



Ex: Plotting two functions in MATLAB®

```
clear, clf, hold off
x1=0:0.1:2;   y1=-2:0.1:2;   [x,y]=meshgrid(x1,y1);   [f1,f2]=funnonlin(x,y);
figure(1)
subplot(1,2,1)
mesh(f1,'linewidth',2),  hold on,   mesh(f2,'linewidth',2),
axis([min(x1) max(x1) min(y1) max(y1)  -10 10]);
xlabel('x'); ylabel('y'); zlabel('z');  grid on;    hold off,
subplot(1,2,2)
[c,h]=contour(x,y,f1,'-r','linewidth',2); clabel(c,h);
hold on
[c,h]=contour(x,y,f2,'linewidth',2); clabel(c,h);
hold off
axis([min(x1) max(x1) min(y1) max(y1)]);  xlabel('x'); ylabel('y'); grid on;  legend('f1','f2‘,2)

x2=0:0.1:20; y2=-2:0.1:20;  [x,y]=meshgrid(x2,y2);   [f1,f2]=funnonlin(x,y);
figure(2)
subplot(1,2,1)
mesh(f1),   hold on,    mesh(f2),  axis([min(x2) max(x2) min(y2) max(y2)  -10 10]);
xlabel('x'); ylabel('y'); zlabel('z');   grid on;    hold off,
subplot(1,2,2)
[c,h]=contour(x,y,f1,'-r','linewidth',2); clabel(c,h);
hold on
[c,h]=contour(x,y,f2,'linewidth',2); clabel(c,h);
hold off
axis([min(x1) max(x1) min(y1) max(y1)]); xlabel('x'); ylabel('y'); grid on;  legend('f1','f2')
```

Plotting two functions in MATLAB®