BCA 603 Mathematics and Physics for Game Engines

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Lecture #1

- Systems of Units

- Addition and Subtraction of Vectors
- Time Derivatives of Vectors
- Position, Velocity, and Acceleration

Systems of Units

To describe the physical world, we need to measure things. We want to know how heavy something is or how fast it is traveling. In order to measure things, a system of units is required. It would be nice if there were one universal system of units.

The International System of Units, or **SI** units, includes three fundamental units of measure for mass, length, and time. The unit of length in the **SI** system is the meter, the unit of mass is the kilogram, and the unit of time is the second. The three quantities are usually abbreviated as **m**, **kg**, and **s**. Most other physical quantities, density, pressure, force, and so on, can be expressed in terms of meters, kilograms, and seconds. The unit of measure for temperature in the SI system is Kelvin (**K**).

Systems of Units

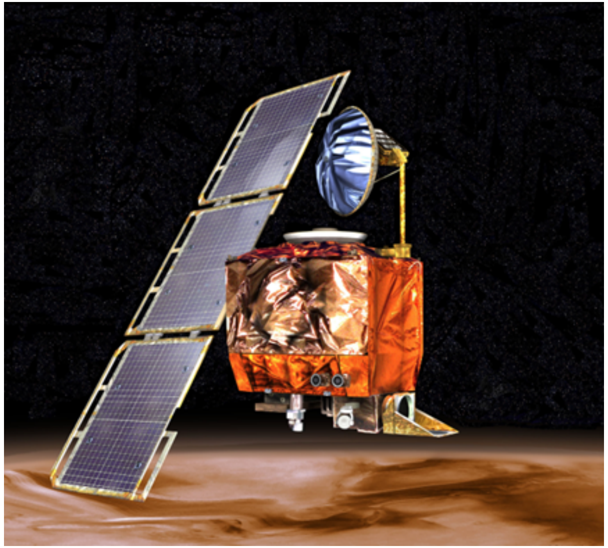
An older of system of units known as the English System of Units (Imperial) is still widely used in some countries. The unit of mass under the English system is the pound-mass (**lbm**) or the slug. The unit of length is the foot (**ft**). One consistent element between the English and SI systems is that time in both systems is measured in seconds. The English system is still widely used in the United States and England. Most of the rest of the world uses SI units. Temperature in the English system is expressed in terms of degrees Fahrenheit (F).

Systems of Units

Quantity	English Units	SI Units	Conversion Factor
Length	foot (*ft*)	meter (*m*)	0.3048
	mile	kilometer (*km*)	1.609
Mass	pound-mass (*lbm*)	kilogram (*kg*)	0.4536
	slug	kilogram (*kg*)	14.593
Force	pound (*lb*)	Newton (*N*)	4.448
Pressure	lb/in^2	N/m^2	6894.7
Density	$slug/ft^3$	kg/m^3	515.379
	lbm/ft^3	kg/m^3	16.018
Temperature	Fahrenheit (^oF)	Kelvin (*K*)	5/9(F + 459.67)
	Rankine (*R*)	Kelvin (*K*)	5/9

Systems of Units

The **Mars Climate Orbiter** was a 338 kilogram robotic space probe launched by NASA on December 11, 1998 to study the Martian climate, atmosphere, and surface changes and to act as the communications relay in the Mars Surveyor '98 program for Mars Polar Lander. However, on September 23, 1999, communication with the spacecraft was lost as the spacecraft went into orbital insertion, due to ground-based computer software which produced output in **non-SI** units of **pound-seconds** instead of the metric units of **newton-seconds** specified in the contract between NASA and Lockheed.



Scientific Notation

Sometimes it is necessary to work with really large or really small numbers. For example, the gravitational constant, a quantity that relates the force two objects exert on each other, is a very small number. Written in standard decimal notation, it is equal to the following:

$$G = 0.0000000000667 \frac{N - m^2}{kg^2}$$

Fortunately, there is something known as scientific notation that can be used to express large or small numbers in a more compact form.

$$G = 6.67\ e - 11 \frac{N - m^2}{kg^2}$$

Roundoff Errors

Roundoff errors arise because digital computers cannot represent some quantities exactly. They are important to engineering and scientific problem solving because they can lead to erroneous results. In certain cases, they can actually lead to a calculation going unstable and yielding obviously erroneous results. Such calculations are said to be ill-conditioned. Worse still, they can lead to subtler discrepancies that are difficult to detect.

Numerical roundoff errors are directly related to the manner in which numbers are stored in a computer. The fundamental unit whereby information is represented is called a word. This is an entity that consists of a string of binary digits, or bits. Numbers are typically stored in one or more words.

Roundoff Errors

Integer Representation : For example, the integer value of 173 is represented in binary as 10101101:

$$(10101101)_2 = 2^7 + 2^5 + 2^3 + 2^2 + 2^0 = 128 + 32 + 8 + 4 + 1 = (173)_{10}$$

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Floating-Point Representation : By default, MATLAB has adopted the IEEE double-precision format in which eight bytes (64 bits) are used to represent floating-point numbers.

Roundoff Errors

```
def sumdemo():
    s = 0
    for _ in range(10000):
        s = s + 0.0001
    return s

print(sumdemo())
```

When this function is executed, the result is

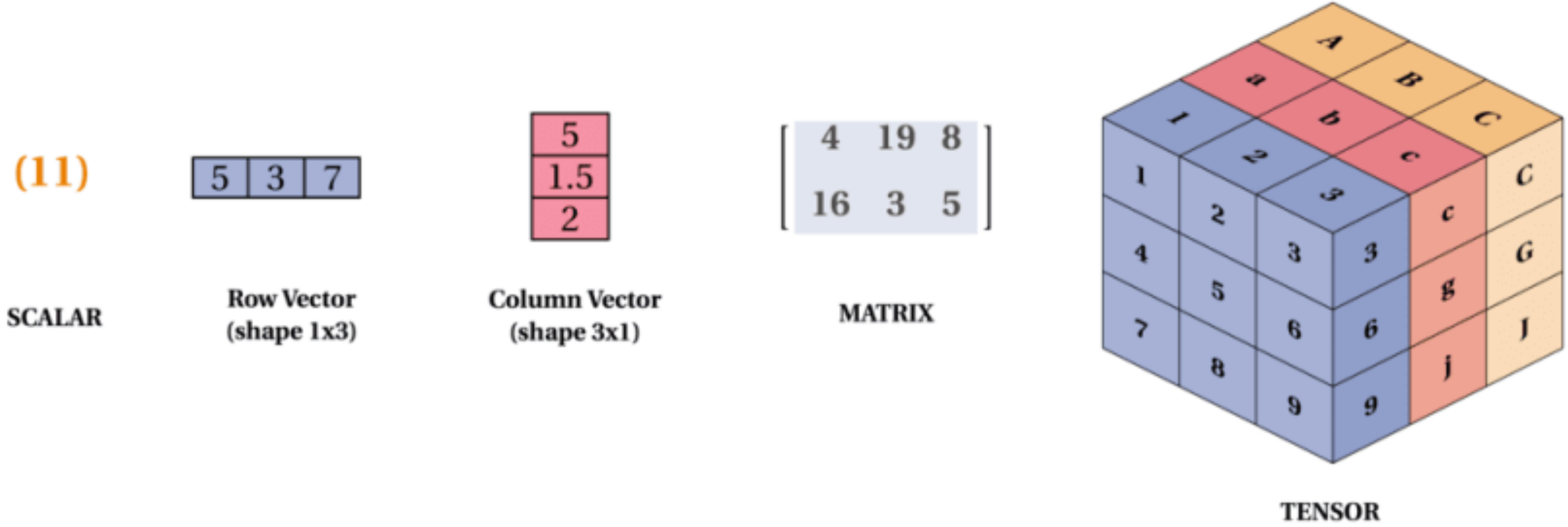
```
>>> sumdemo()

0.9999999999999062
```

Roundoff Errors

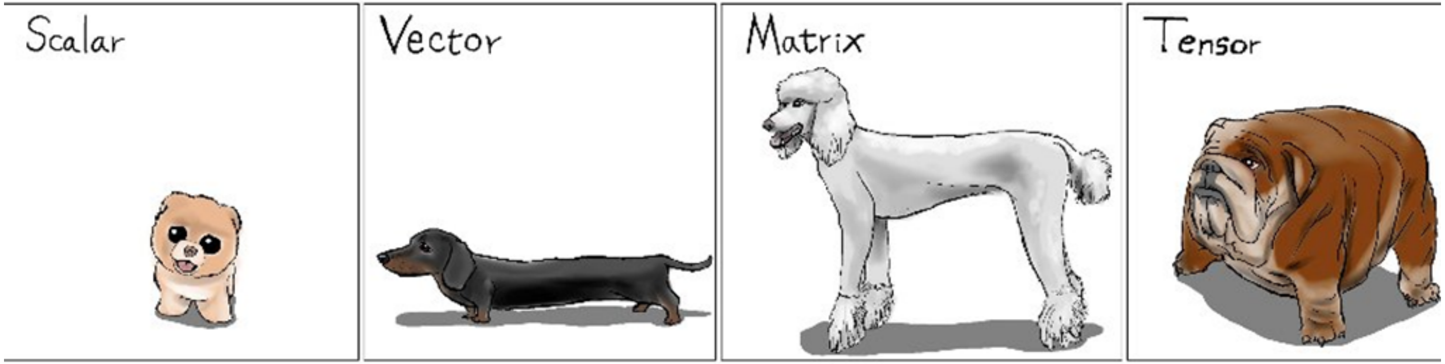
```
>>> 1.0023 - 1.0567
  -0.05440000000000004
>>> 1000.0023 - 1000.0567
  -0.05439999999999869
```

Scalar, Vector, Matrix and Tensor

In mathematics, a tensor is an algebraic object that describes a (multilinear) relationship between sets of algebraic objects related to a vector space. Objects that tensors may map between include vectors and scalars, and even other tensors.



Scalar, Vector, Matrix and Tensor



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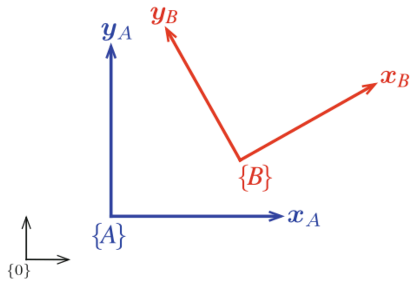


1D – 1-dimensional

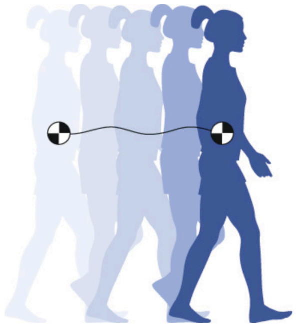
2D – 2-dimensional

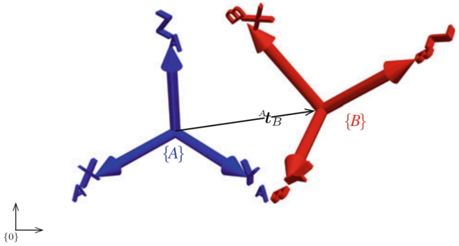
3D – 3-dimensional

CoM – Center of mass

DoF – Degrees of freedom

***n*-tuple** – A group of n numbers, it can represent a point or a vector



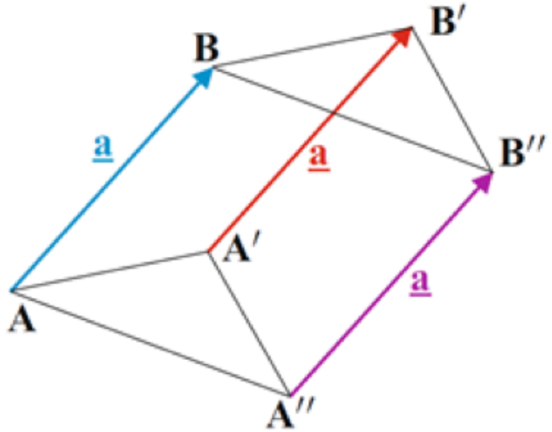


Euler (1707–1783) was a Swiss mathematician and physicist who dominated eighteenth century mathematics. He was a student of Johann Bernoulli and applied new mathematical techniques such as calculus to many problems in mechanics and optics.

Vector

There are three types of vectors, namely, **free vector**, **sliding vector** and **fixed vector**.

A **free vector** is a vector whose point of application and direction are free to move in space. In general, a free vector is a vector that can move without any constraint.



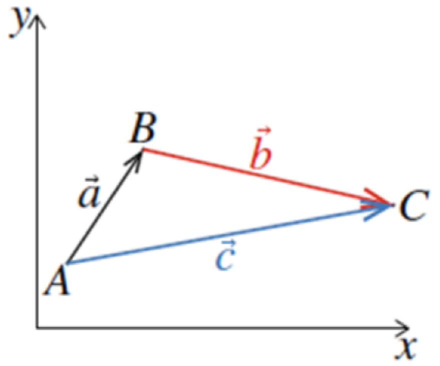
A **sliding vector** is a vector whose point of application is not fixed but can slide along its line of action.

A bound **vector** is a **fixed vector**.

Vector Addition

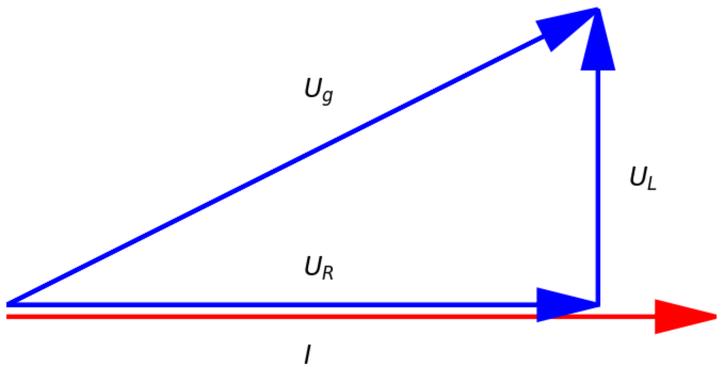
Vector addition is intuitive for the addition of displacements: If you first move along the vector $\vec{a}$ from **A** to **B**, and then along the vector b from B to C, the net displacement is the vector:

$$\vec{c} = \vec{a} + \vec{b}$$



from point **A** to **C**. This geometric definition of vector addition is general, and we use it also for vectors that are not displacements: We find the sum of two vectors **a** and **b** geometrically by placing the tail of vector **b** at the tip of vector **a**. The sum is called the **<u>resultant vector</u>**.

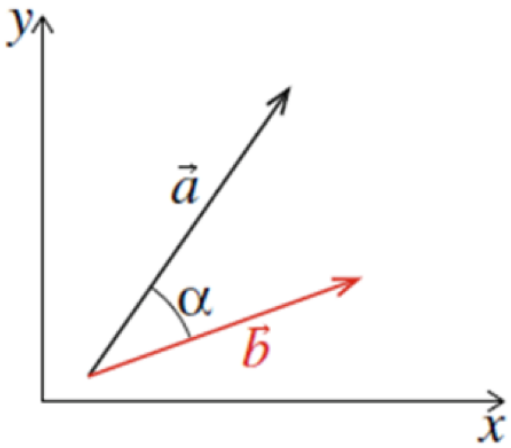
```python
import matplotlib.pyplot as plt

x1, x2 = 0, 12
y1, y2 = 0, 8
lb = 2        # line width
pb = 0.5      # arrow width
pl = 1        # arrow length
U_R = 10      # ohmic voltage drop
U_L = 5       # inductive voltage drop
I = 12        # current

fig, ax = plt.subplots()
ax.axis([x1,x2,y1,y2])

# Arrows: x, y, x+dx, y+dy
ax.arrow(0, 1.8, I, 0, color = 'r', lw = lb, length_includes_head = True, head_width = pb, head_length = pl)
ax.arrow(0, 2, U_R, 0, color = 'b', lw = lb, length_includes_head = True, head_width = pb, head_length = pl)
ax.arrow(U_R, 2, 0, U_L, color = 'b', lw = lb, length_includes_head = True, head_width = pb, head_length = pl)
ax.arrow(0, 2, U_R, U_L, color = 'b', lw = lb, length_includes_head = True, head_width = pb, head_length = pl)

#Labels
ax.annotate("$I$", xy = (5, 1), xytext = (5, 1), fontsize = 12)
ax.annotate("$U_g$", xy = (5, 5), xytext = (5, 5.5), fontsize = 12)
ax.annotate("$U_L$", xy = (10.5, 4), xytext = (10.5, 4), fontsize = 12)
ax.annotate("$U_R$", xy = (5, 3), xytext = (5, 2.5), fontsize = 12)

ax.set_xticks([])
ax.set_yticks([])
ax.set_frame_on(False)
ax.set_aspect('equal')
plt.show()
```



The Dot Product

The dot product between two vectors A and B is defined as:

$$\vec{A} \,.\, \vec{B} = \left|A\right|\left|B\right| cos\ \alpha$$



where α is the angle between the two vectors,

The dot product is linear:

$$\left(\vec{A} + \vec{B}\right) . \vec{C} = \vec{A} \,.\, \vec{C} + \vec{B} \,.\, \vec{C}$$

And commutative:

$$\vec{A} \,.\, \vec{B} = \vec{B} \,.\, \vec{A}$$

Homework #1

Solve Problems 1.1 to 1.8

1.13 Problems

1.1 The force **F** shown in the Fig. 1.14 has the vector components $\mathbf{F}_x$, $\mathbf{F}_y$, and $\mathbf{F}_z$ withe the magnitudes F_x, F_y, and F_z respectively. Find the direction angles θ_x, θ_y and θ_z made by the vectorial force **F** with the positive x, y, and z axes. For the numerical application use $F_x = 140$ units, $F_y = 170$ units, and $F_z = 190$ units.

1.2 The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied as shown in the Fig. 1.15. The force $\mathbf{F}_2$ has the magnitude F_2 and makes the angle β with the horizontal axis and the force $\mathbf{F}_1$ has the magnitude F_1. The angle between the segment AB and the force F_1 is φ and the angle between BA and the horizontal axis is denoted by θ. Determine

Homework Assignment: Due NO LATER than Wednesday 7th March 2025

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Subject : BCA603 HW1