Two-Dimensional Kinematics

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Biyomekanik Araştırma Grubu

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- **Two-Dimensional Kinematics**
- Three-Dimensional Kinematics
- Body Segment Parameters
- Three-Dimensional Kinetics
- Muscle Modelling
- Computer Simulation of Human Movement
- Musculoskeletal Modelling

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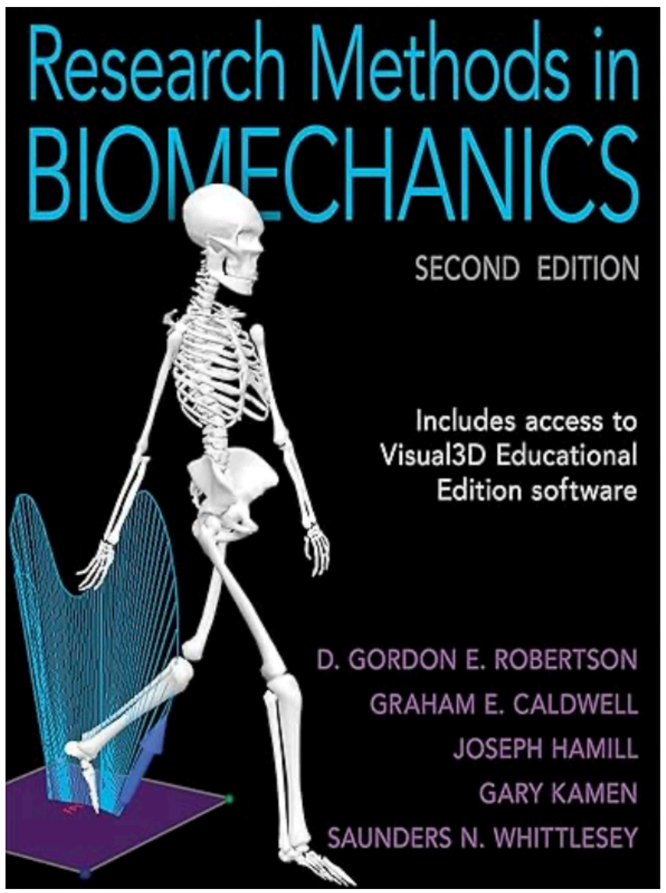
Textbooks recommended for the course



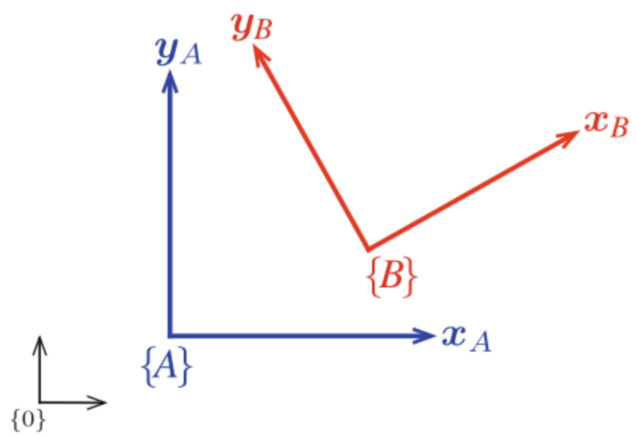
- **Publisher : Human Kinetics; Second edition (November 1, 2013)**
- **Language : English**
- **Hardcover : 440 pages**
- **ISBN-10 : 0736093400**
- **ISBN-13 : 978-0736093408**

Chapter 1

Planar Kinematics

D. Gordon E. Robertson and Graham E. Caldwell

Kinematics is the study of bodies in motion without regard to the causes of the motion. It is concerned with describing and quantifying both the linear and angular positions of bodies and their time derivatives.



1D – 1-dimensional

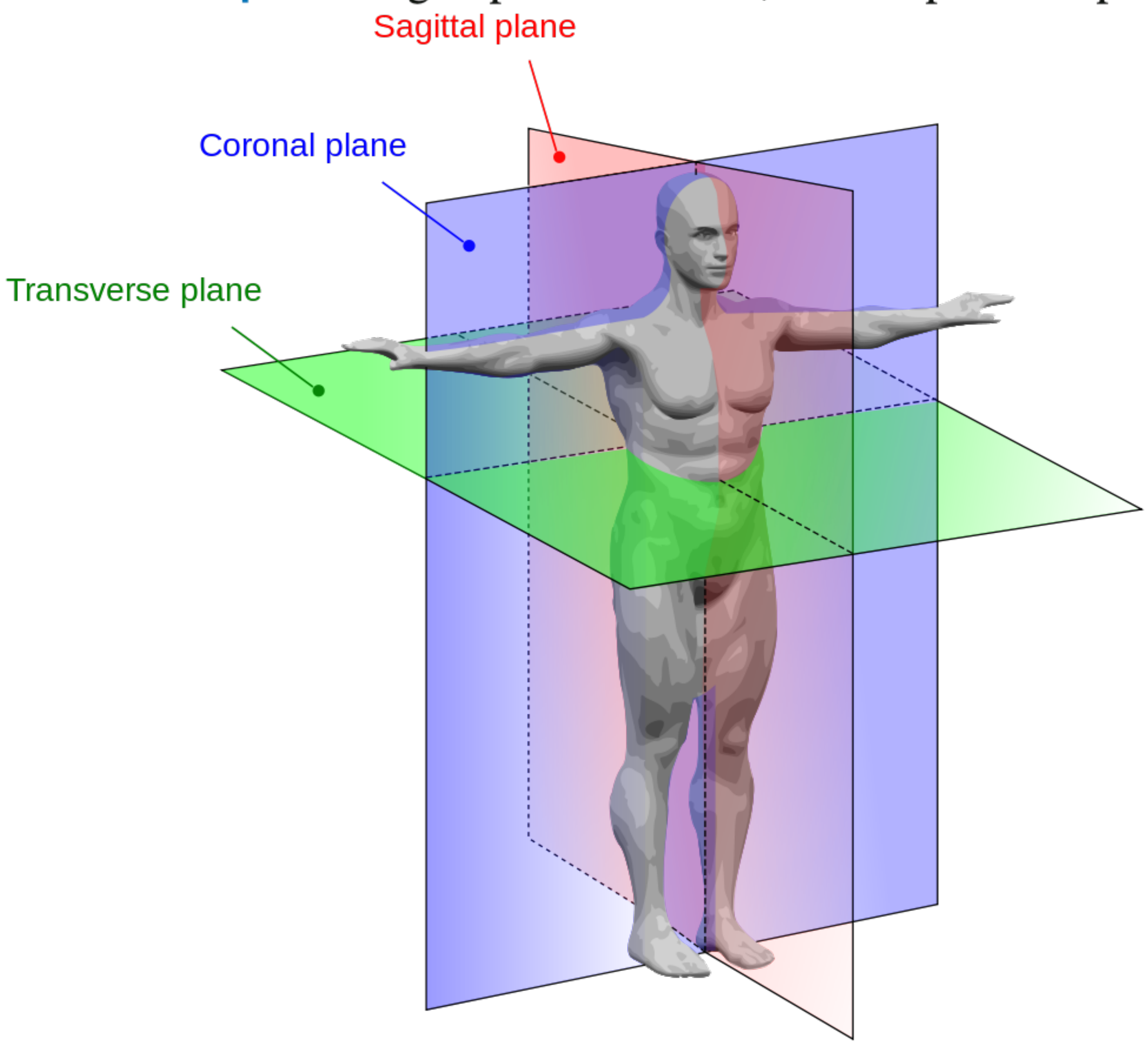
2D – 2-dimensional

3D – 3-dimensional

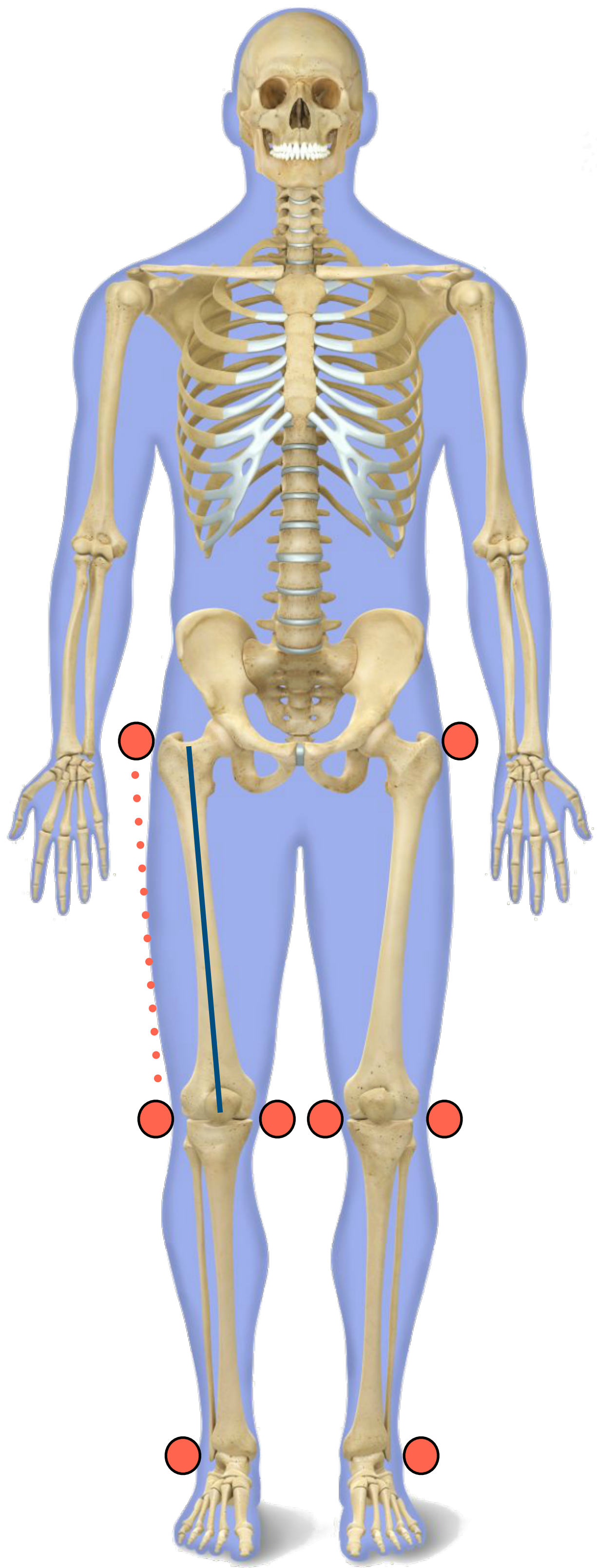
CoM – Center of mass

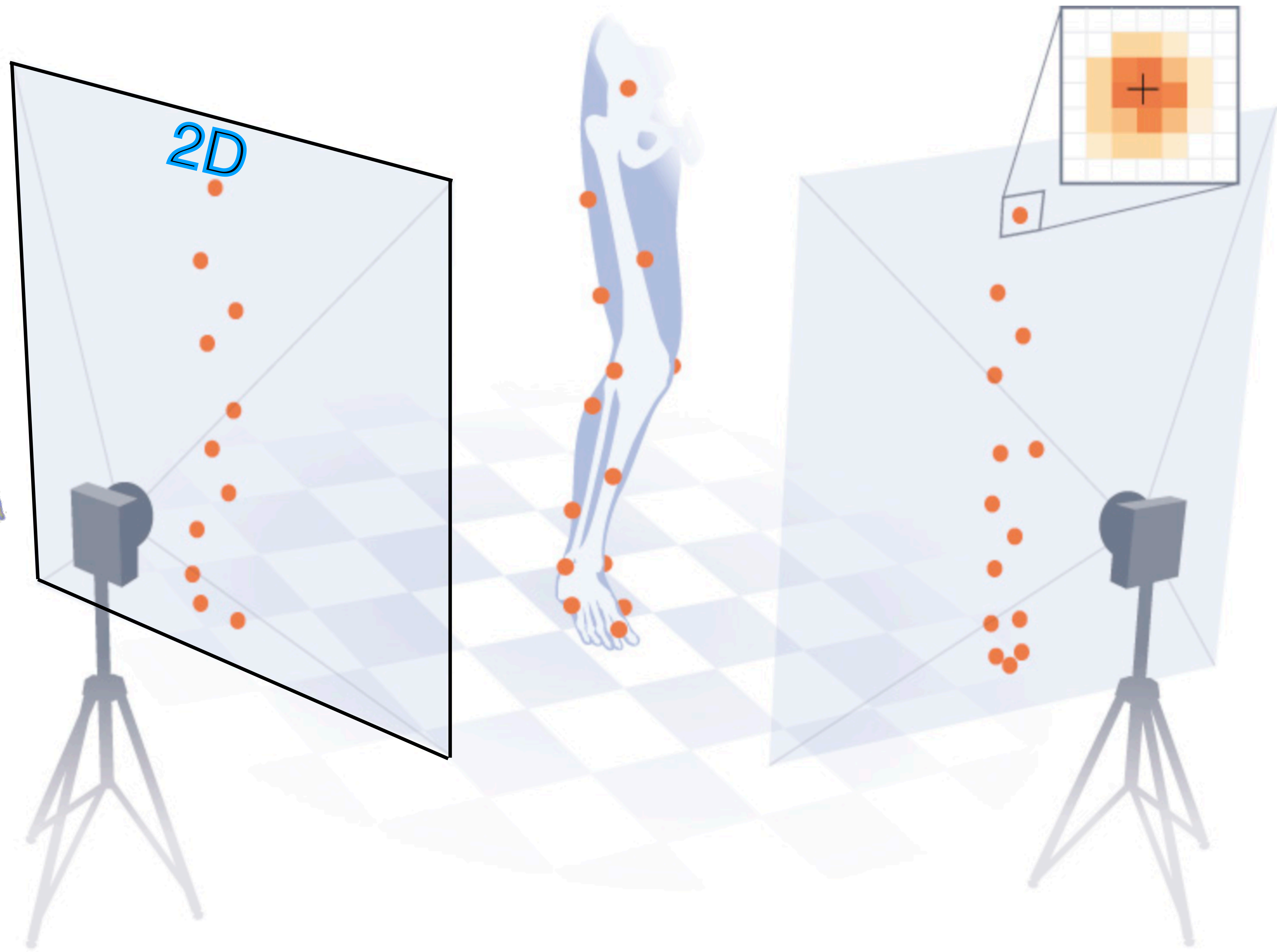
DoF – Degrees of freedom

***n*-tuple** – A group of n numbers, it can represent a point or a vector



Motion Capture





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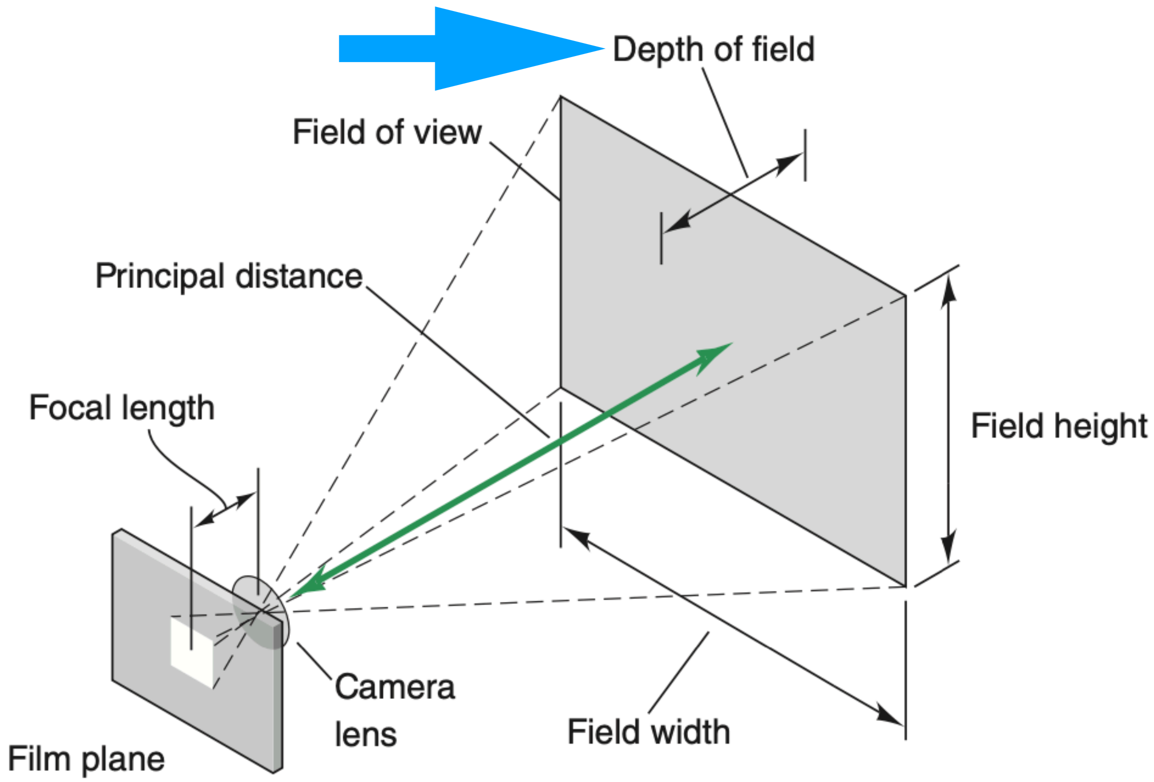

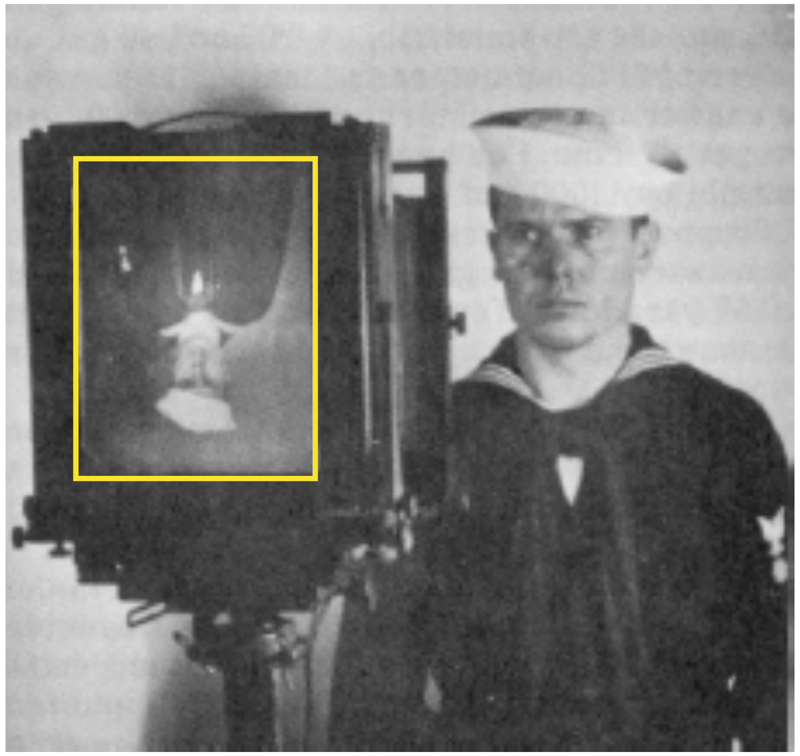

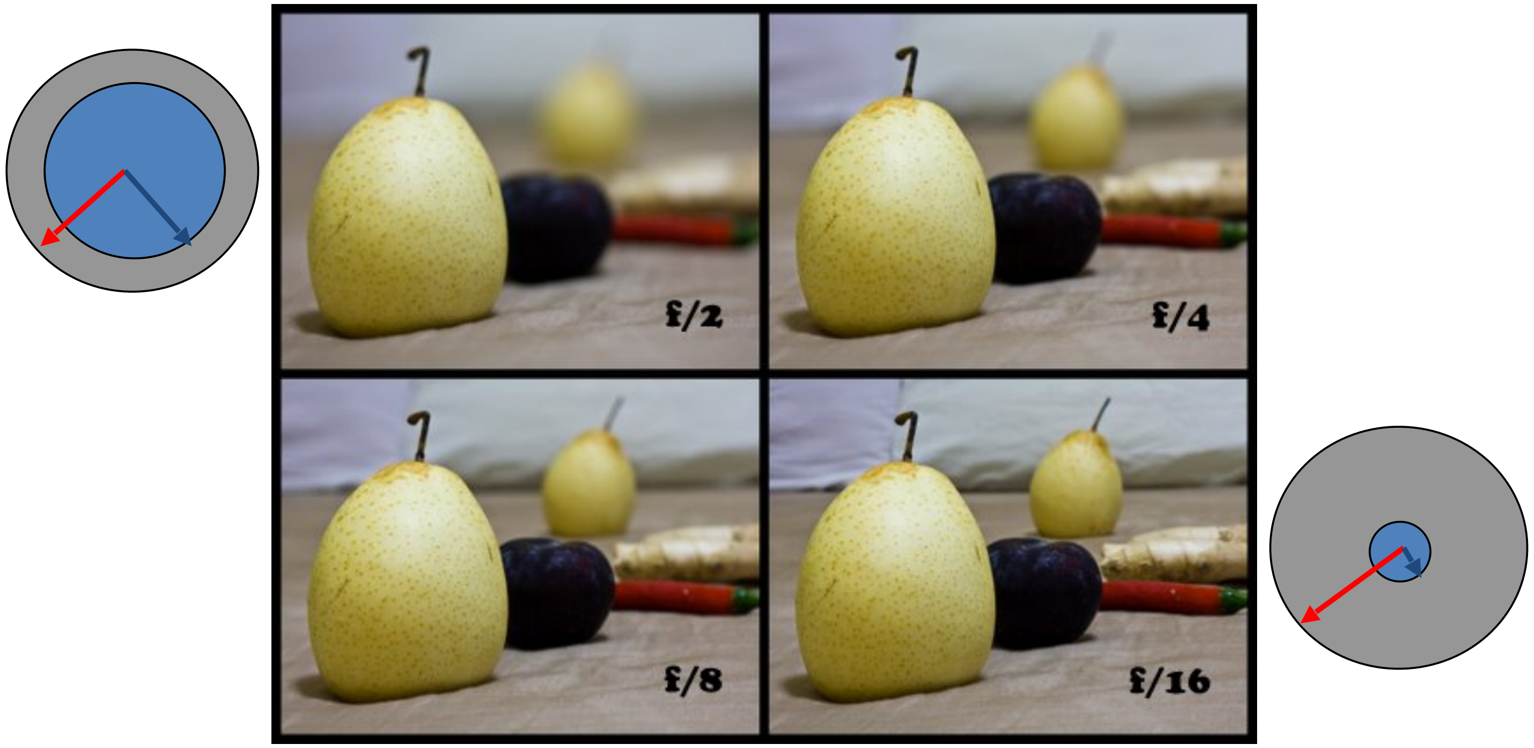

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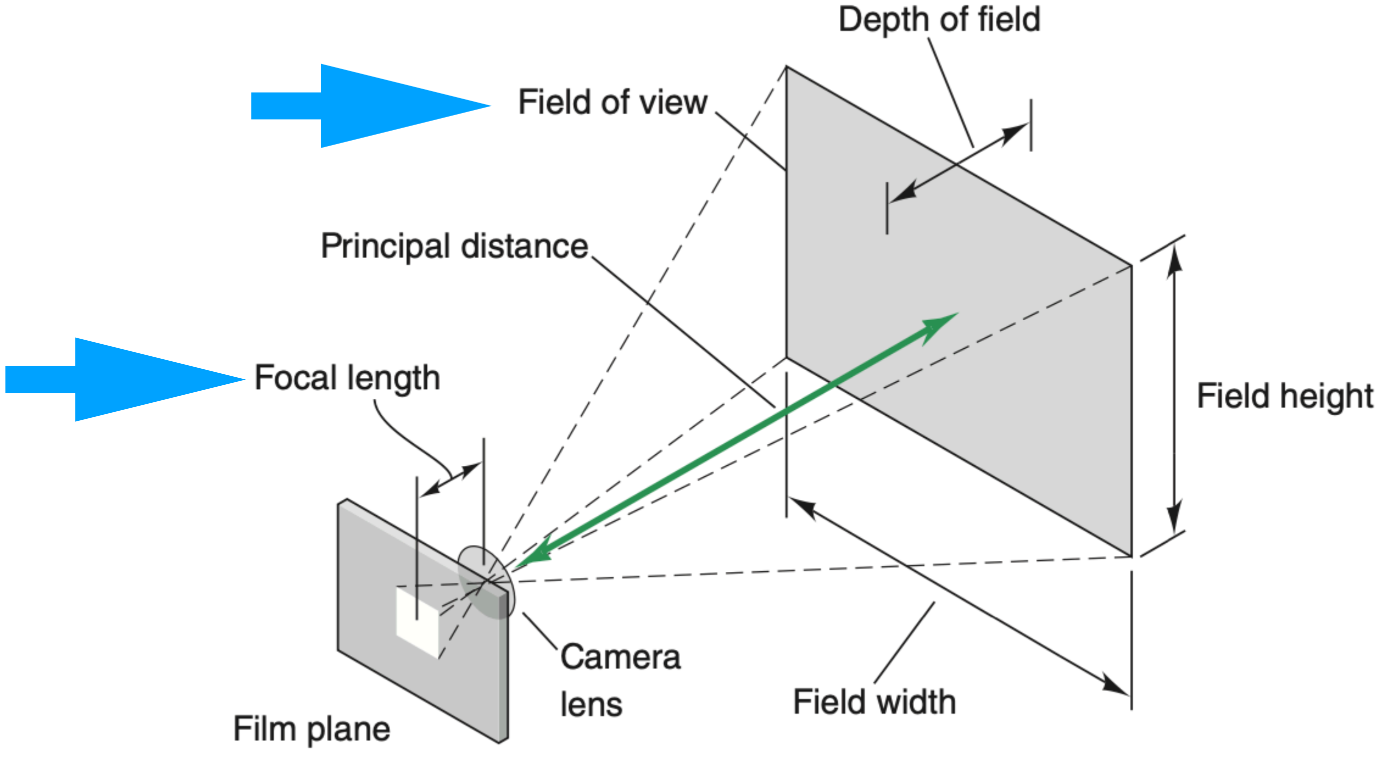

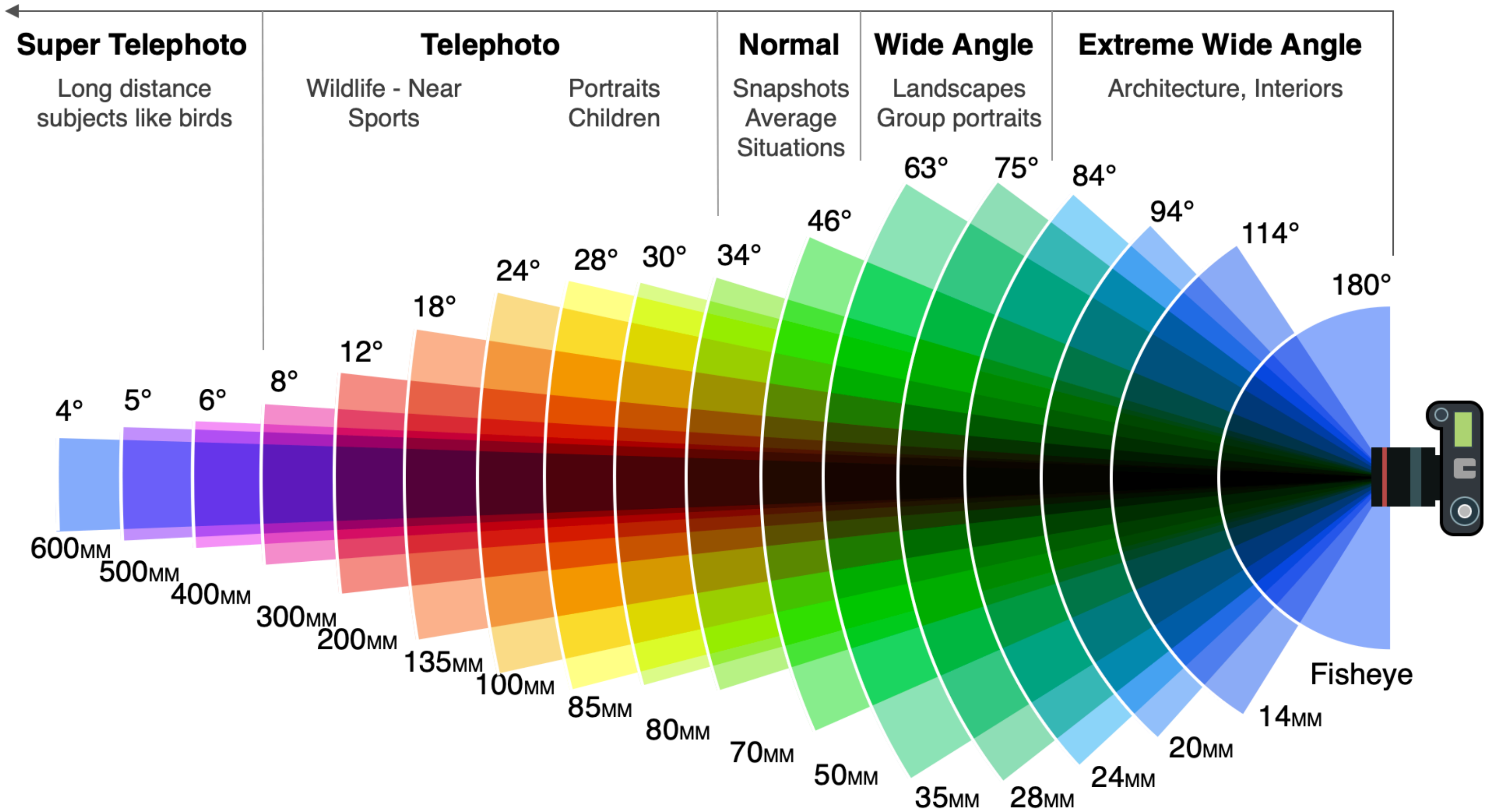

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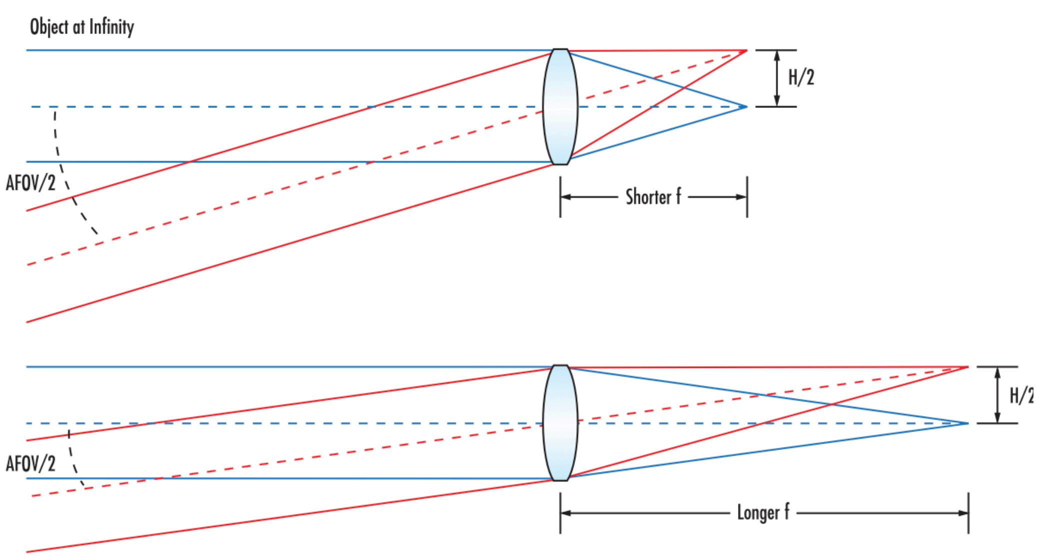
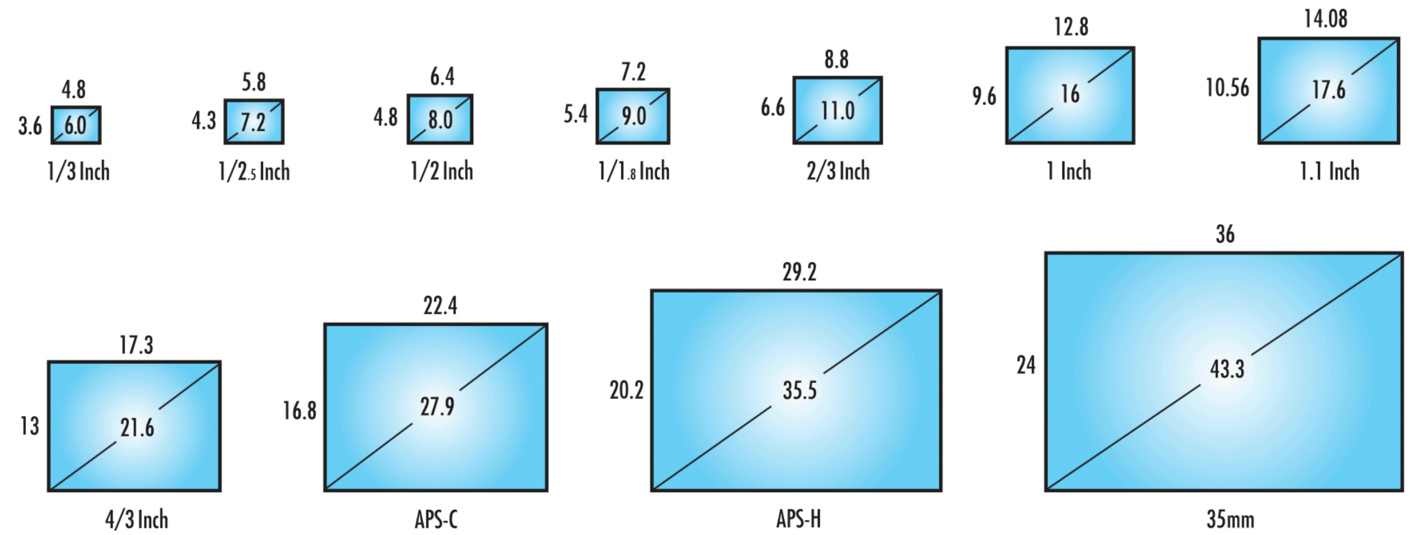
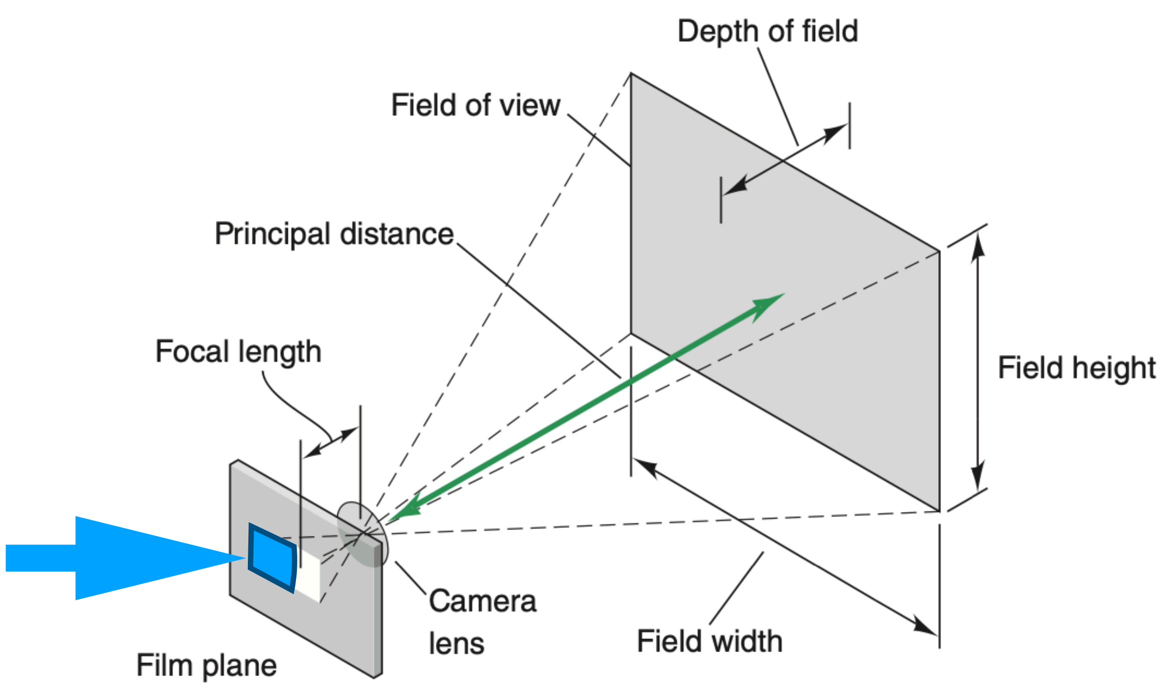
The **larger the sensor**, the **larger the obtainable Angular Field Of View** (AFOV) for the same focal length. For example, a 25mm lens could be used with a ½” (6.4mm horizontal) sensor or a 35mm lens could be used with a 2/3” (8.8mm horizontal) sensor as they would both approximately produce a 14.5° AFOV on their respective sensors.

$$\mathrm{AFOV} = 2 \times \tan^{-1}\left(\frac{H}{2f}\right)$$





CROP FRAME	9.3MM	18MM	55MM	200MM	300MM
FULL FRAME	14MM	27MM	83MM	300MM	450MM



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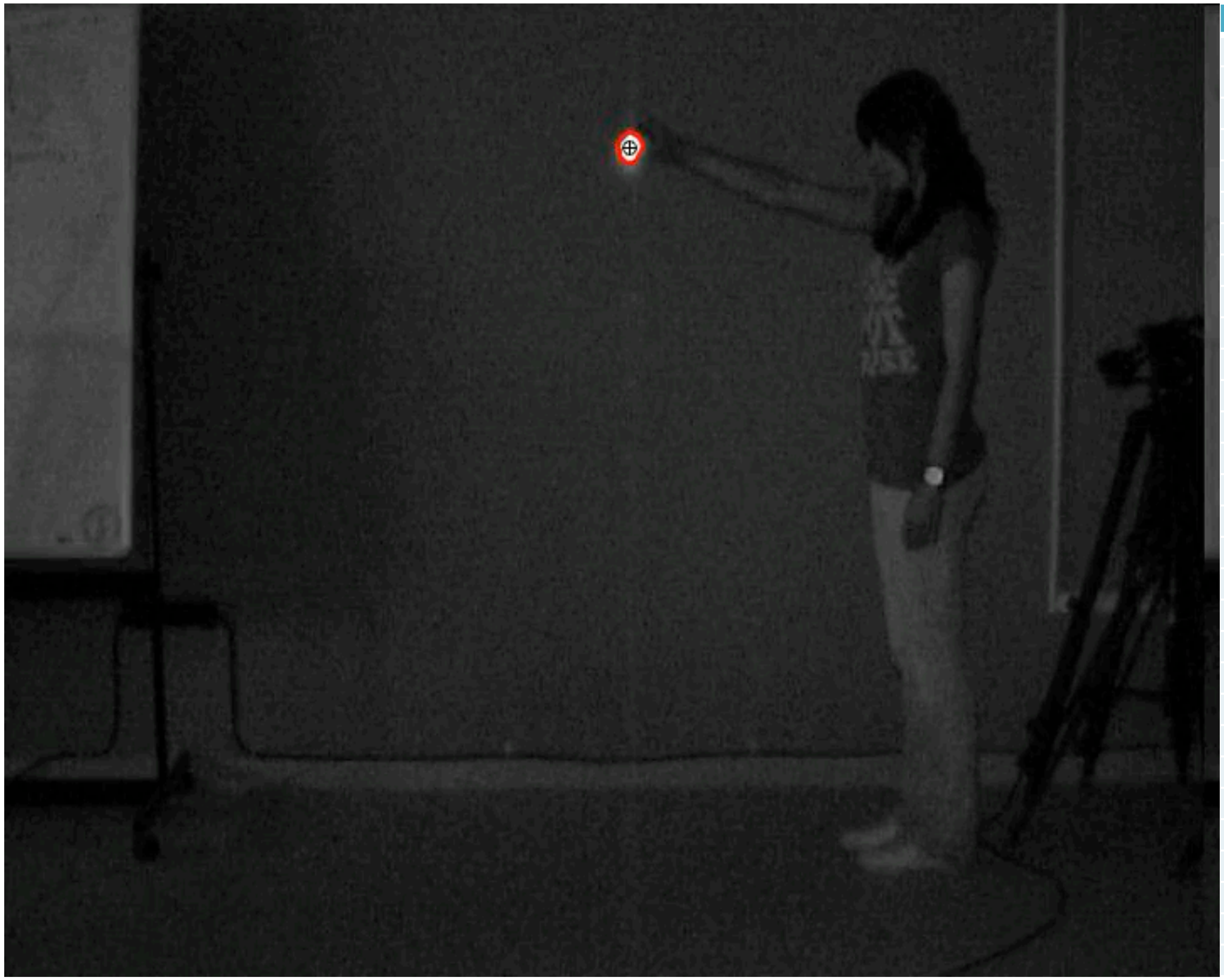
Two-Dimensional Kinematics



X	Y
56.77	159.06
56.74	159.10
56.70	158.85
56.64	158.37
56.53	157.37
56.47	156.11
56.38	154.33
56.30	152.21
56.21	149.65
56.12	146.74
56.03	143.33
55.94	139.58
55.87	135.39
55.80	130.86
55.72	125.80
55.63	120.48
55.56	114.61
55.50	108.47
55.41	101.82
55.38	94.85
55.27	87.41
55.17	79.67
55.07	71.47
55.00	63.06
54.92	54.11
54.79	44.90
54.73	35.26
54.69	25.35
54.61	15.05
54.58	4.53

Two-Dimensional Kinematics

The equations presented here to obtain velocity and acceleration values are examples of ***difference calculus***. The method used is the ***central difference*** method.

	Y
1	159.06
2	159.10
3	158.85
	158.37
	157.37
	156.11
	154.33
	152.21
	149.65
	146.74
	143.33
	139.58
	135.39
	130.86
	125.80
	120.48
	114.61
	108.47
	101.82
	94.85
	87.41
	79.67
	71.47
	63.06
	54.11
	44.90
	35.26
	25.35
	15.05
	4.53

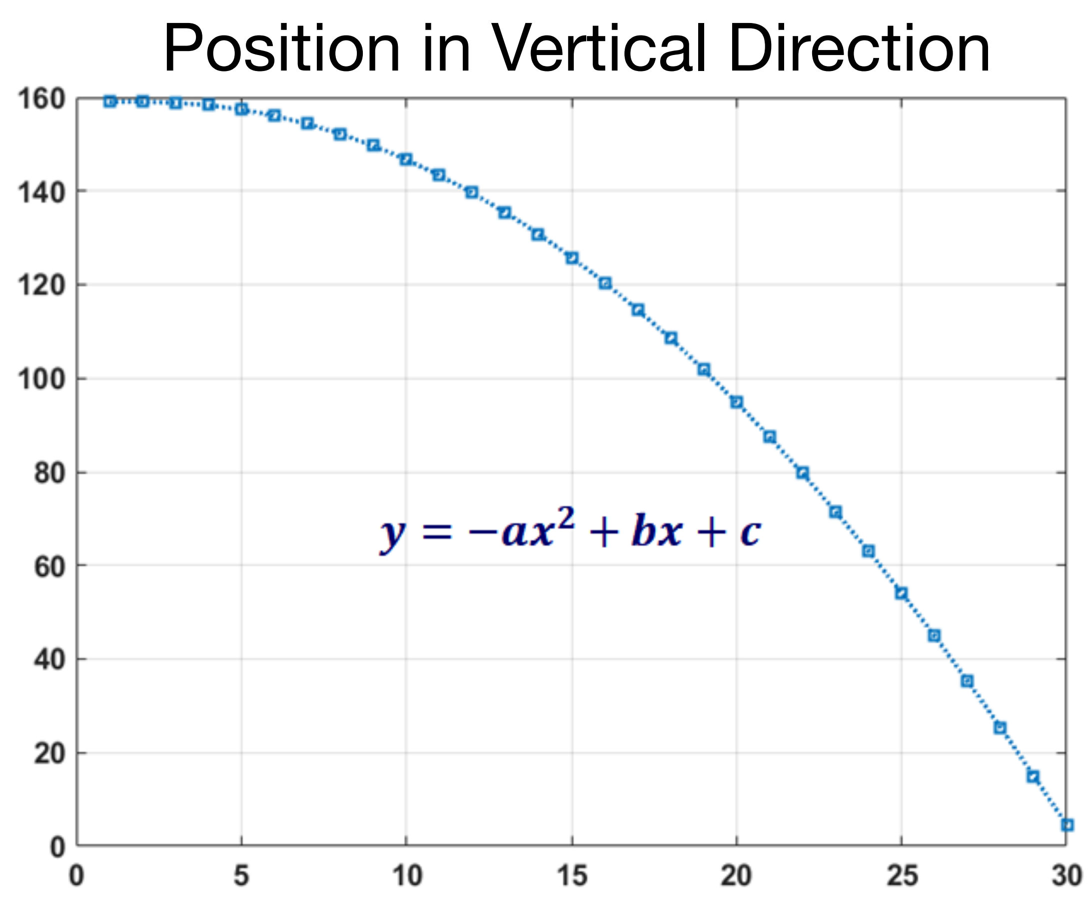
v_2

$$v_i = \frac{s_{i+1} - s_{i-1}}{2(\Delta t)}$$

$$a_i = \frac{v_{i+1} - v_{i-1}}{2(\Delta t)} = \frac{s_{i+2} - 2s_i + s_{i-2}}{4(\Delta t)^2}$$

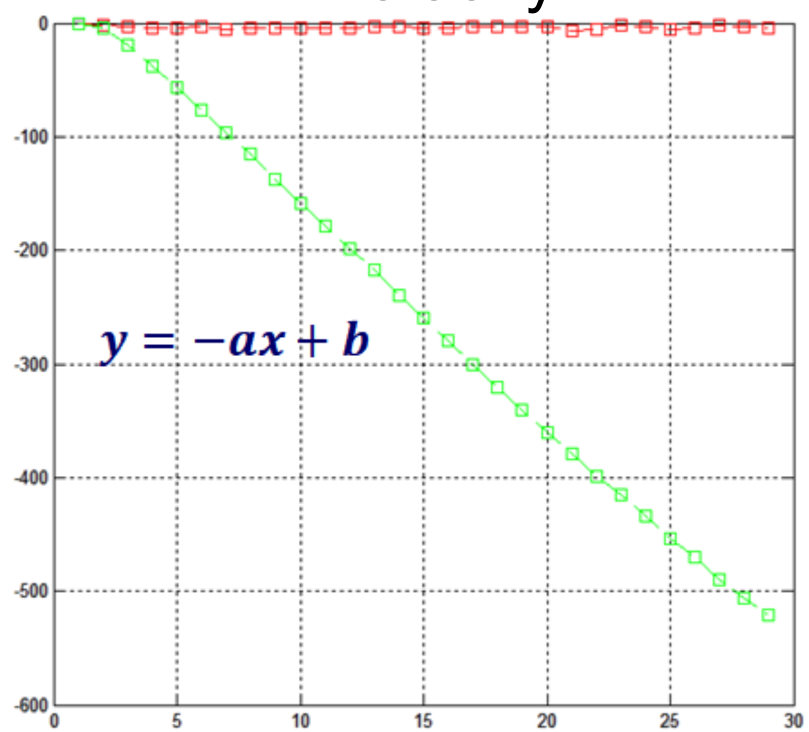
$$\text{or} \quad a_i = \frac{s_{i+1} - 2s_i + s_{i-1}}{(\Delta t)^2}$$

Two-Dimensional Kinematics

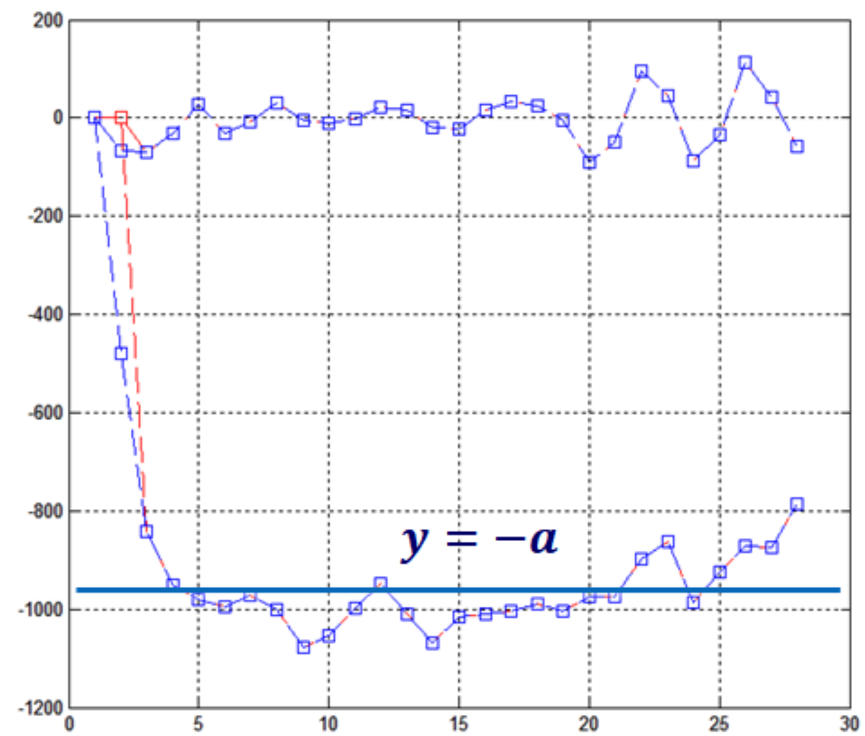


Two-Dimensional Kinematics

Velocity



Acceleration



Two-Dimensional Kinematics

Curve fitting is the process of constructing a curve, or mathematical function, that has the best fit to a series of data points, possibly subject to constraints. Curve fitting can involve either interpolation, where an exact fit to the data is required, or smoothing.

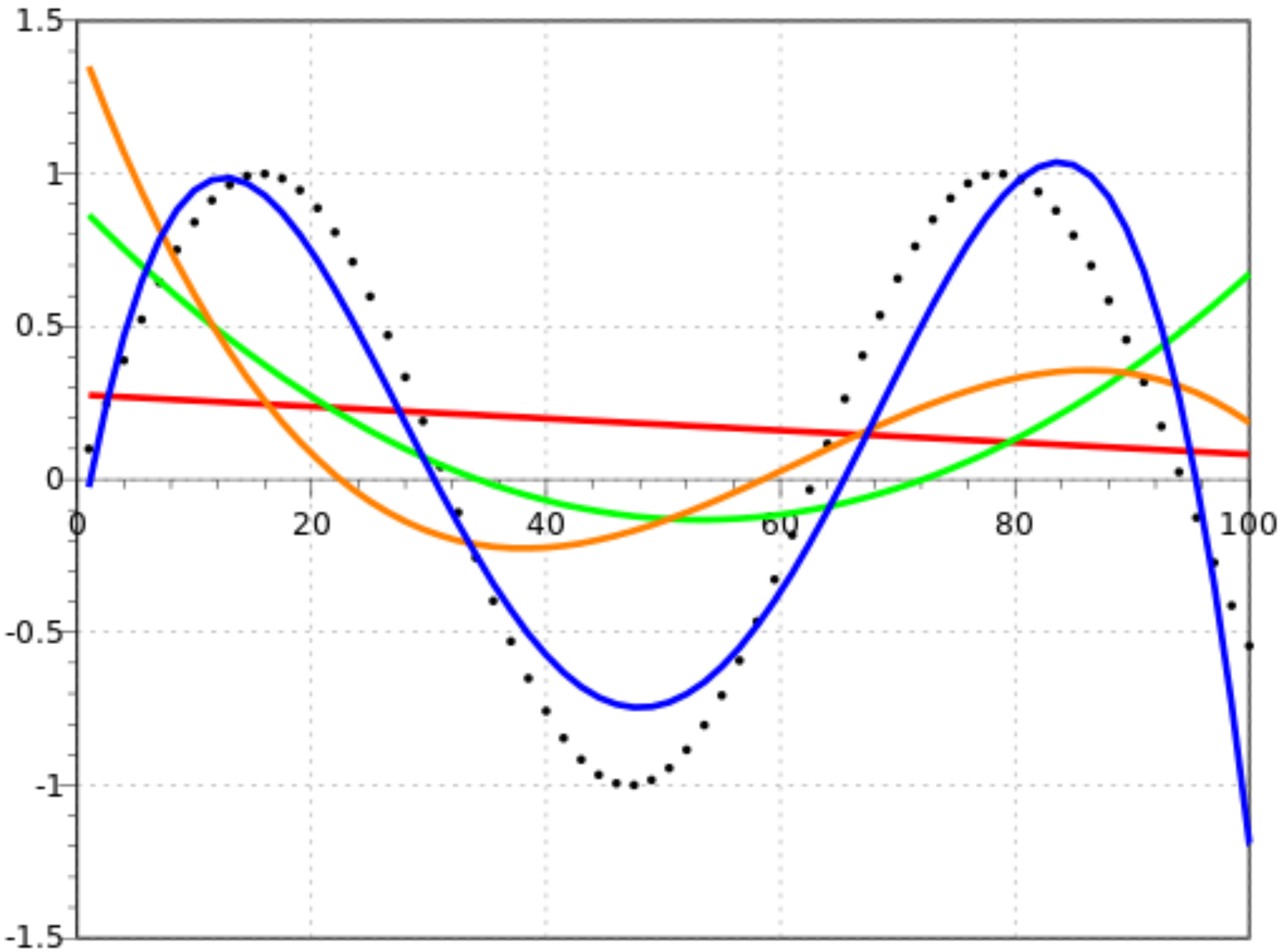
Most commonly, one fits a function of the form $y=f(x)$.

First degree polynomial equation $$y = ax + b$$

Second degree polynomial equation $$y = ax^2 + bx + c$$

Third degree polynomial equation $$y = ax^3 + bx^2 + cx + d$$

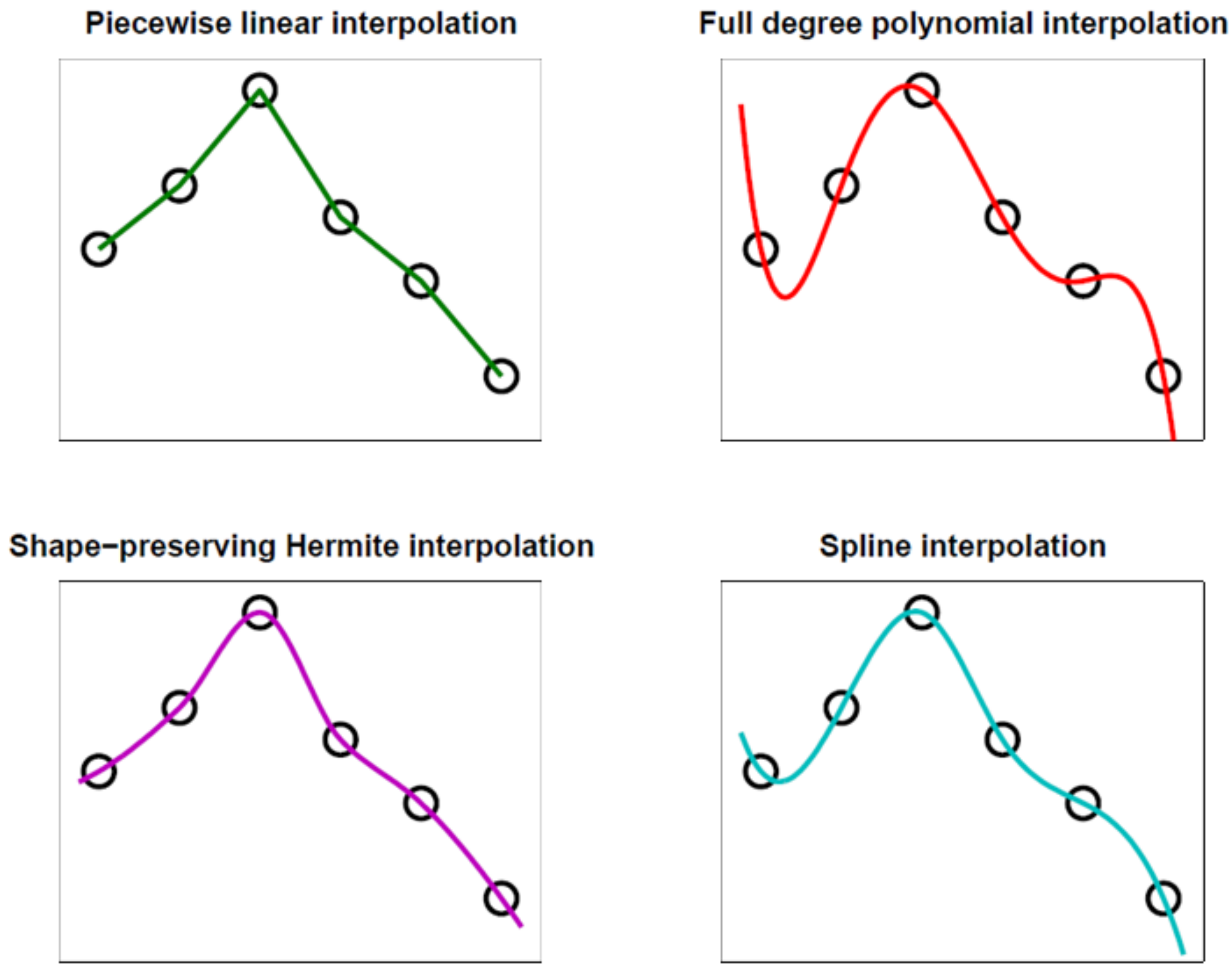
Curve fitting : Fitting lines and polynomial functions to data points



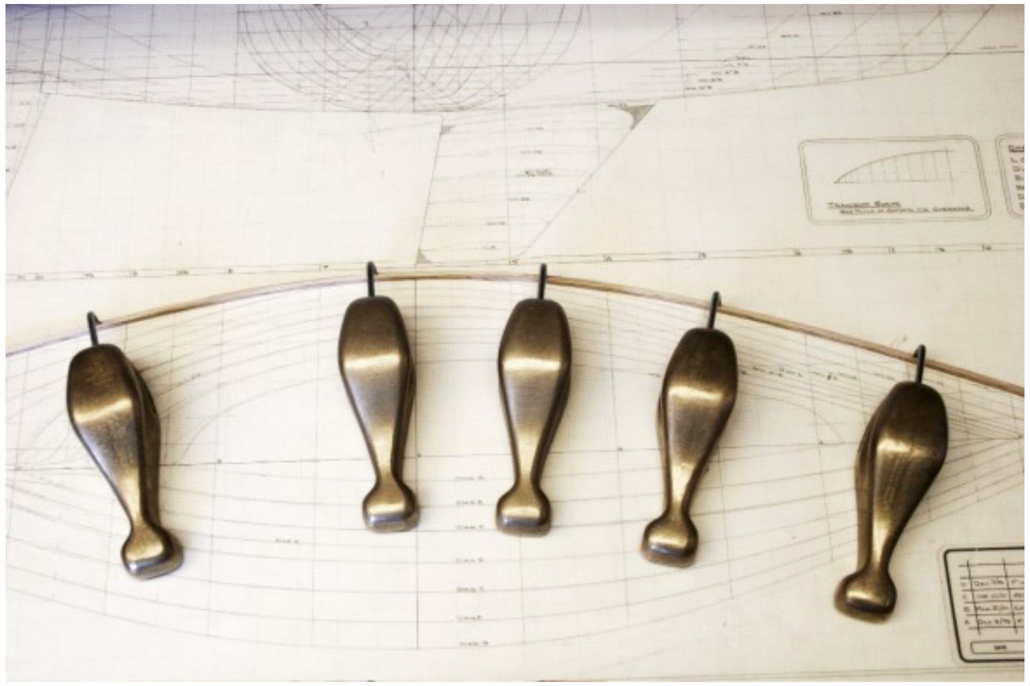
Polynomial curves fitting points generated with a sine function.

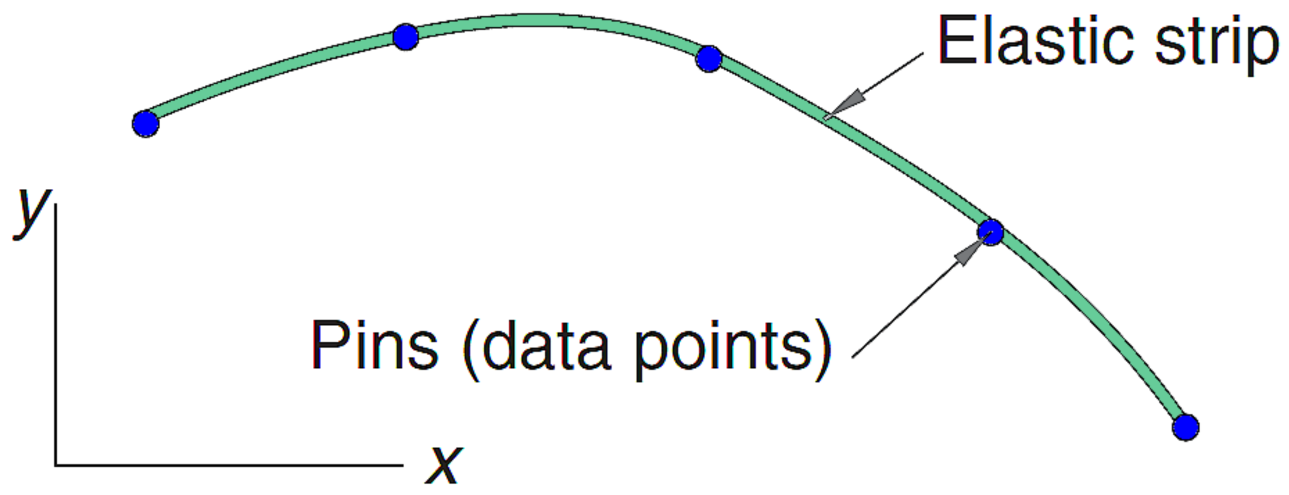
Red line is a first degree polynomial, green line is second degree, orange line is third degree and blue is fourth degree.

Curve fitting : Fitting lines and polynomial functions to data points



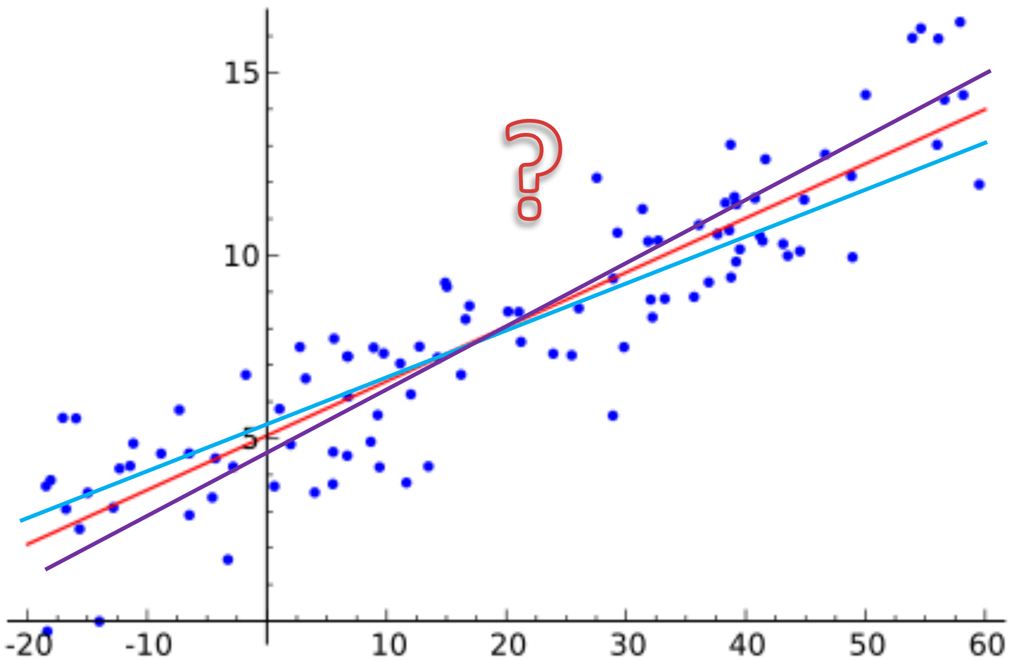
Curve fitting : Fitting lines and polynomial functions to data points





In mathematics a **spline** is a function defined piecewise by polynomials. In interpolating problems, spline is often preferred to polynomial because it yields similar results, even when using low degree polynomials.

Goodness of fit: Coefficient of determination



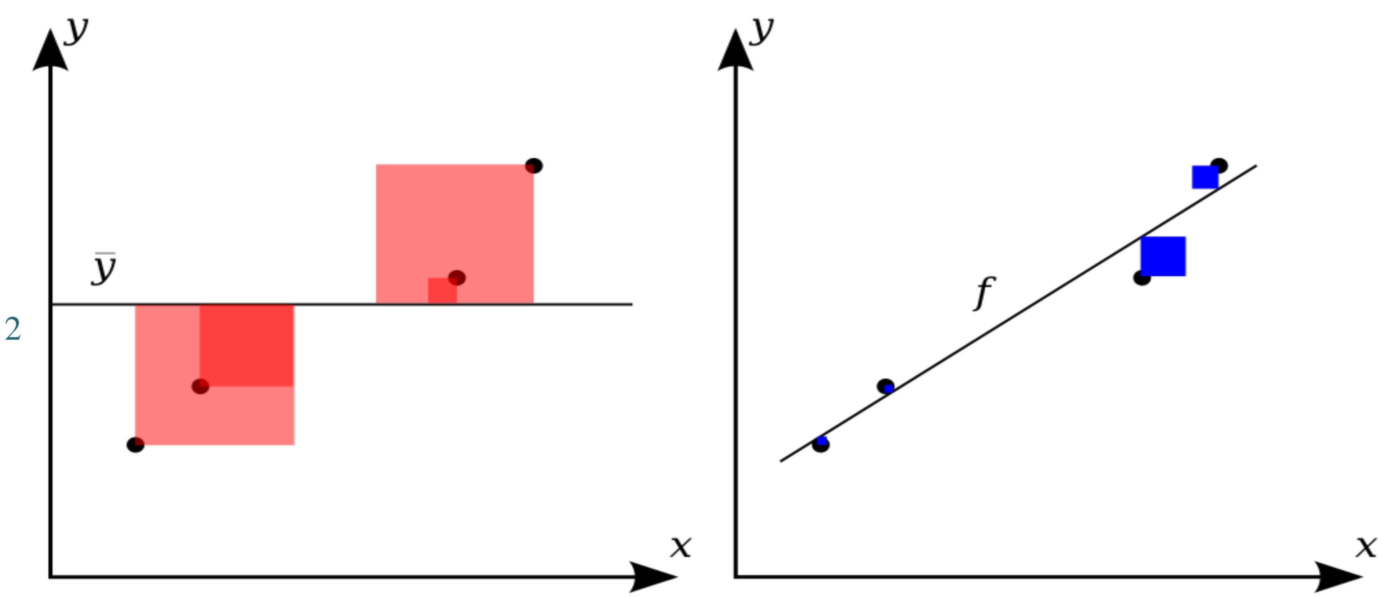
The goodness of fit of a statistical model describes how well it fits a set of observations. Measures of goodness of fit typically summarize the discrepancy between observed values and the values expected under the model in question.

The coefficient of determination, denoted R^2 or r^2 and pronounced ***R squared***, is a number that indicates how well data fit a statistical model.

Goodness of fit: Coefficient of determination

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

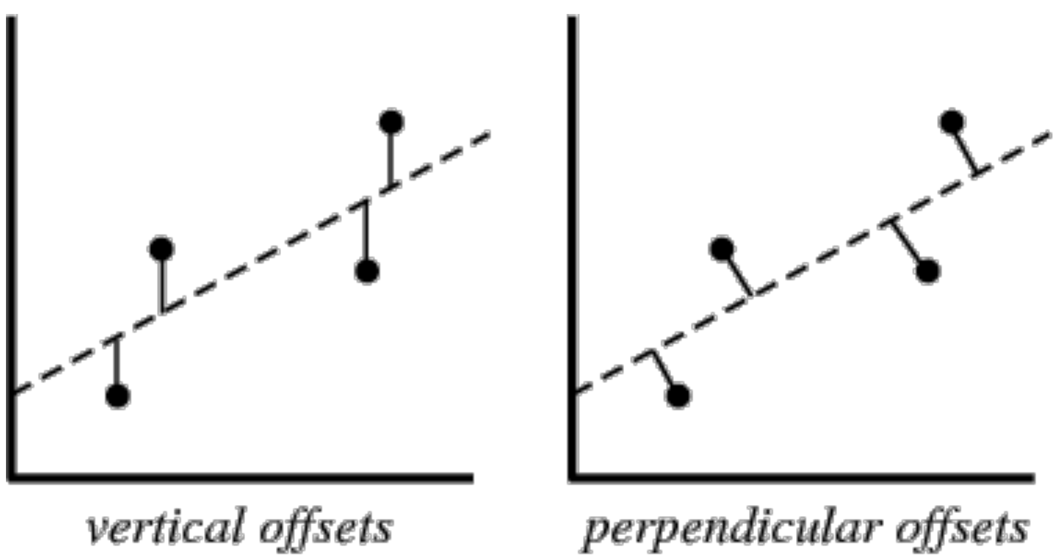
$$SS_{tot} = \sum_i (y_i - \bar{y})^2$$

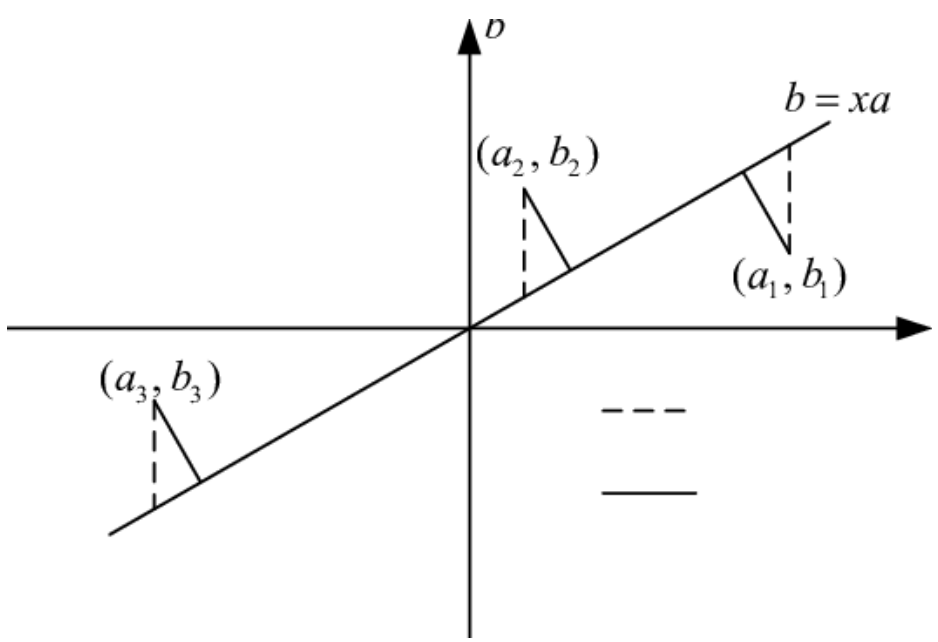


$$SS_{res} = \sum_i (y_i - f_i)^2$$

The better the linear regression (on the right) fits the data in comparison to the simple average (on the left), the closer the value of R^2 is to 1. The areas of the blue squares represent the squared residuals with respect to the linear regression. The areas of the red squares represent the squared residuals with respect to the average value.

Ordinary Least Squares (OLS) versus Total Least Squares (TLS)





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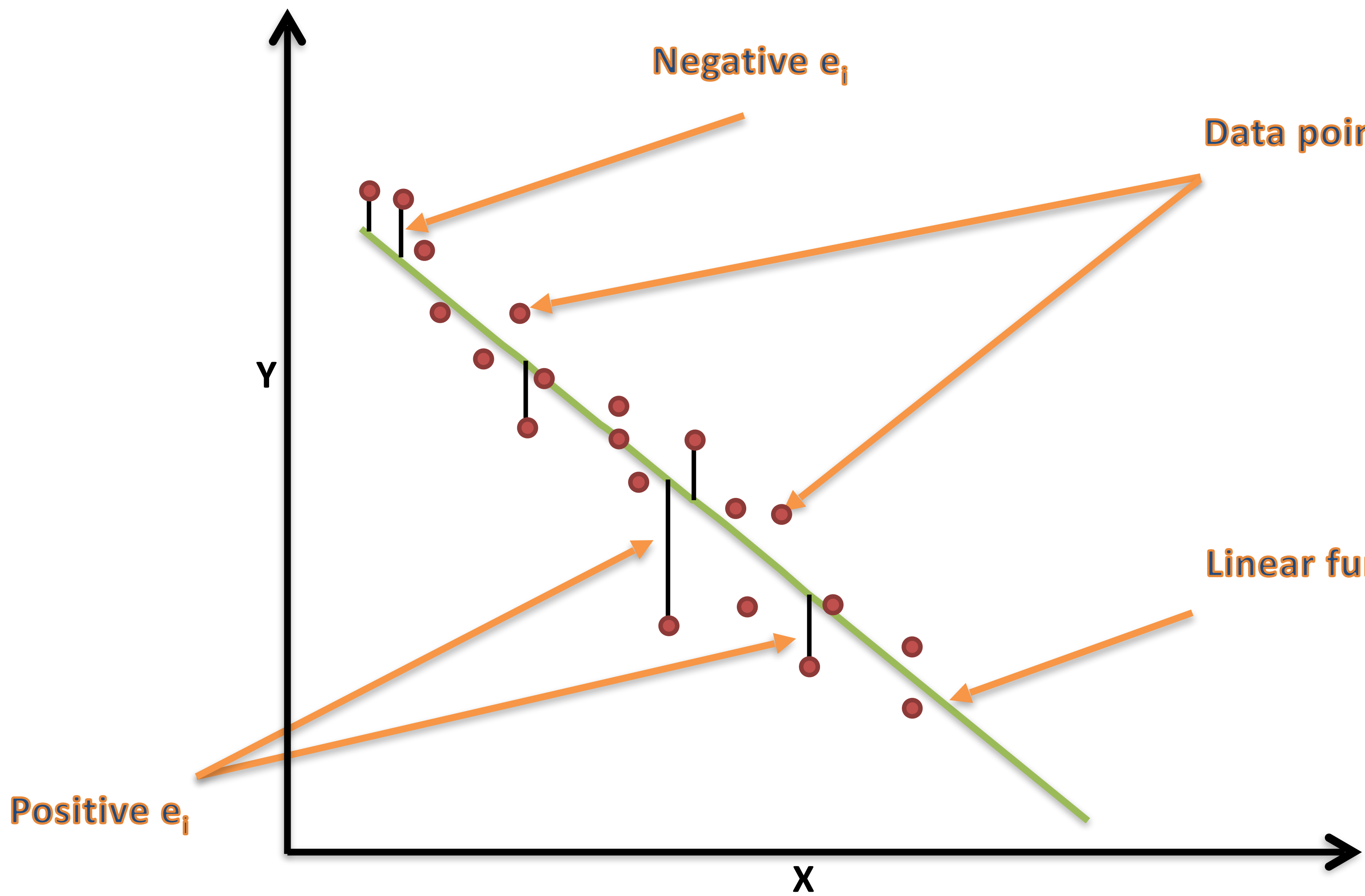
Curve fitting : Fitting lines and polynomial functions to data points



Curve fitting : Fitting lines and polynomial functions to data points

How does it work if the function f is to be of the form

$\boldsymbol{f(x) = ax + b}$

for <u>to-be-chosen</u> parameters ***a*** and ***b***?

Given (x,y) data pairs $(x_1,y_1), (x_2,y_2), \dots , (x_N,y_N)$

For fixed values of a and b, the mismatch (**error**) is

$e_1 = ax_1+b - y_1$

$e_2 = ax_2+b - y_2$

$\dots$

$e_N = ax_N+b - y_N$

by choosing

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix} \begin{bmatrix} \boxed{a} \\ \boxed{b} \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Goal: make this small

"data"

Curve fitting : Fitting lines and polynomial functions to data points

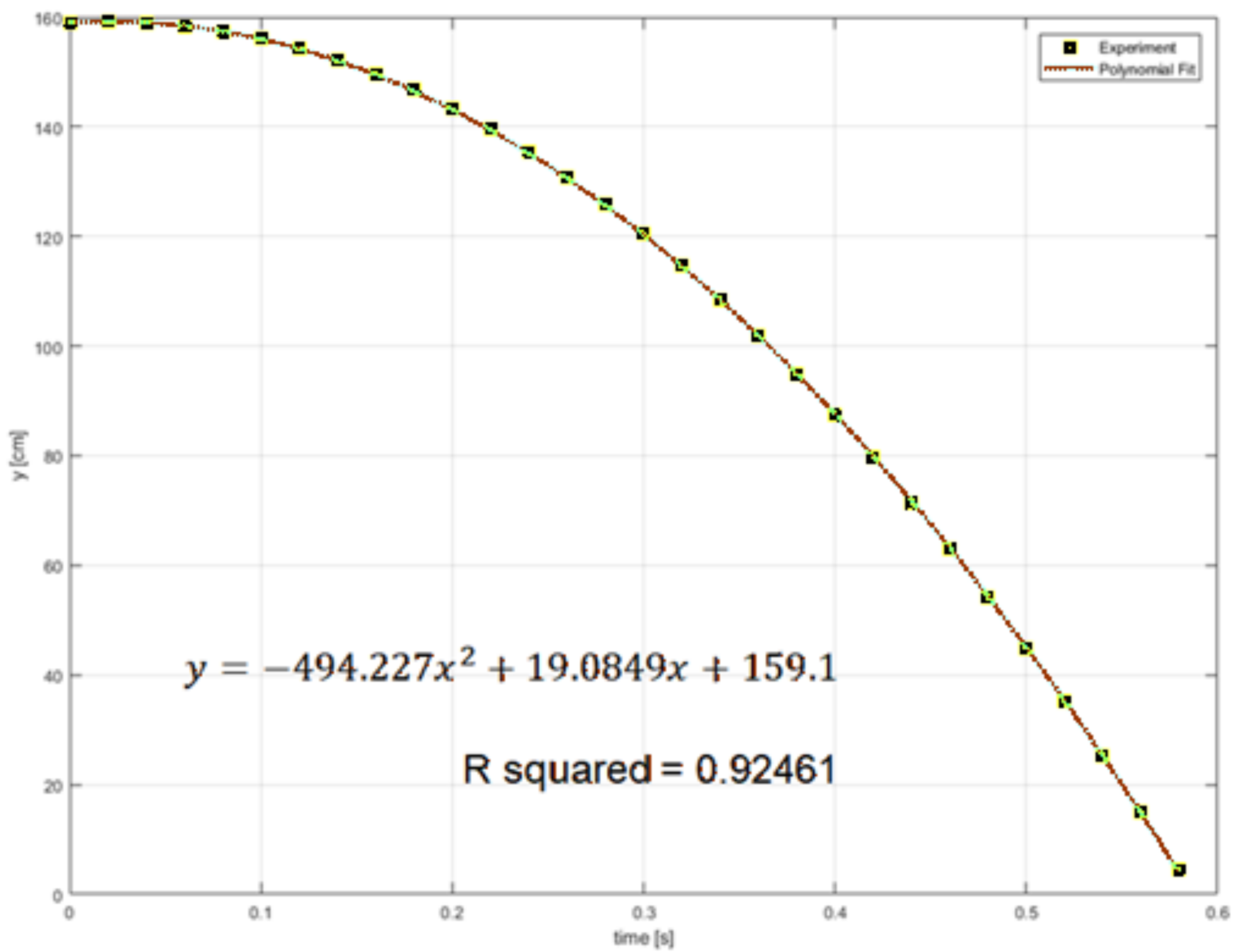
How does it work if the function *f* is to be of the form

$$f(x) = a_1x^n + a_2x^{n-1} + \dots + a_nx + a_{n+1}$$

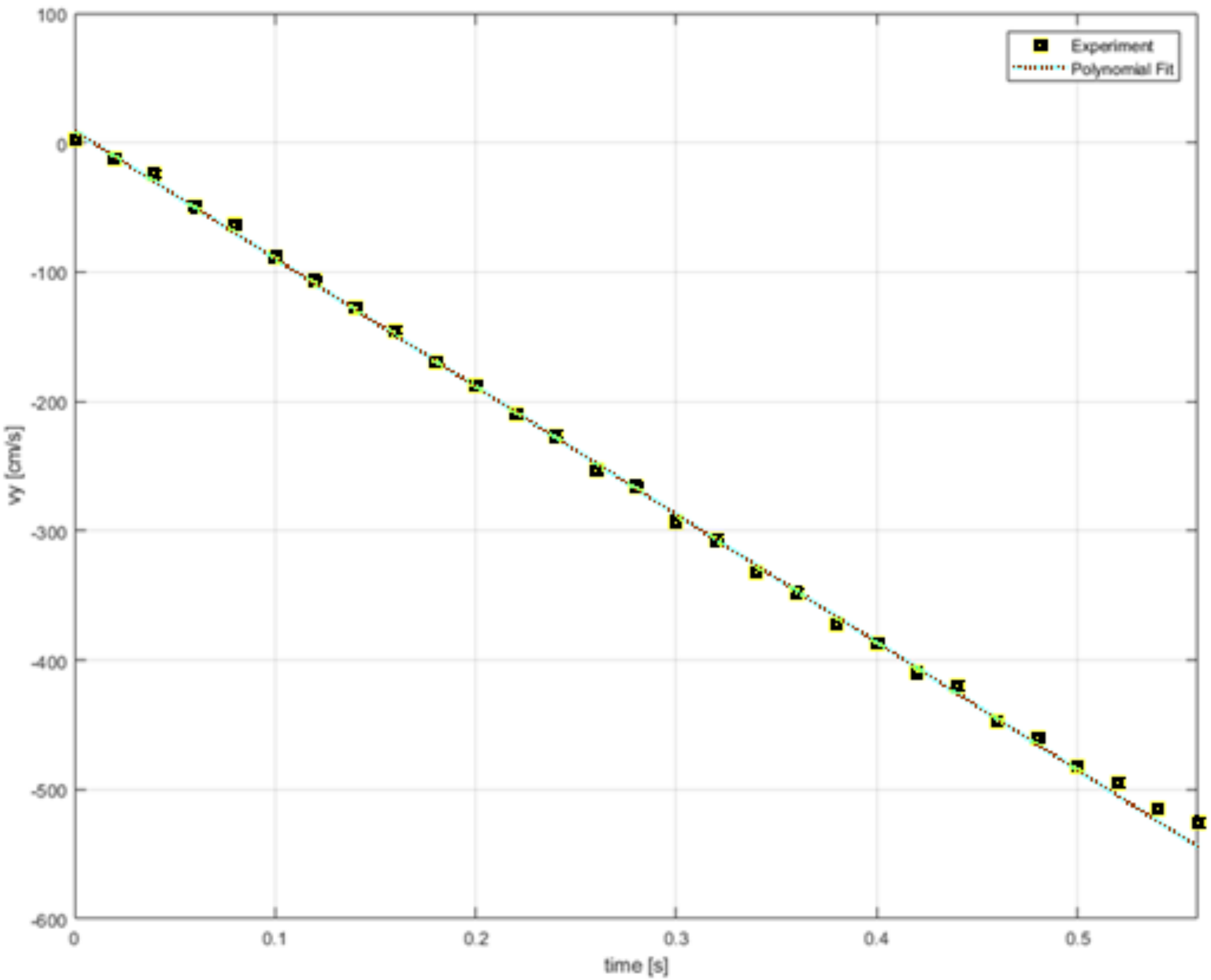
for to-be-chosen parameters $a_1, a_2, \dots, a_{n+1}$?
For fixed values of $a_1, a_2, \dots, a_{n+1}$, the error at (x_k, y_k) is

$$\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} = \begin{bmatrix} x_1^n & x_1^{n-1} & \cdots & x_1 & 1 \\ x_2^n & x_2^{n-1} & \cdots & x_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_N^n & x_N^{n-1} & \cdots & x_N & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ a_{n+1} \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}$$

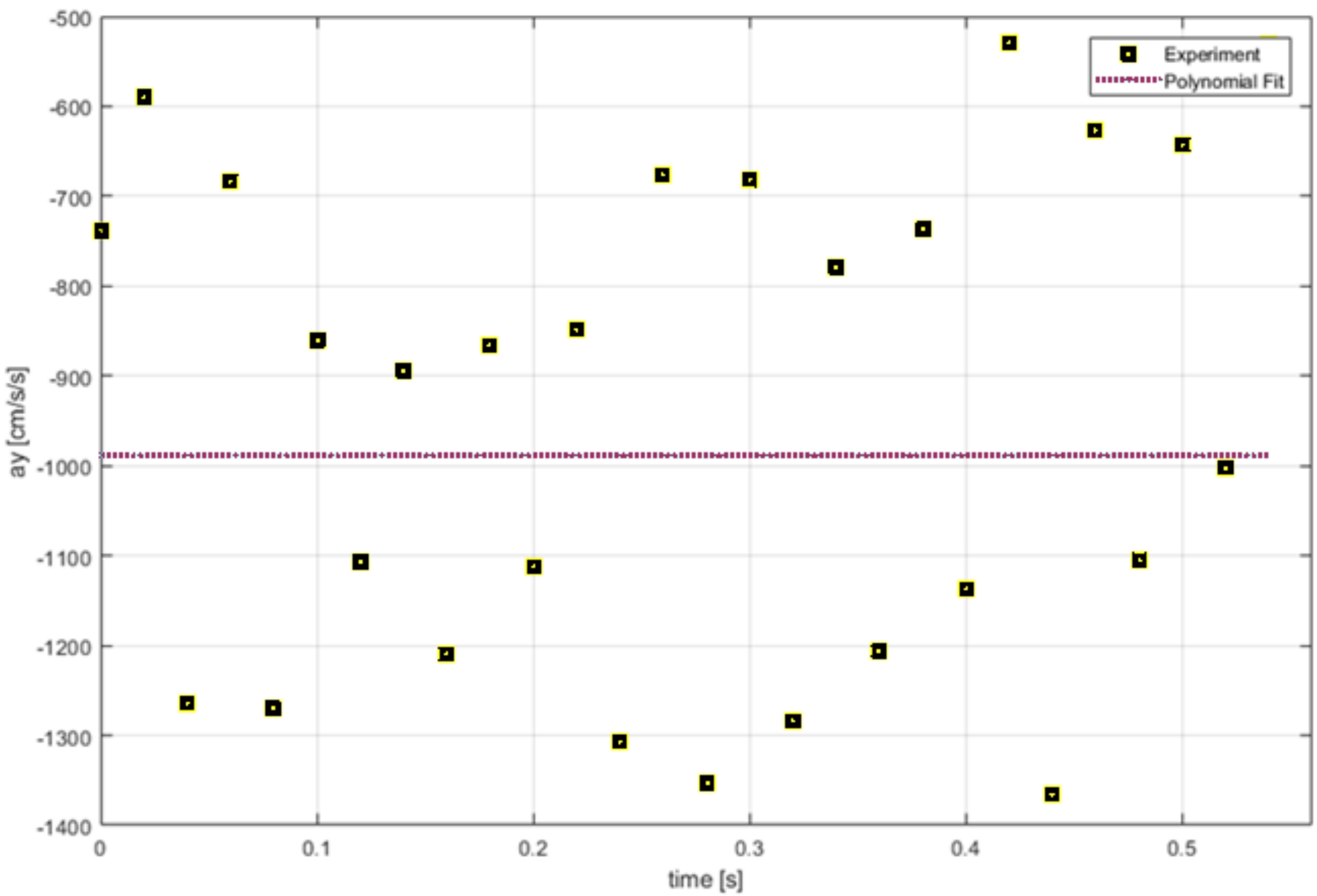
Curve fitting : 2nd degree polynomial functions to data points



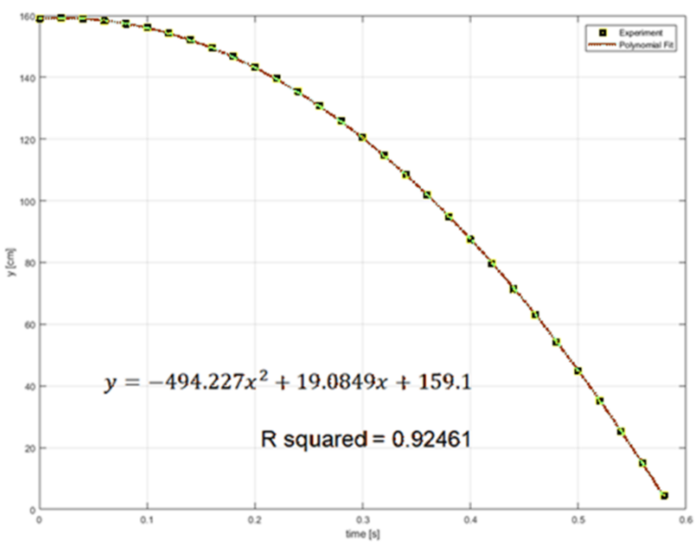
Velocity from the Experiment and Polynomial functions

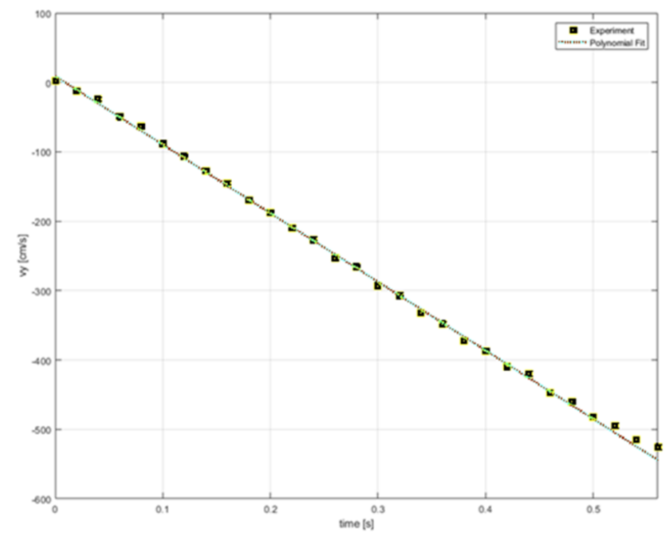


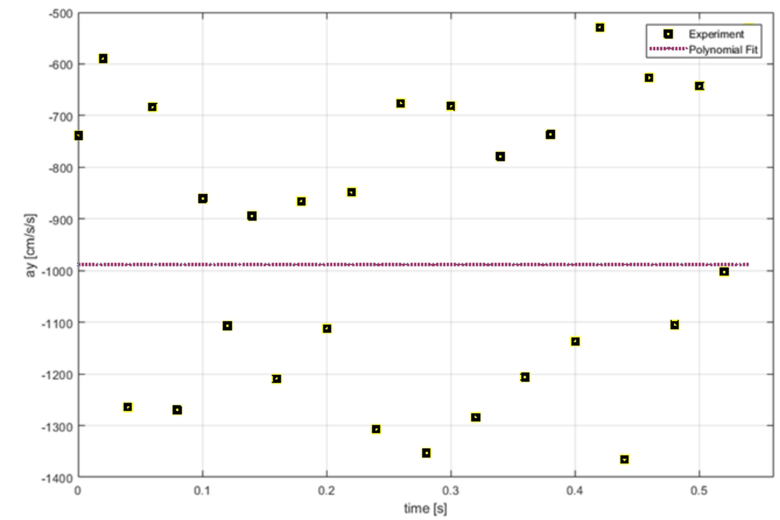
Acceleration from the Experiment and Polynomial functions



Calculate the Velocity and Acceleration from the Mathematical Model







polyfit
Polynomial curve fitting

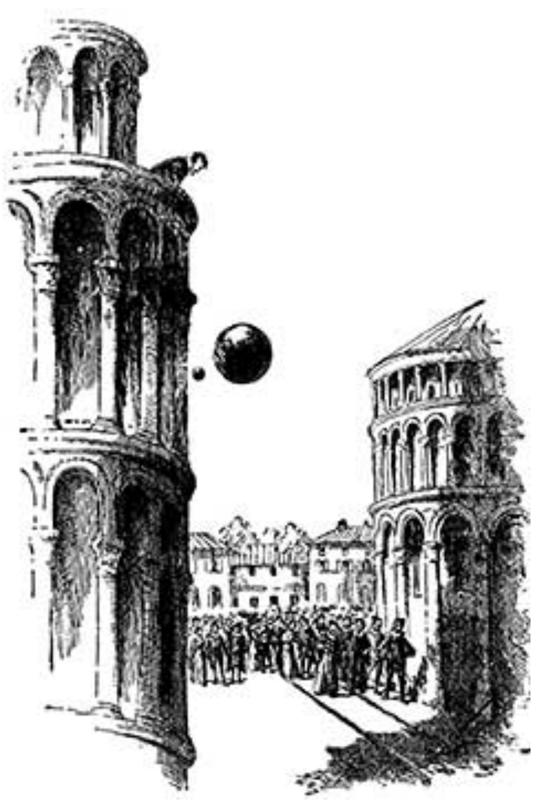
numpy.polynomial
A sub-package for efficiently dealing with polynomials.

Polynomials.jl

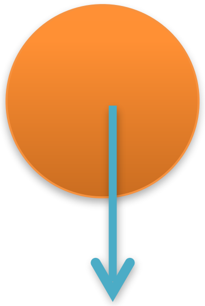
polynomial: Polynomials
Description
Construct, coerce to, test for, and print polynomial objects.

LINEST function

What is the Mathematical Model???

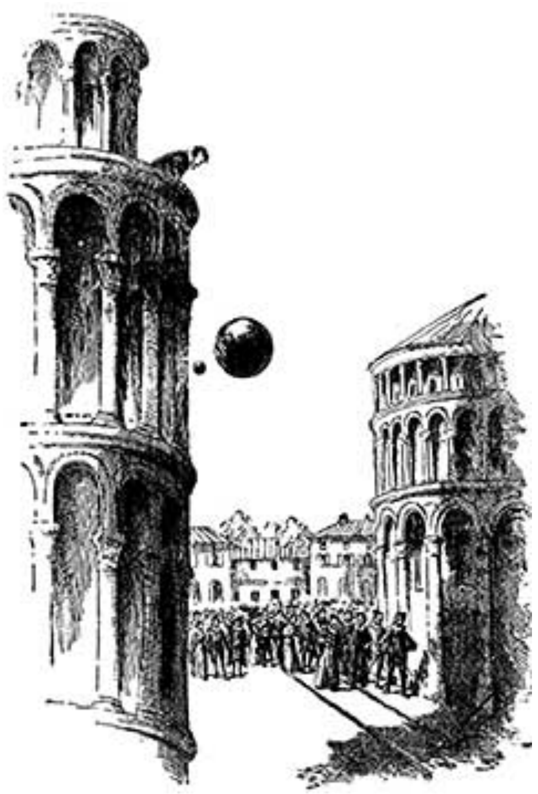


Galileo didn't know calculus (because Newton and Leibniz hadn't discovered it yet) so he couldn't derive the equation mathematically. Since we do know calculus we know that acceleration is the variation of velocity with time.



$$\vec{F}_{gravity} = m\vec{g}.$$

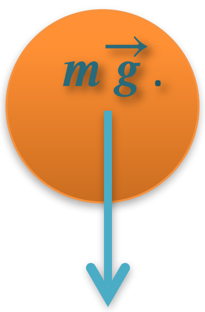
What is the Mathematical Model???



Galileo didn't know calculus (because Newton and Leibniz hadn't discovered it yet) so he couldn't derive the equation mathematically. Since we do know calculus we know that acceleration is the variation of velocity with time.

We assume no drag/air resistance

$$\vec{F}_{gravity} = m\vec{g}$$



$$-\vec{g} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$dv = -\vec{g}\,dt$$

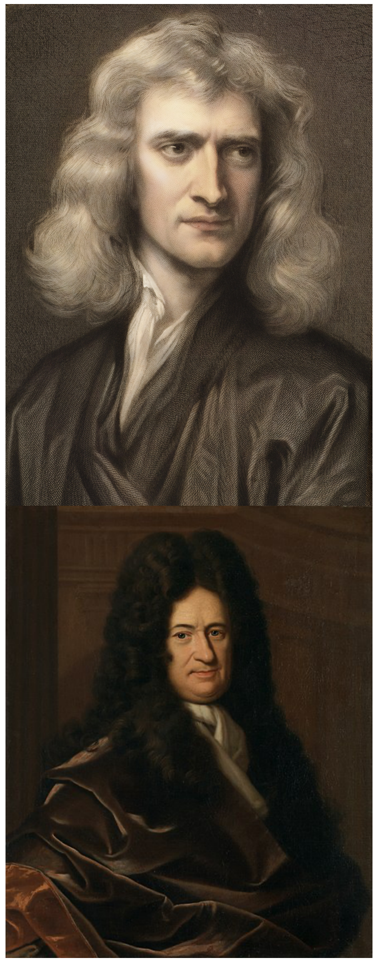
$$v = \int -\vec{g}\,dt$$

$$v = -\vec{g}\,t$$

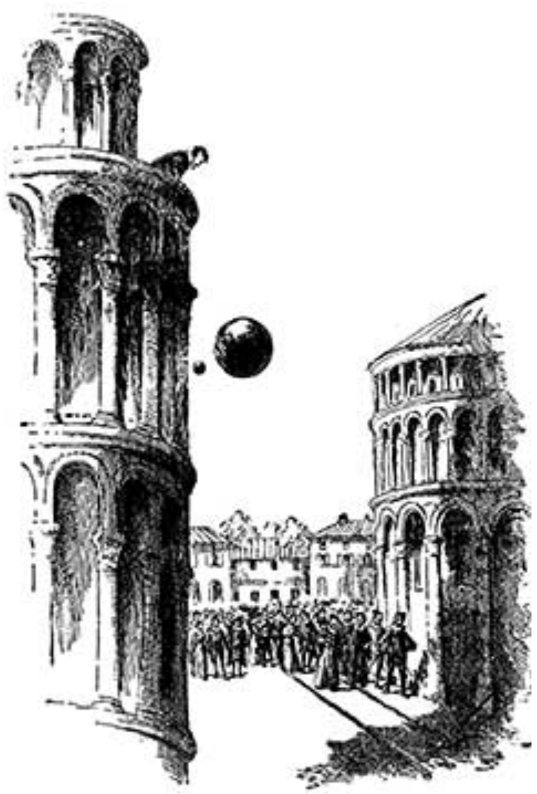
Newton, Leibniz, Euler, Lagrange

$$\dot{f} = \frac{df}{dt} = \partial_t f = f'$$

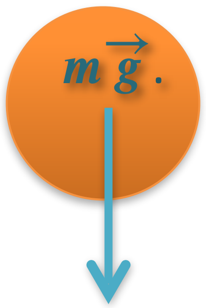
$$\ddot{f} = \frac{d^2 f}{dt^2} = \partial_{tt} f = f''$$



What is the Mathematical Model???



We assume no drag/air resistance

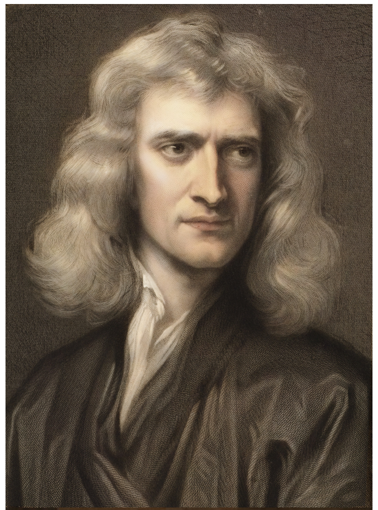


$$\int v dt = \int -\vec{g}\, t dt$$

$$\boxed{x = -\frac{1}{2}\vec{g}\, t^2}$$

$$x = v_0 t + \frac{1}{2} g t^2$$

$$y = bx + ax^2$$



What is the Mathematical Model???

We assume no drag/air resistance

At the end of **the last Apollo 15 moon walk**, Commander David Scott performed a live demonstration for the television cameras. He held out a geologic hammer (1.32 kg) and a feather (0.03 kg) and dropped them at the same time.

Because they were essentially in a vacuum, there was no air resistance and the feather fell at the same rate as the hammer, as **Galileo** had concluded hundreds of years before - ***all objects released together fall at the same rate regardless of mass*** *of mass*.

What is the Mathematical Model???

We assume no drag/air resistance