HAB718 Spor Biyomekaniğinde Hareket Analizi

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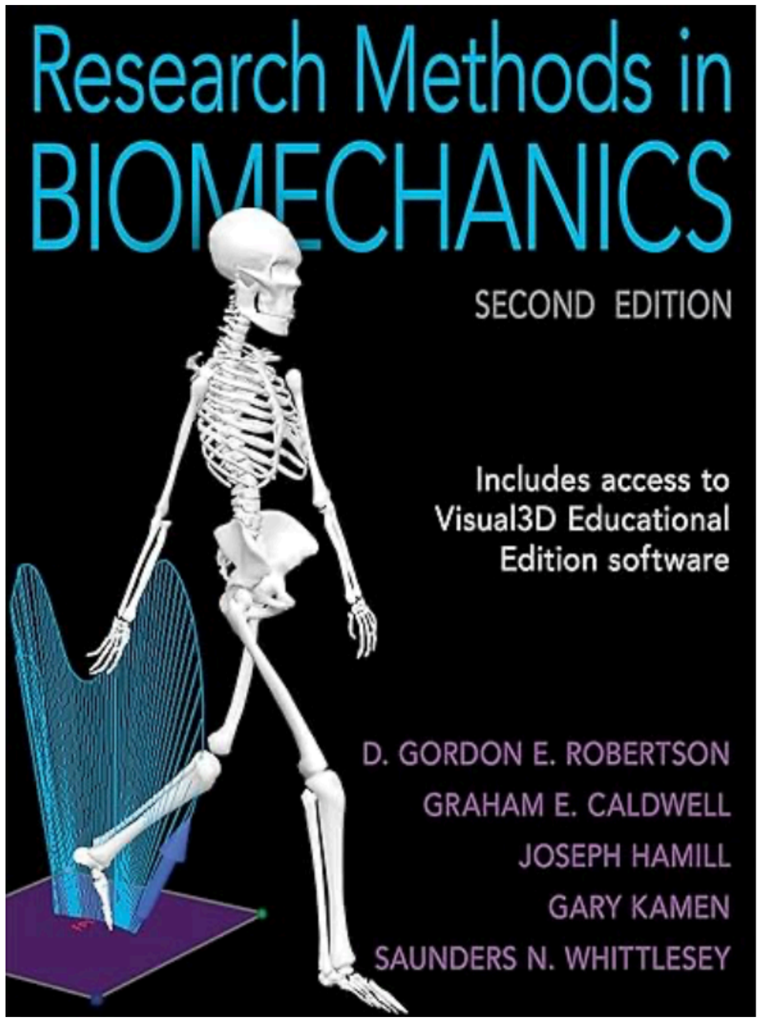
- Transformation Matrix
- Two-Dimensional Kinematics

Transformation Matrix



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TRANSFORMATIONS BETWEEN COORDINATE SYSTEMS

We have identified two types of coordinate systems (GCS and LCS) that exist in the same 3-D motion-capture volume. The descriptions of a rigid segment moving in space in different coordinate systems can be related by means of a *transformation* between the coordinate systems (see figure 2.3). A transformation allows one to convert coordinates expressed in one coordinate system to those expressed in another coordinate system. In other words, we can look at the same location in different ways based on which coordinate system we are using. At first glance this may seem redundant because we have not added new information by describing the same point in different ways. It is, however, convenient because objects move in the GCS but attributes of a segment, such as an anatomical landmark (e.g., segment end-point), are constant in the LCS. We generally refer to transformations as linear or rotational.

Linear Transformation

In figure 2.4, a point is described by the vector $\vec{P}'$ in the LCS and by $\vec{P}$ in the GCS. The linear transformation between the LCS and the GCS can be defined by a vector $\vec{O}$, which specifies the origin of the LCS relative to the GCS. The components of $\vec{O}$ can be written as a column matrix in the form

$$\vec{O} = \begin{bmatrix} O_x \\ O_y \\ O_z \end{bmatrix} \tag{2.2}$$

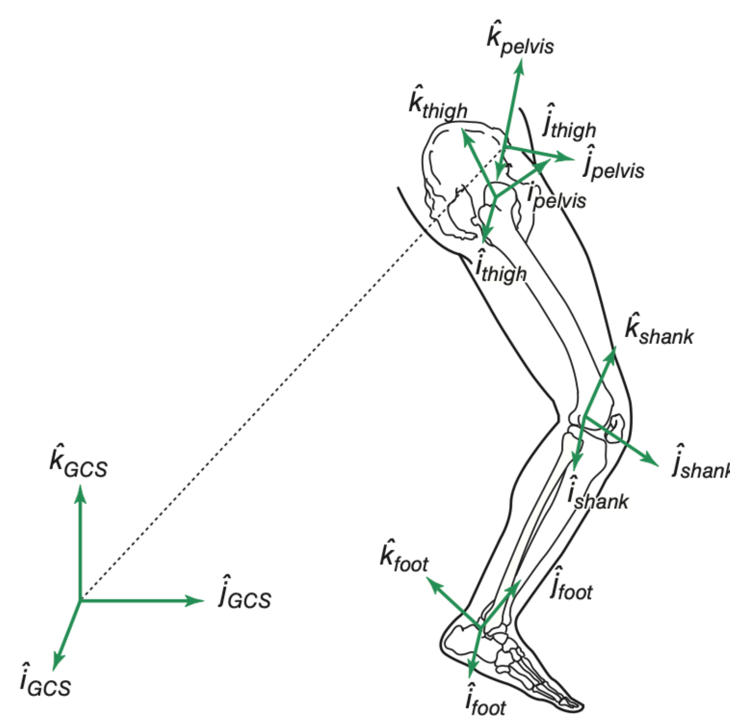
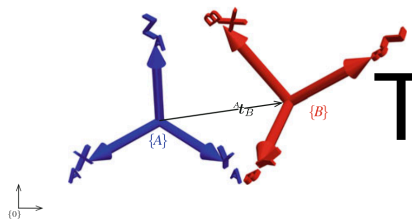
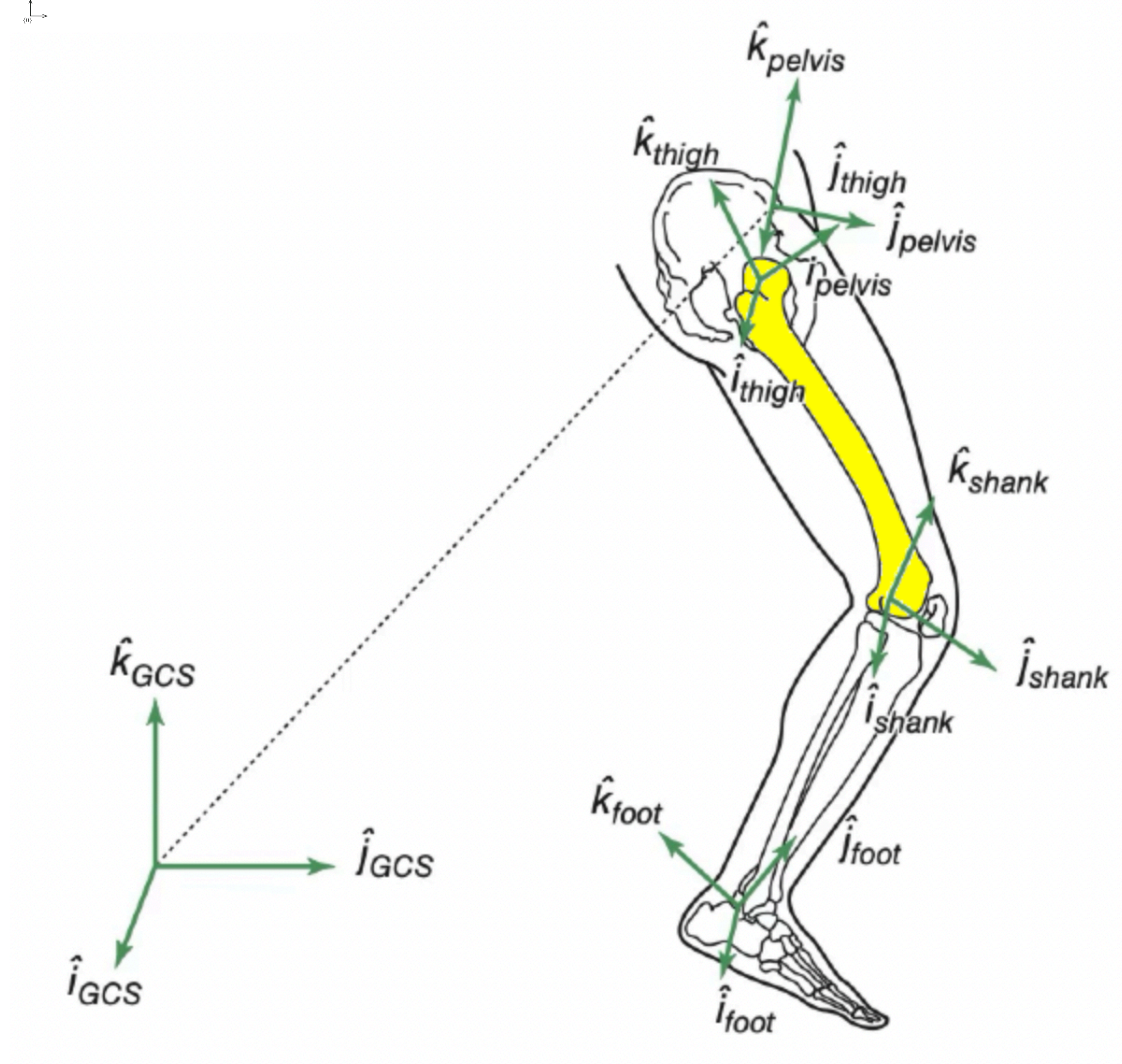


Figure 2.3 The global coordinate system and the local coordinate systems of the right-side lower extremity.



Transformation Matrix



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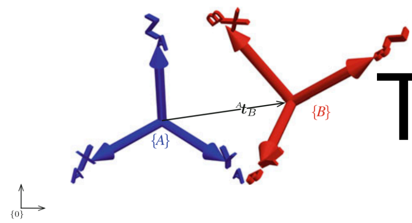
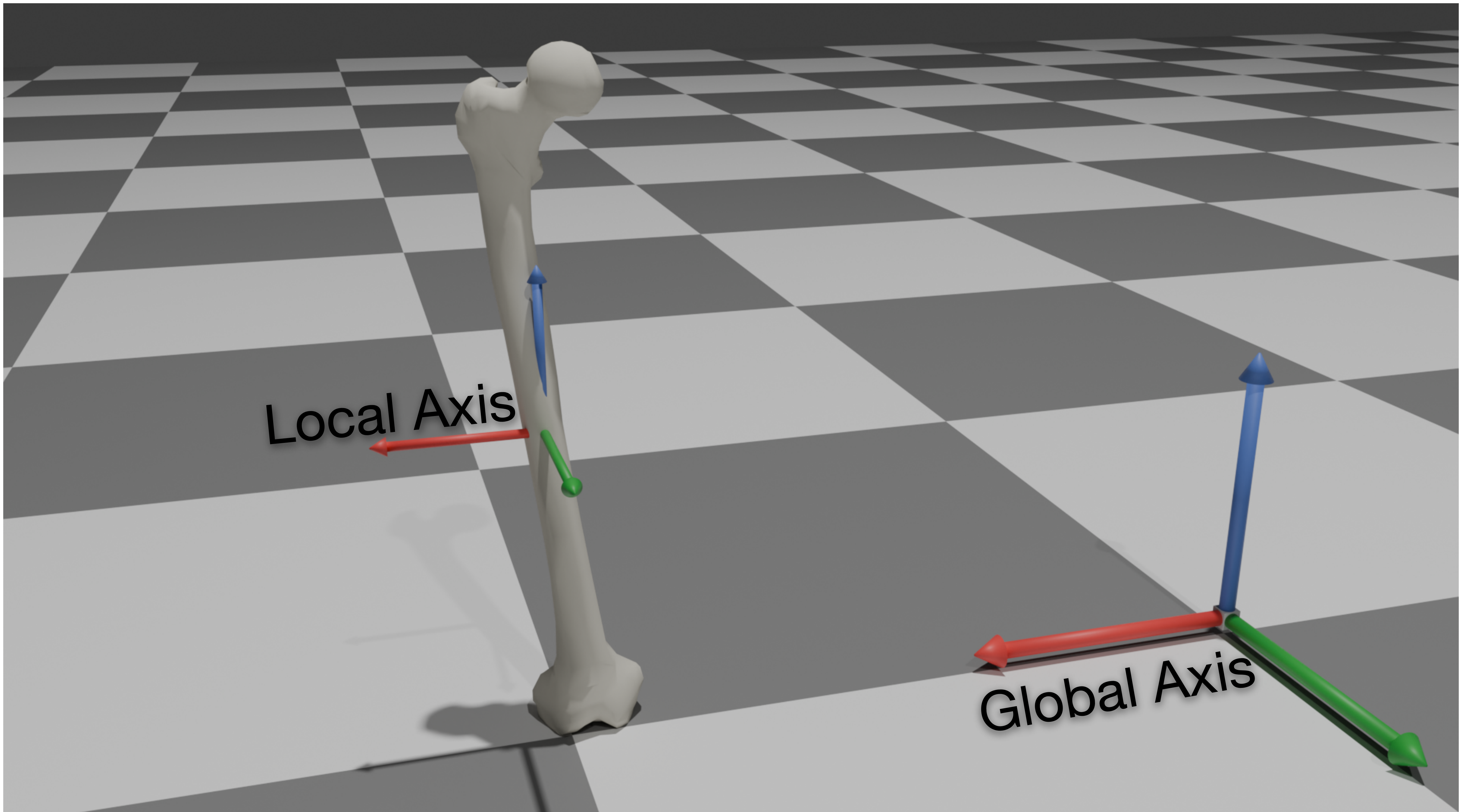
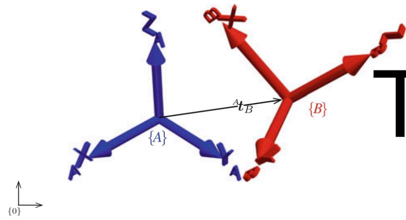
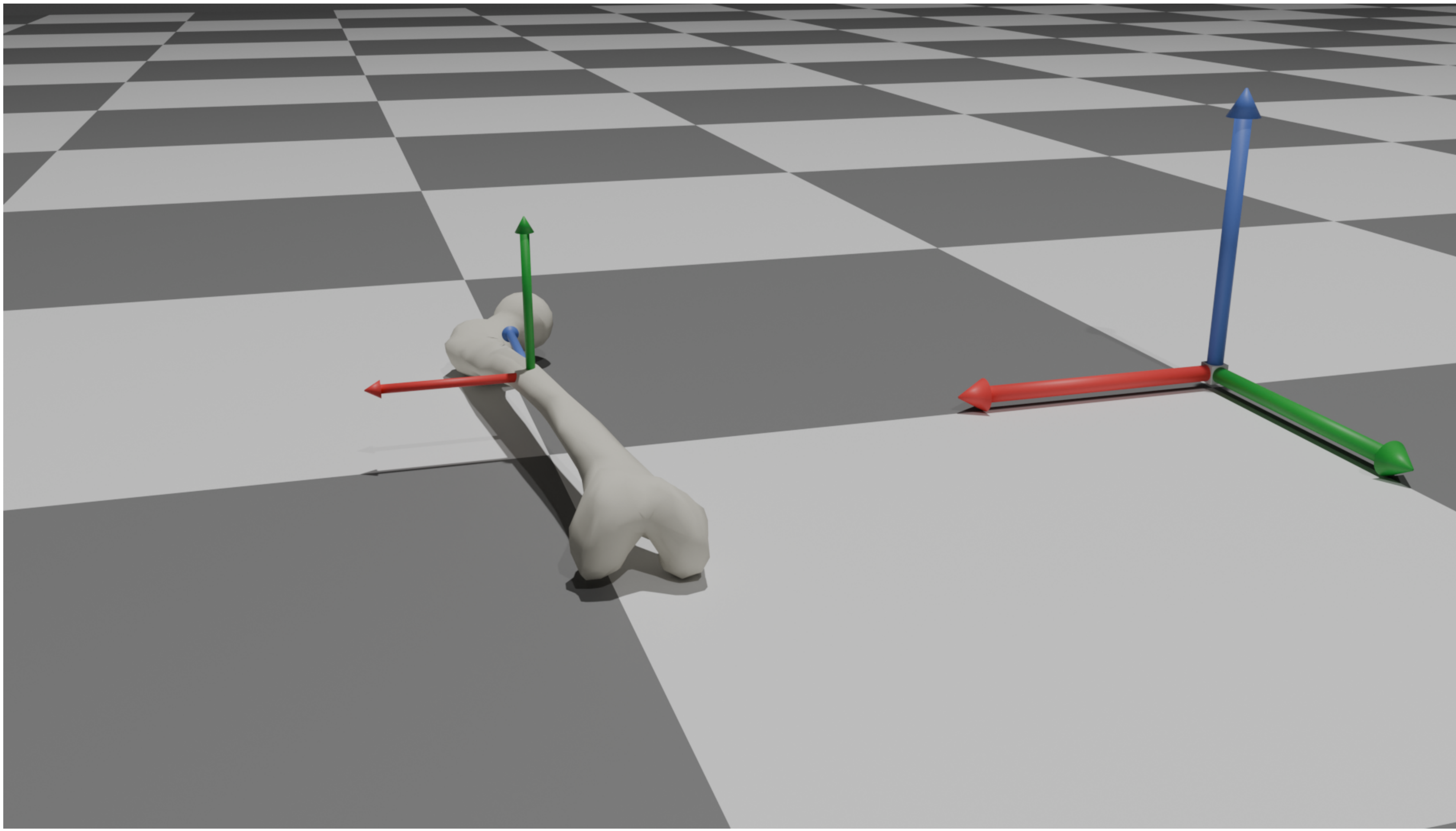


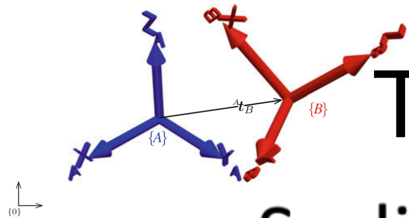
Transformation Matrix





Transformation Matrix





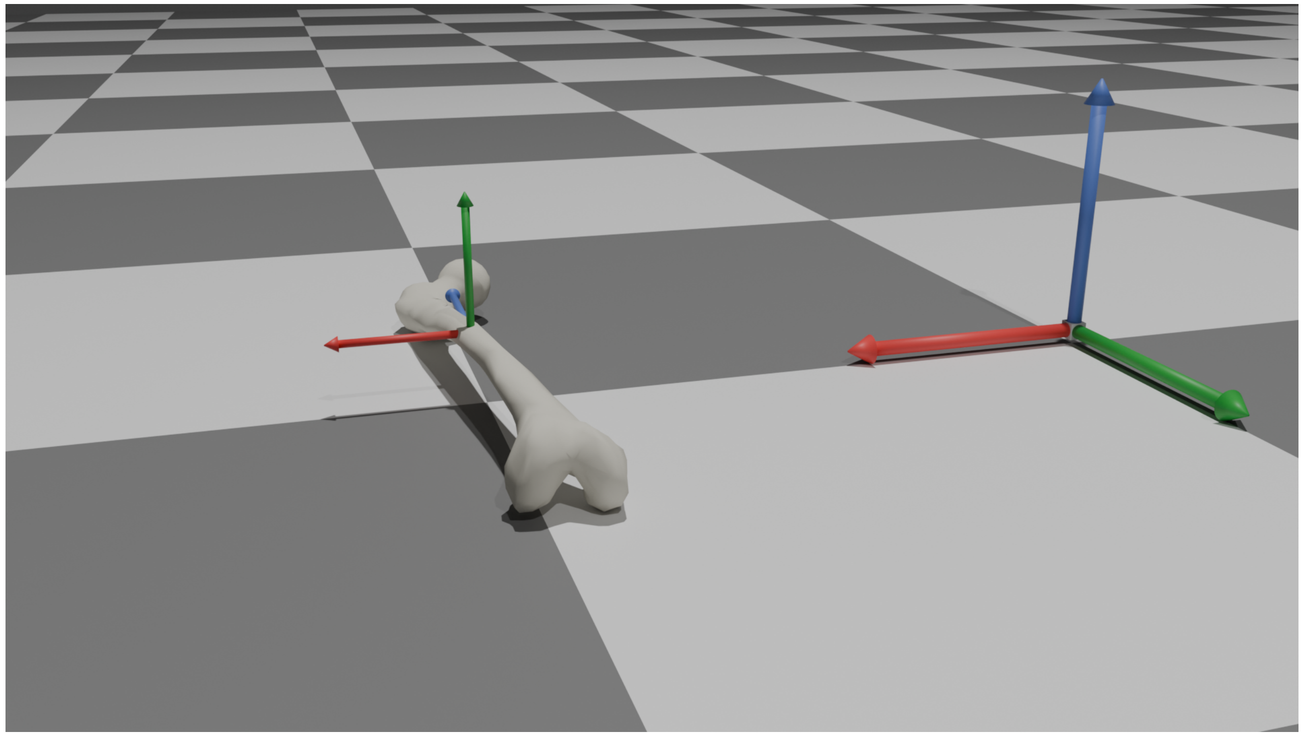
Transformation Matrix

Scaling + Rotation + Translation

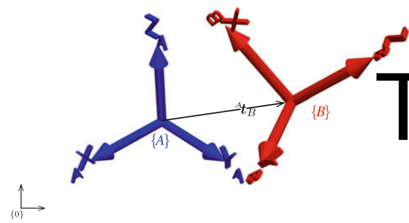
$$\mathbf{p}' = (\mathbf{T}\cdot\mathbf{R}\cdot\mathbf{S})\cdot\mathbf{p} = \begin{bmatrix}1 & 0 & t_x\\ 0 & 1 & t_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}s_x & 0 & 0\\ 0 & s_y & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix}$$

$$= \begin{bmatrix}\cos\theta & -\sin\theta & t_x\\ \sin\theta & \cos\theta & t_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}s_x & 0 & 0\\ 0 & s_y & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix}$$

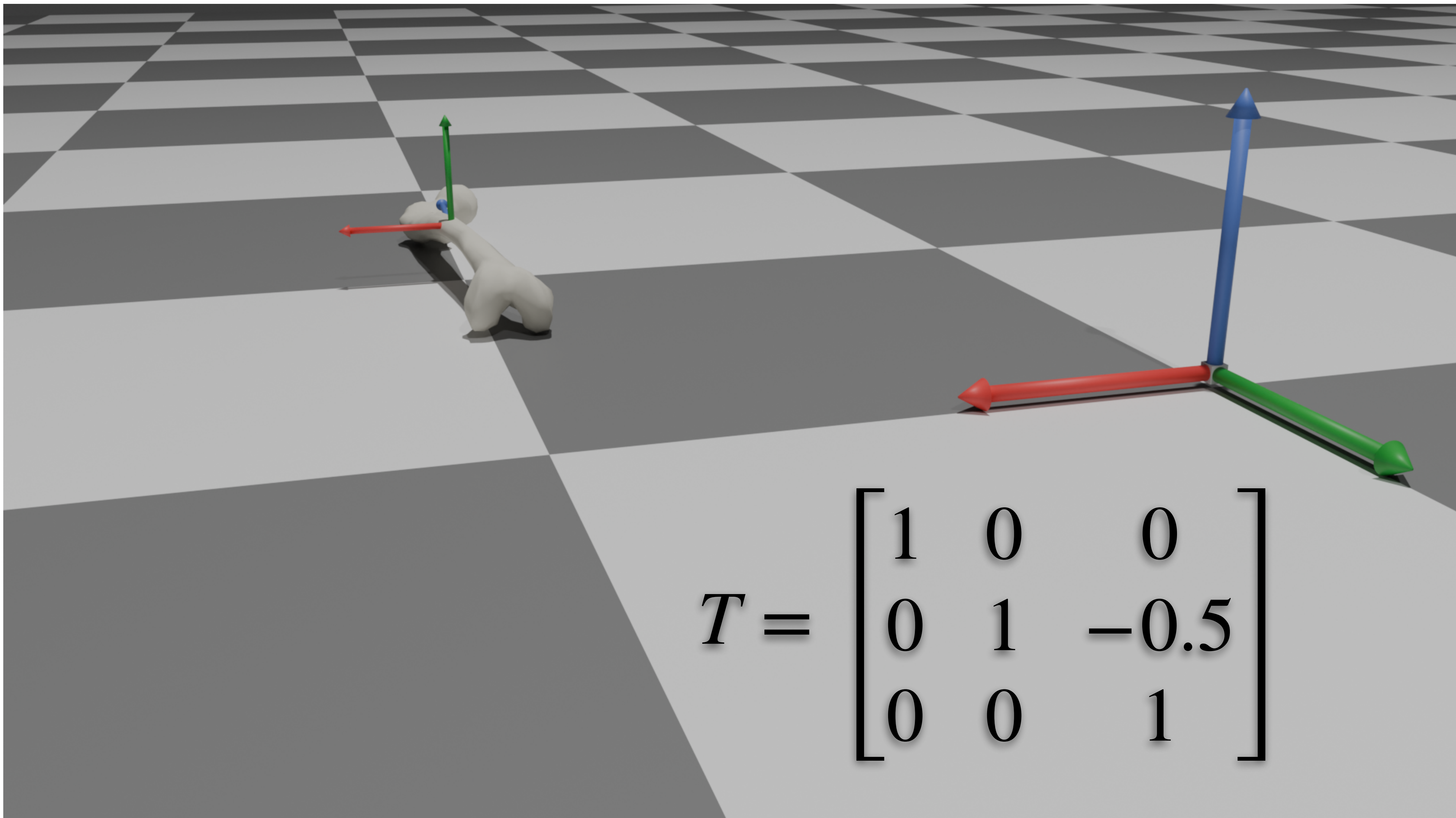
$$= \begin{bmatrix}\mathbf{R}' & \mathbf{t}\\ \mathbf{0} & 1\end{bmatrix}\begin{bmatrix}\mathbf{S}' & \mathbf{0}\\ \mathbf{0} & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix}$$



$$T = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$



Transformation Matrix



$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

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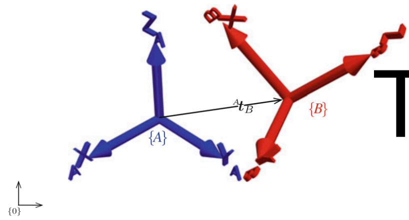
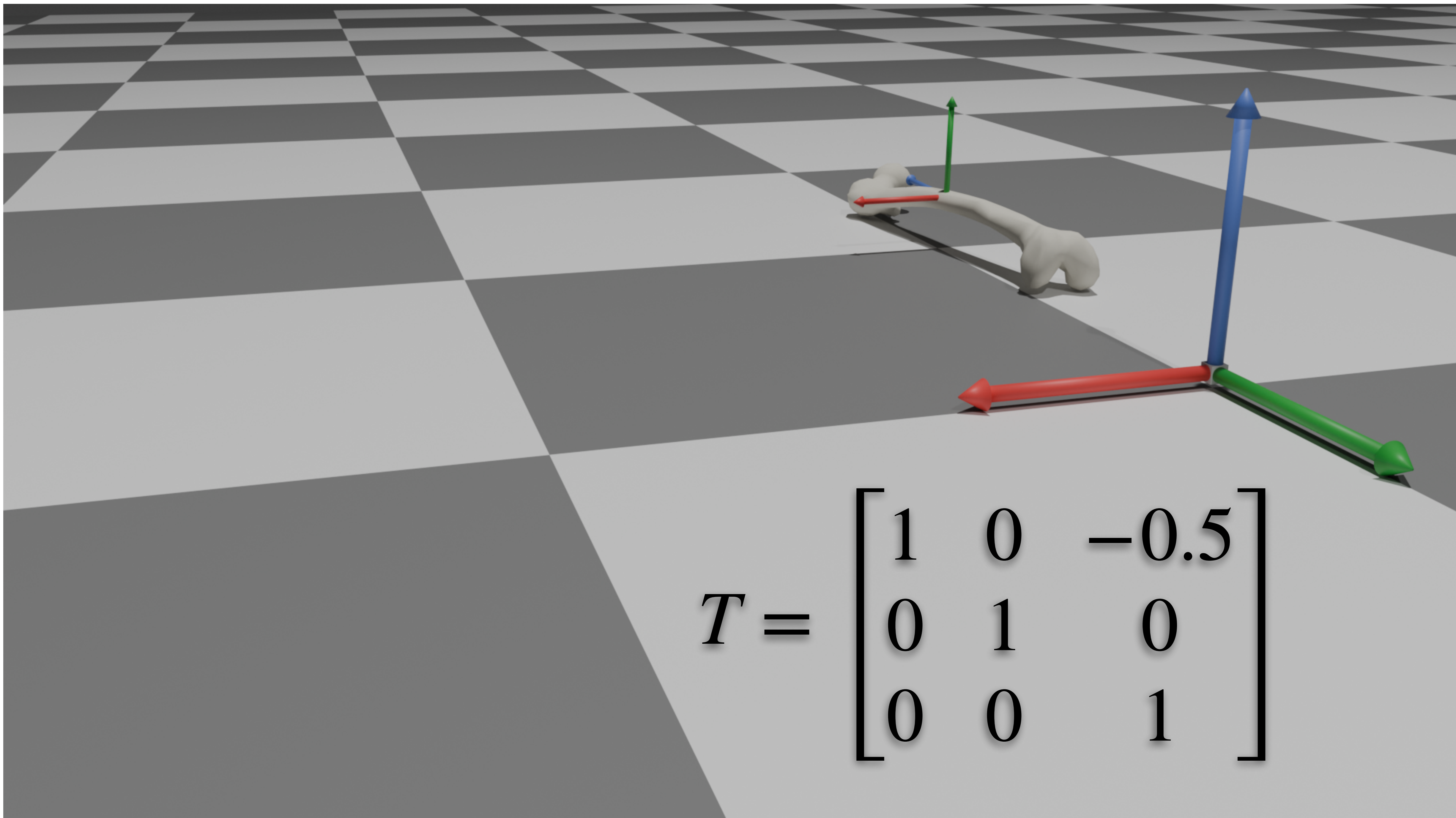
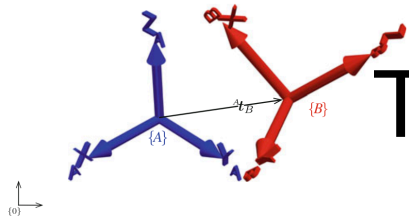


Transformation Matrix





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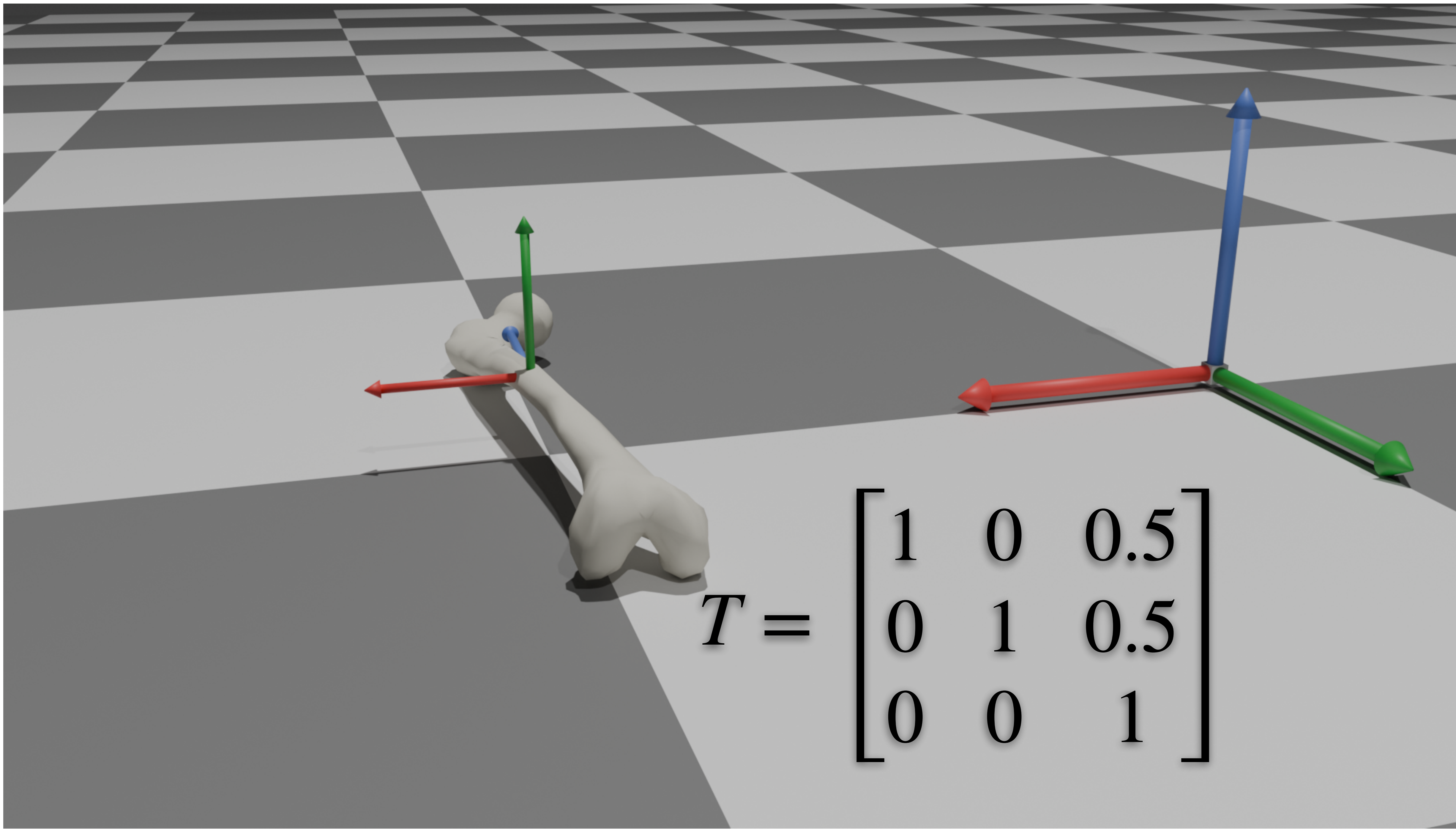
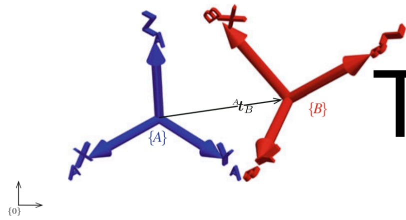
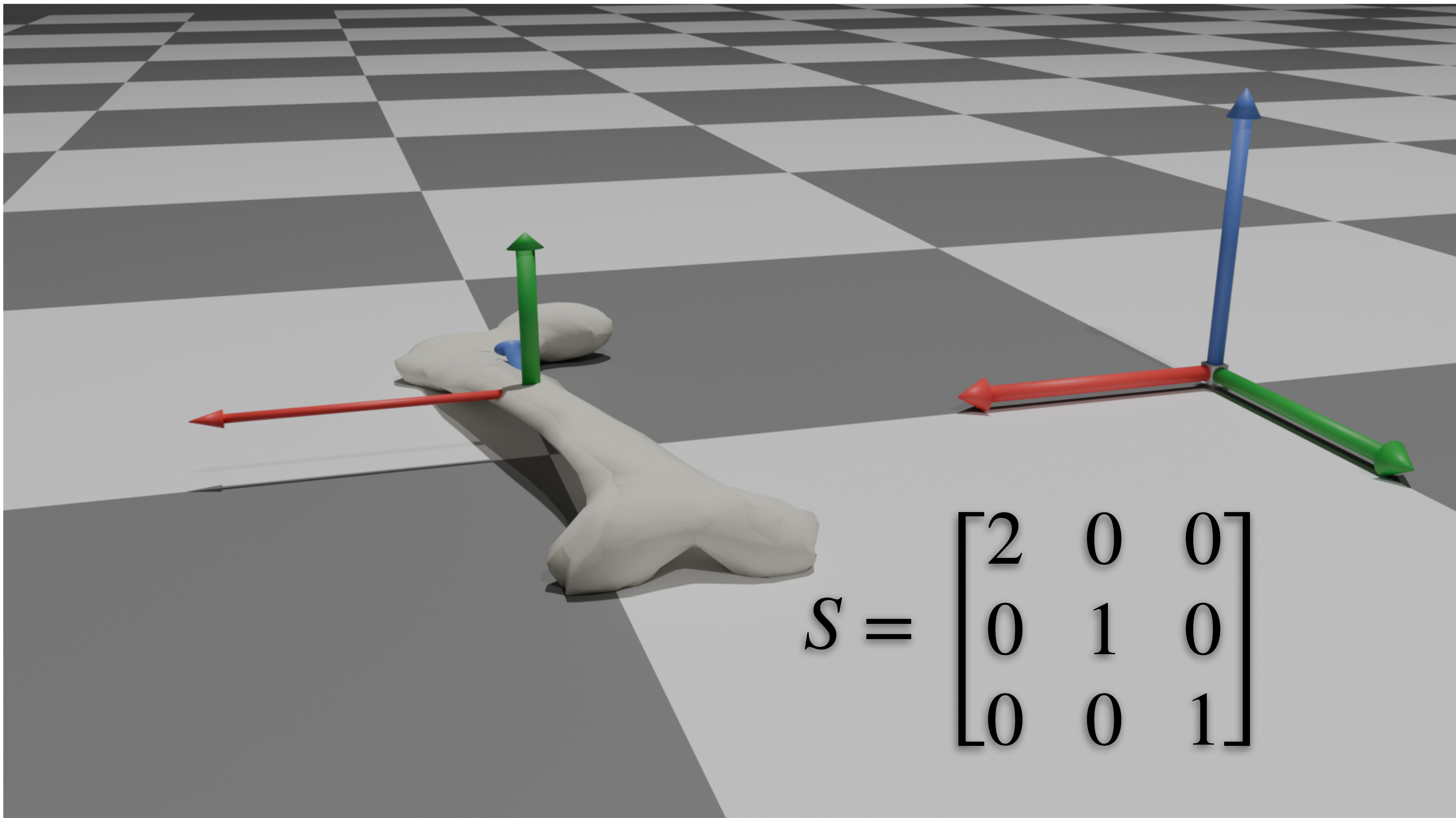


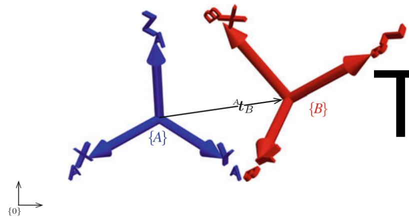
$$T = \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$$



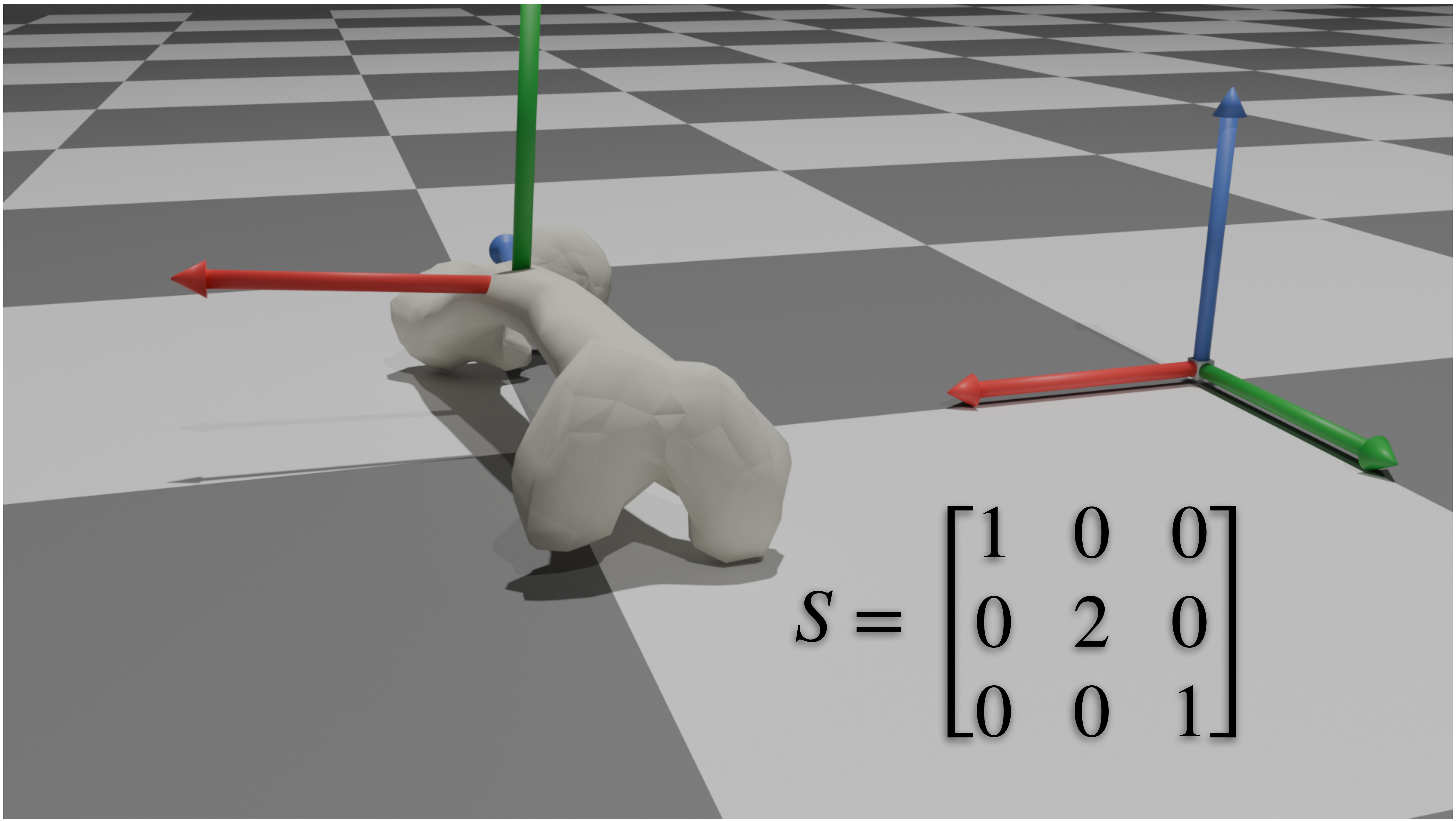
Transformation Matrix



$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Transformation Matrix



$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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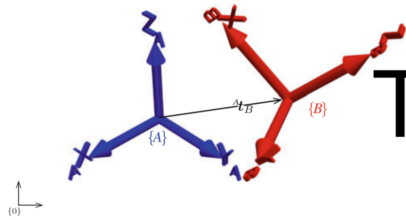

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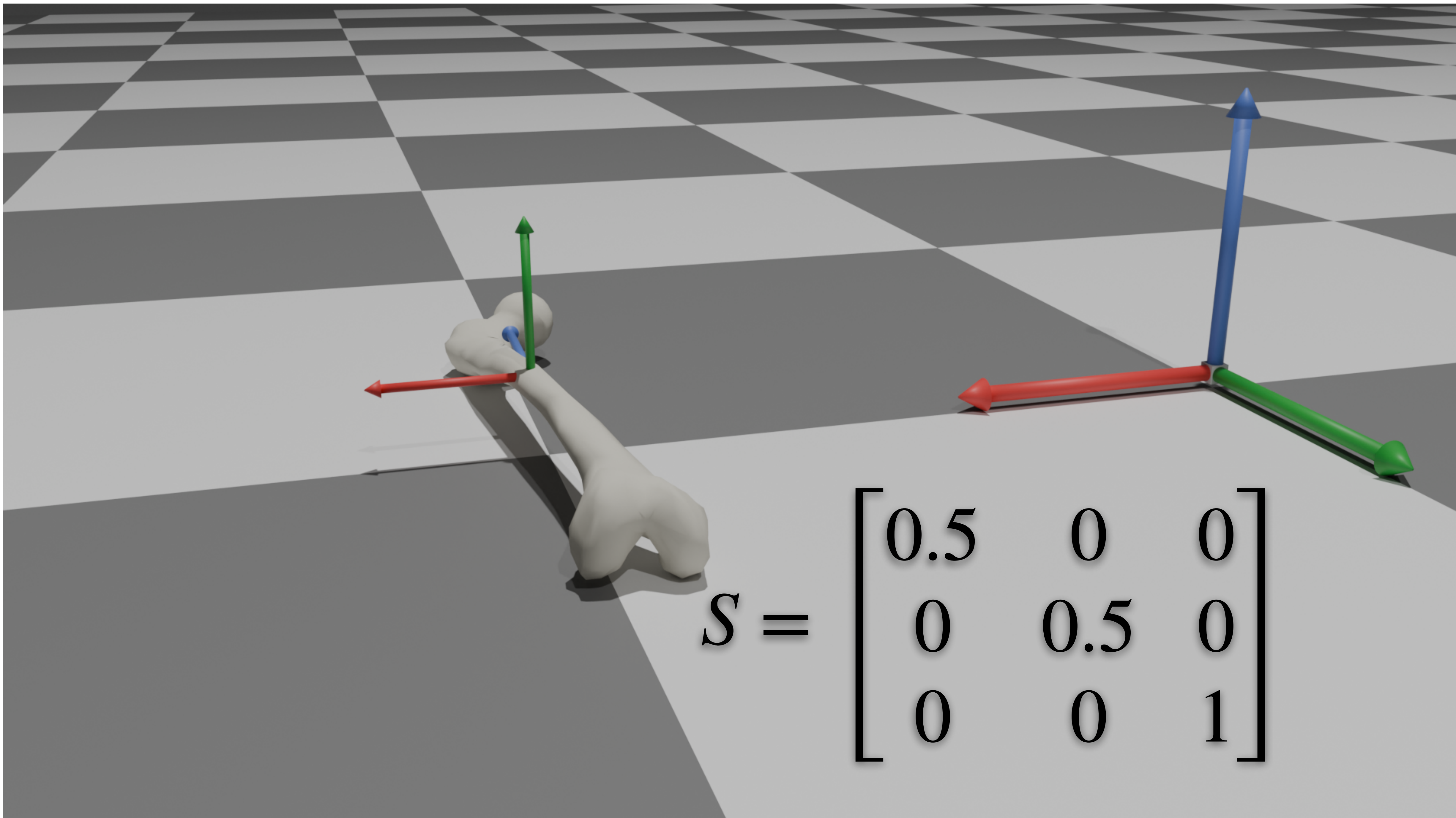

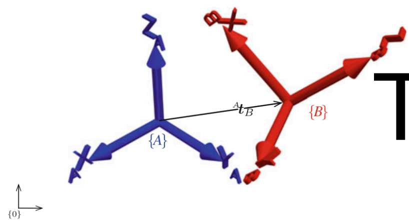

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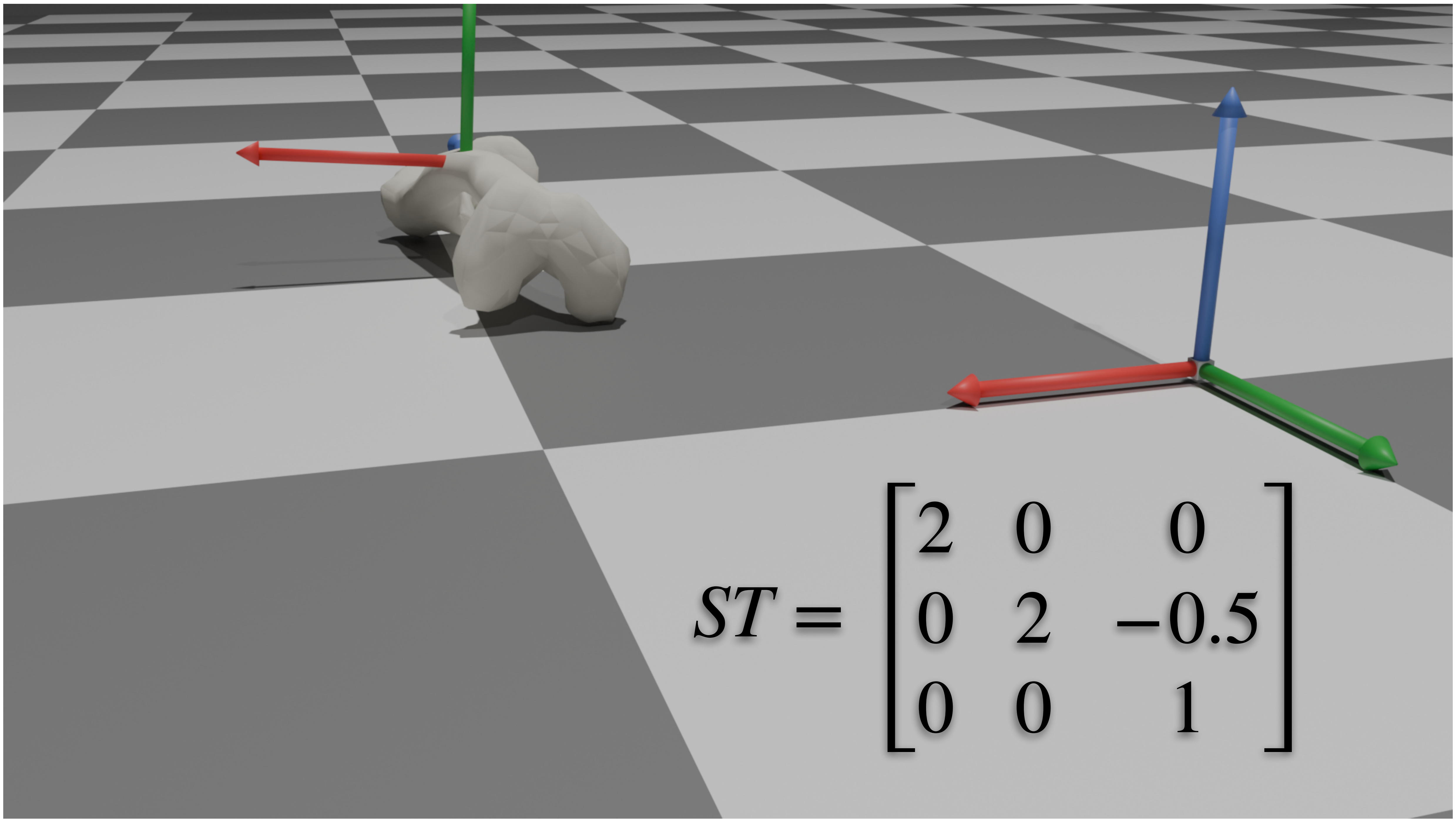
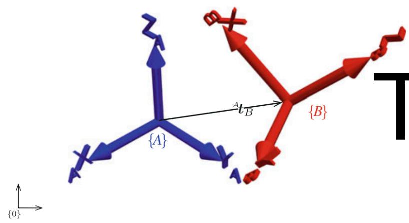


$$ST = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & -0.5 \\ 0 & 0 & 1 \end{bmatrix}$$



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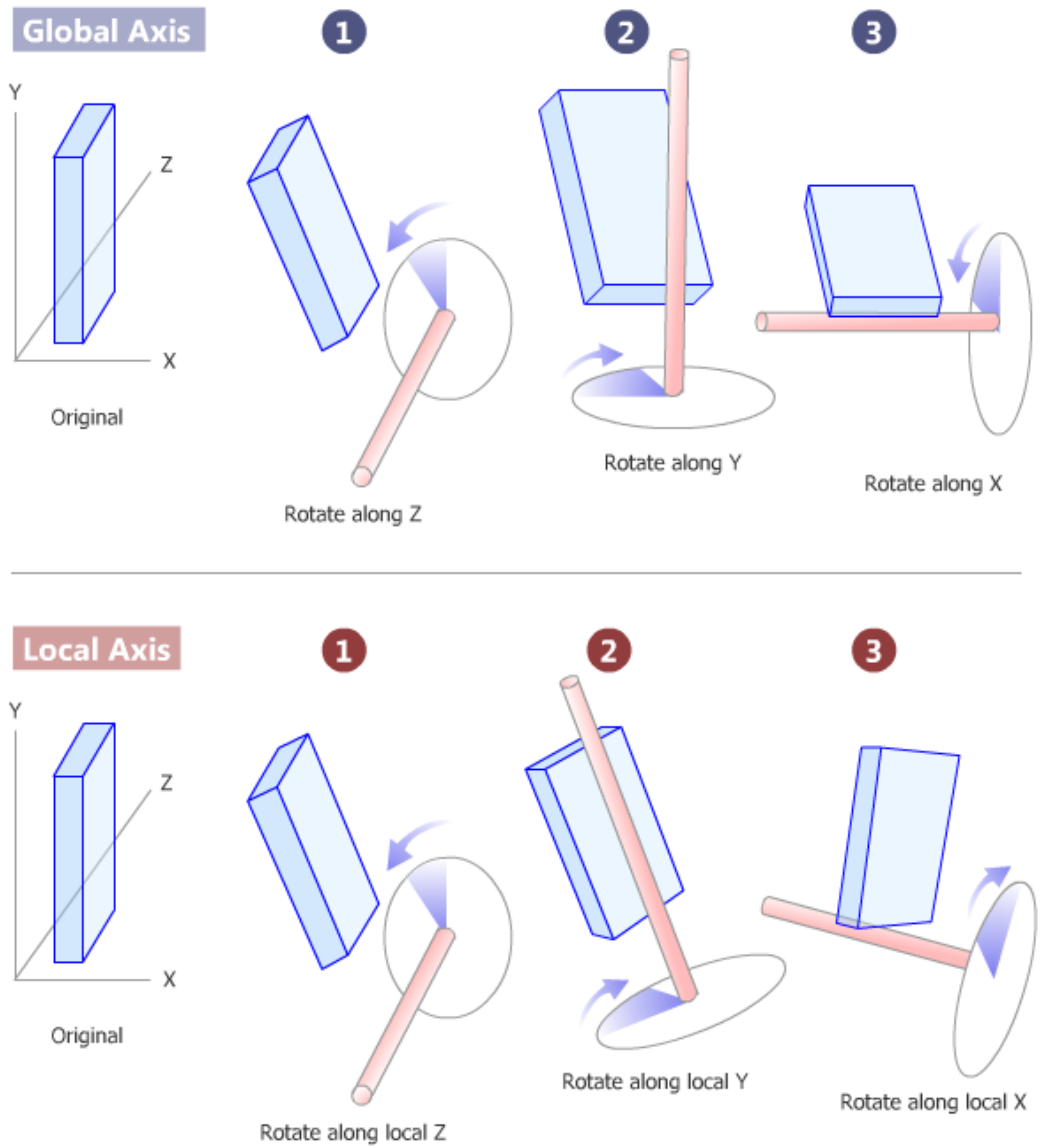
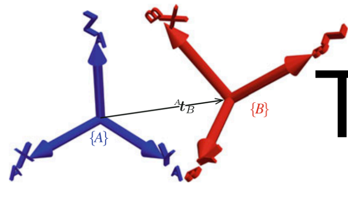
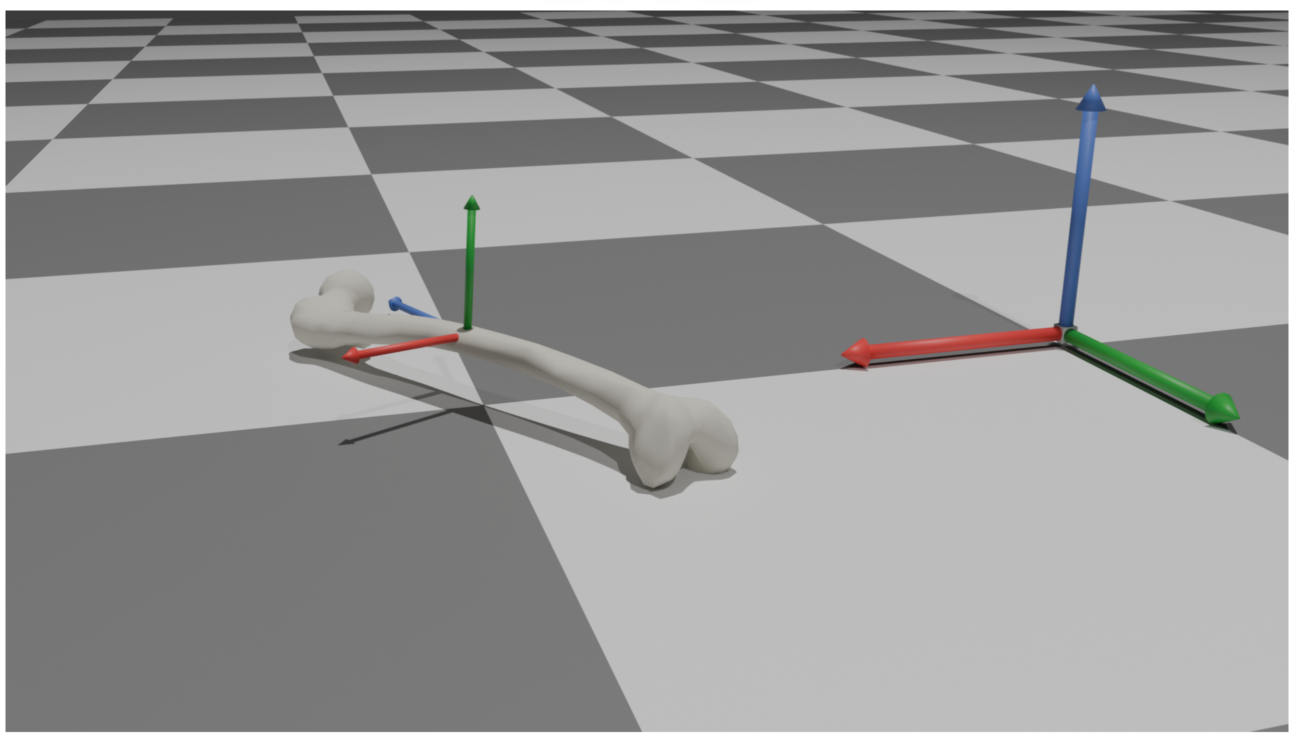


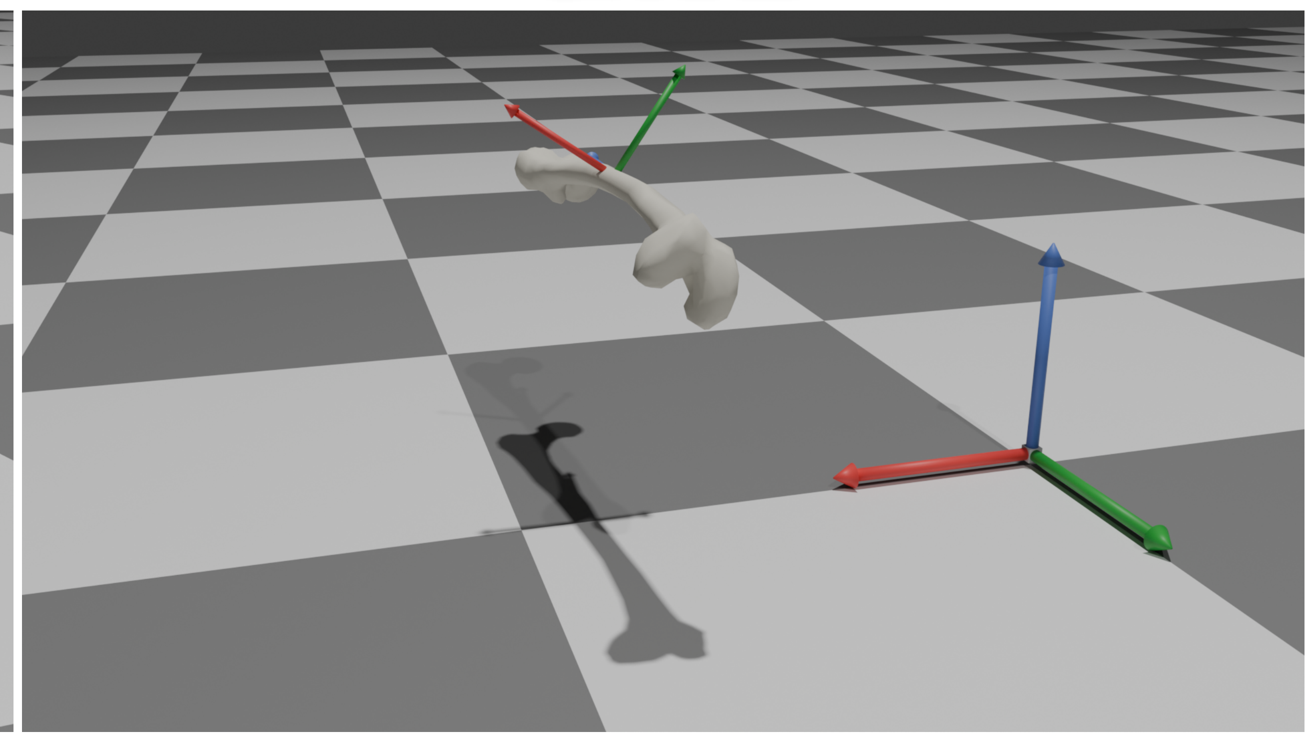


Transformation Matrix

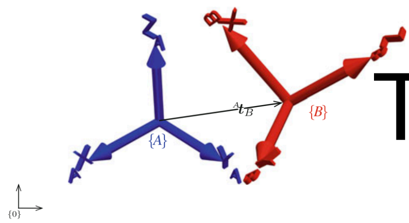
Local



Global



A Rotation Through ***an Angle -30 Degrees*** About y-axis



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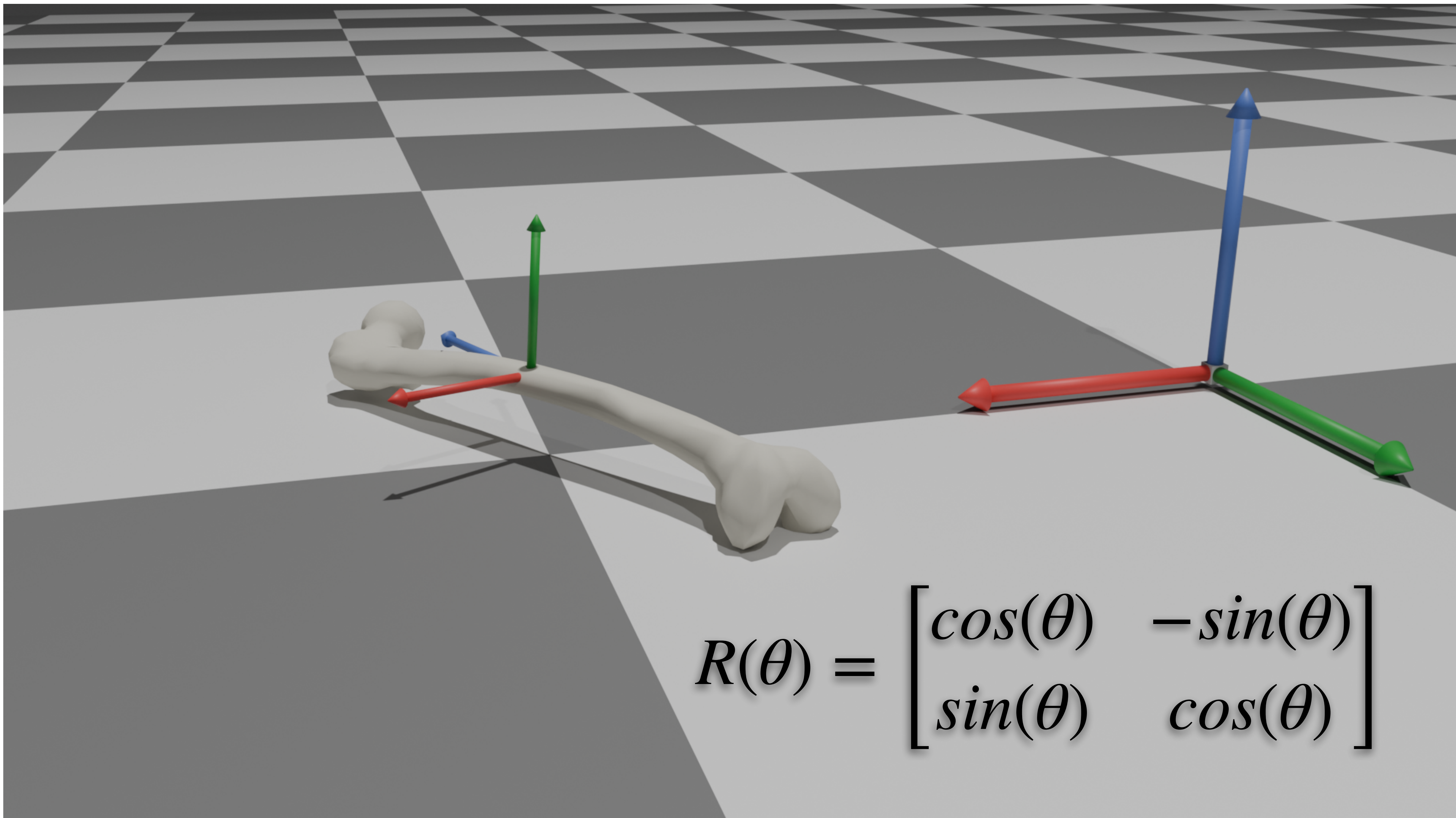
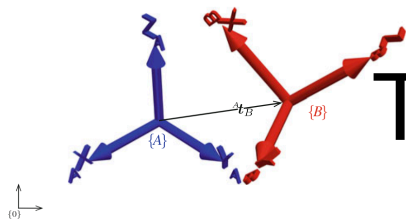


$$R(\theta) = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$



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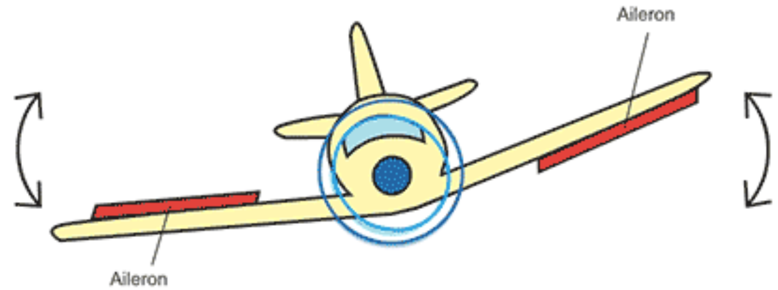
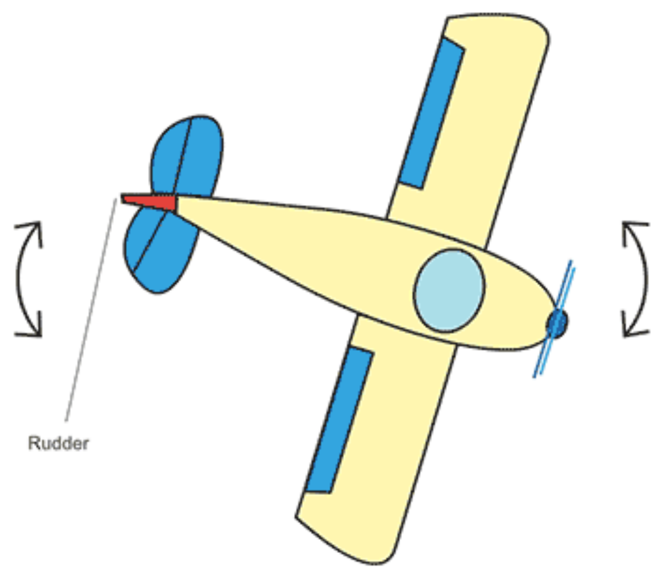
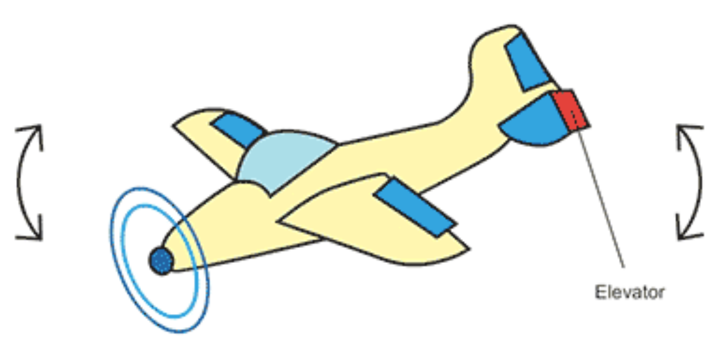


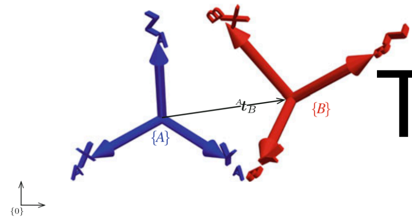
Use the ailerons to control
Roll



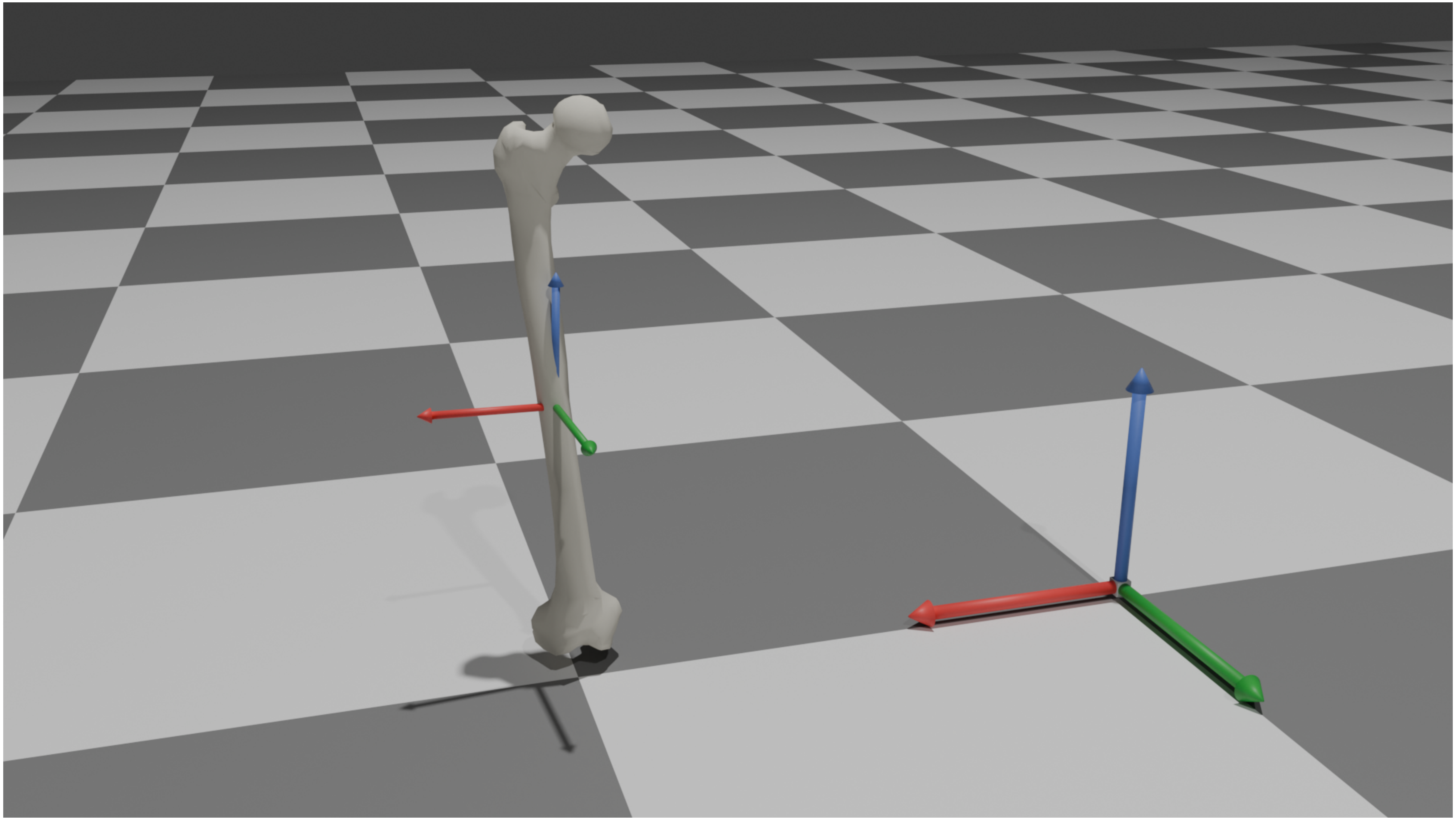
Use the rudder to control
Yaw



Use the elevator to control
Pitch



Transformation Matrix



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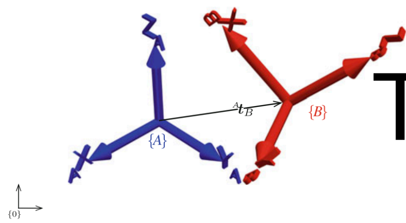

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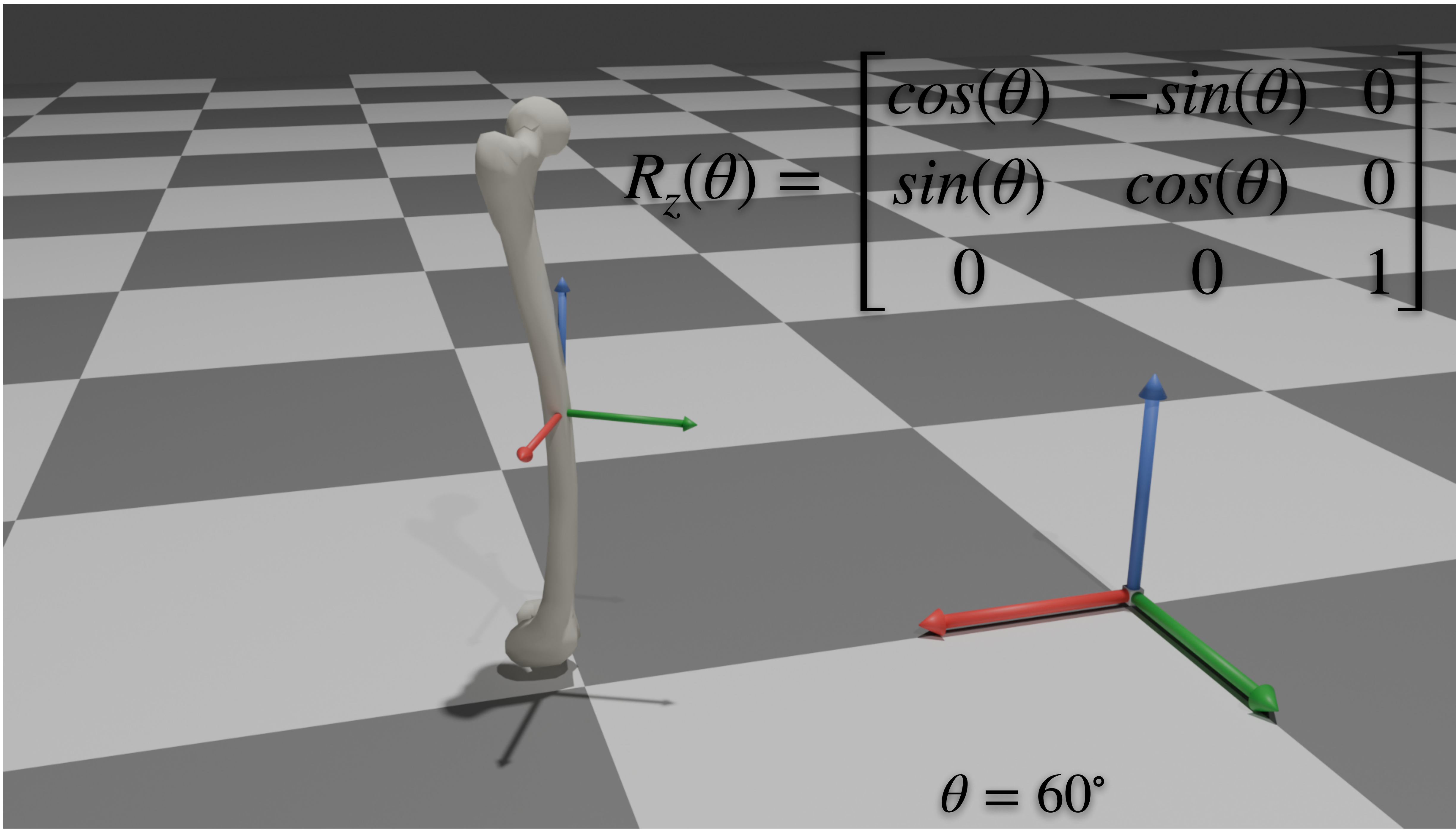

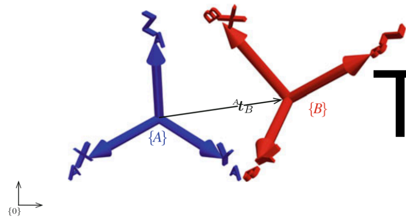

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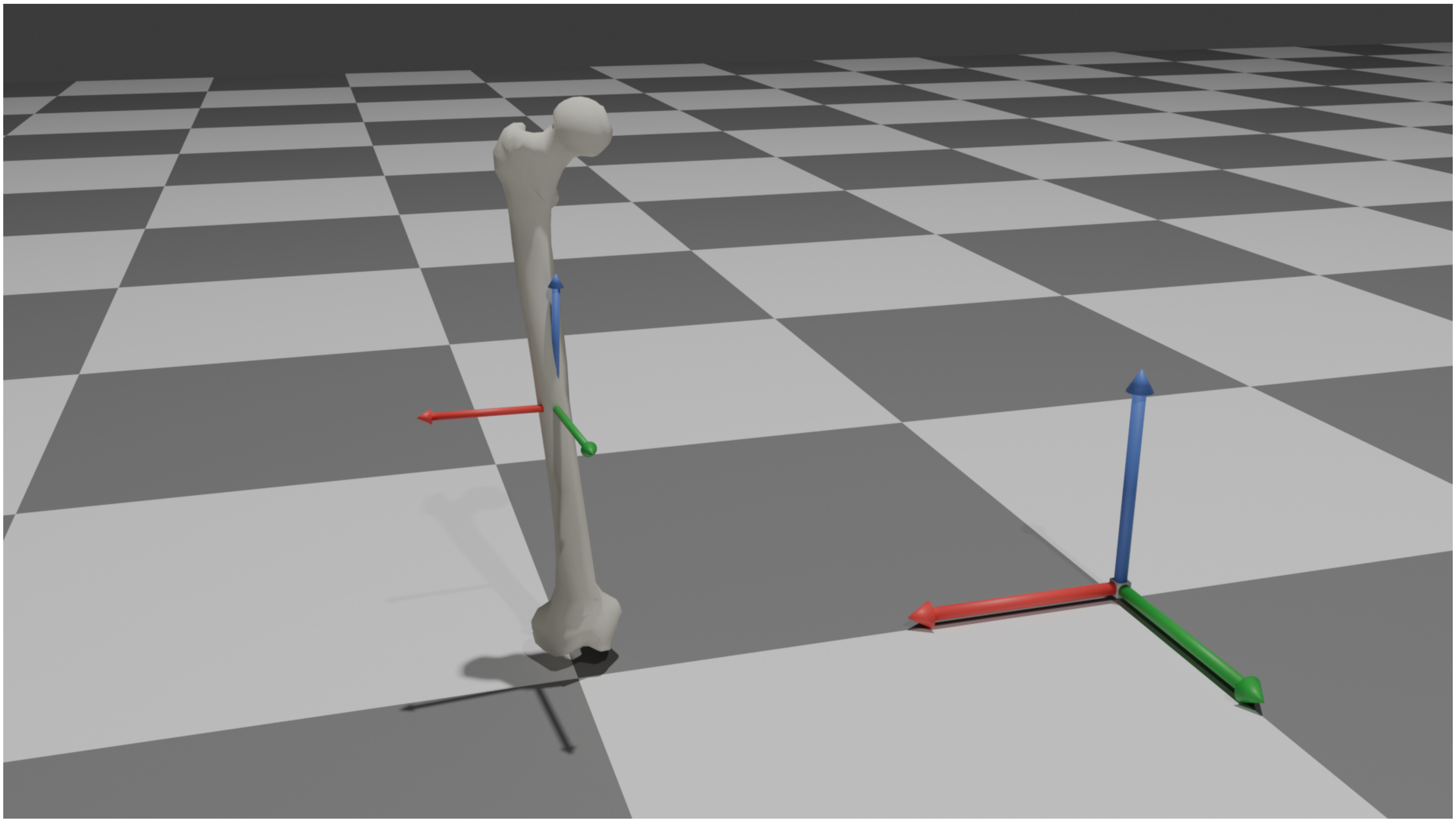

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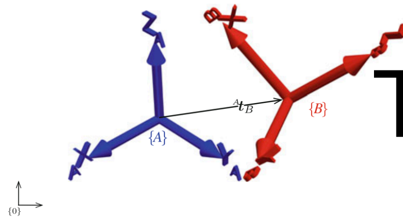

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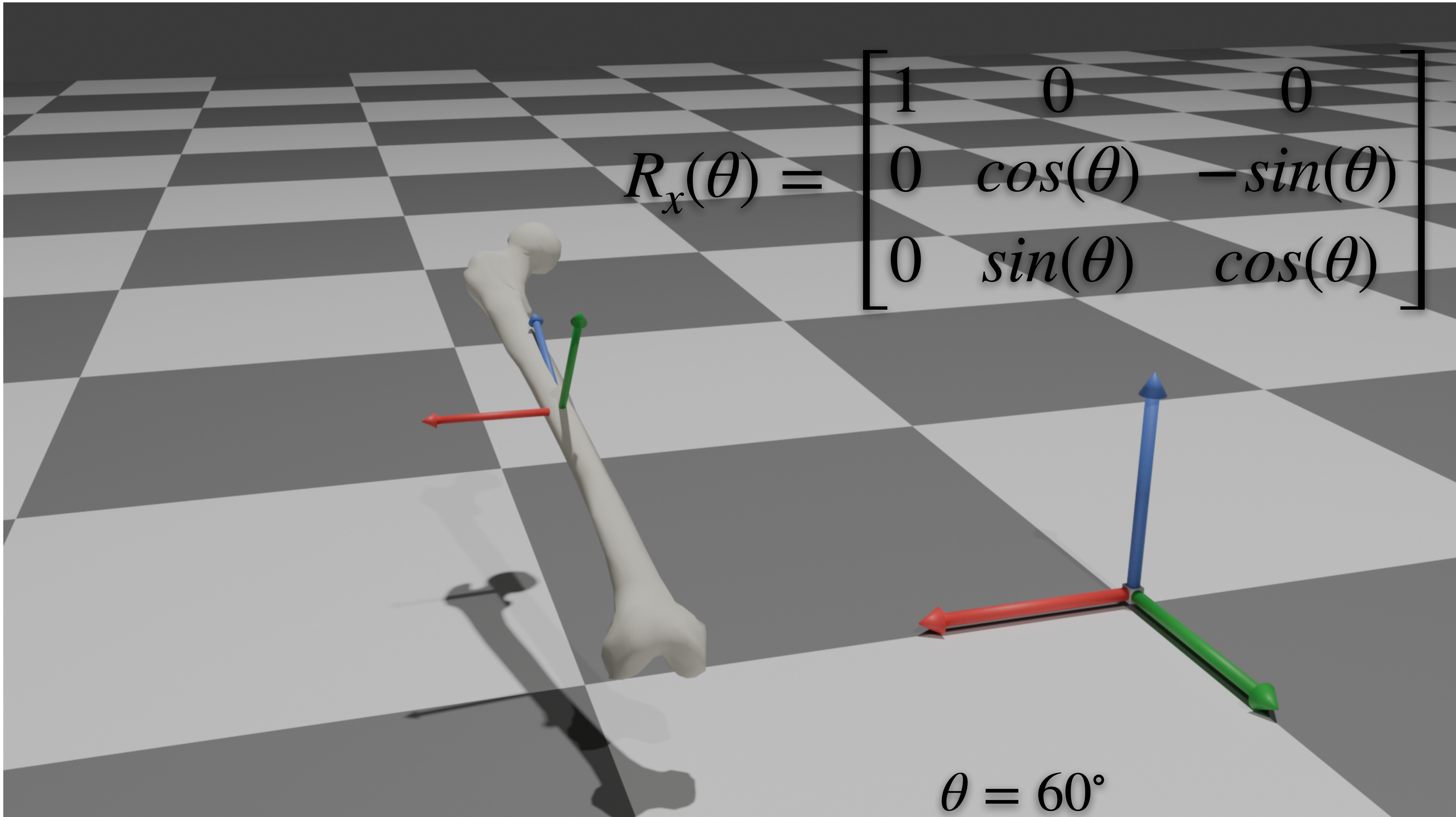

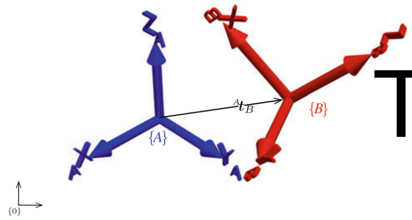

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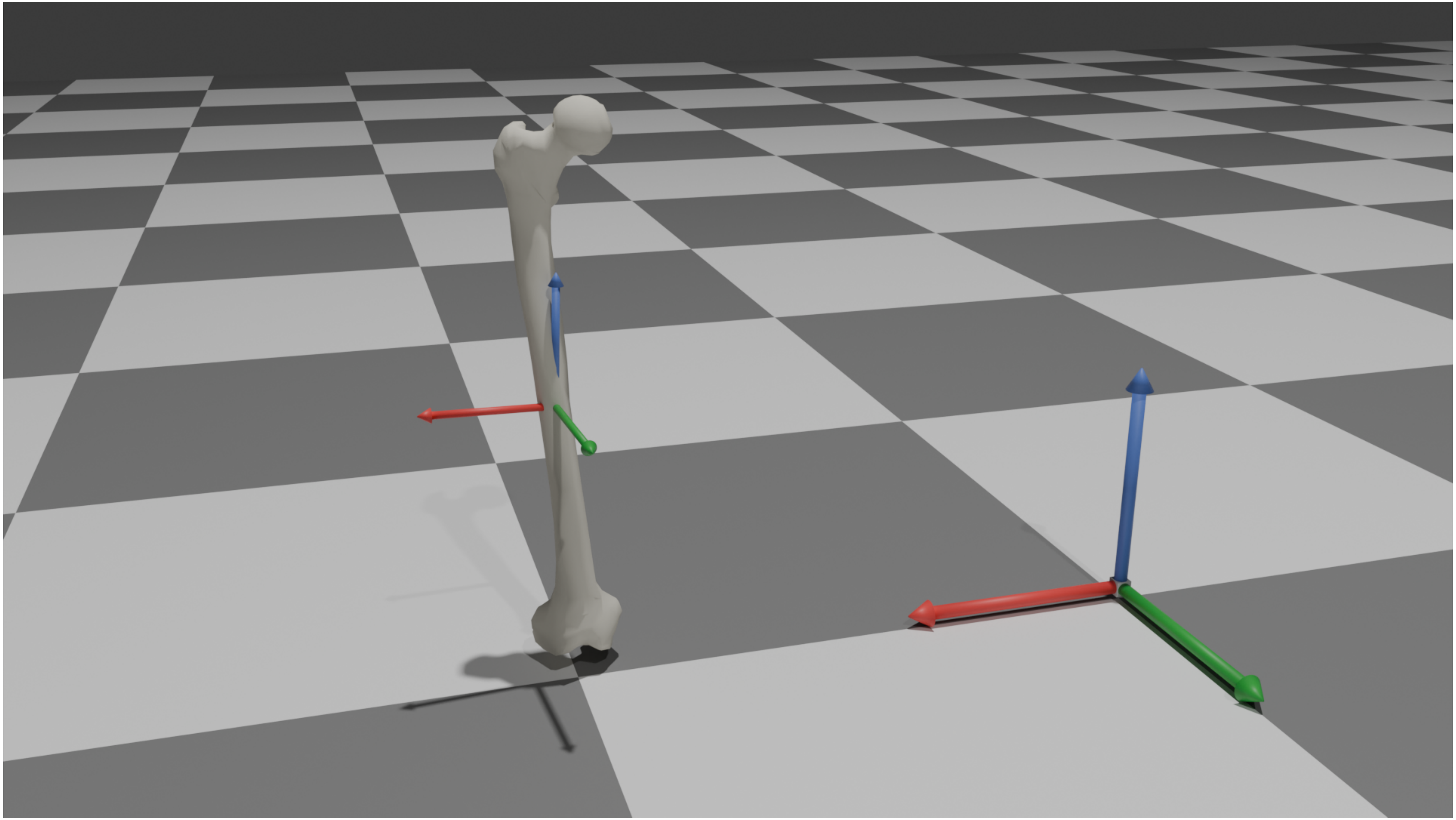

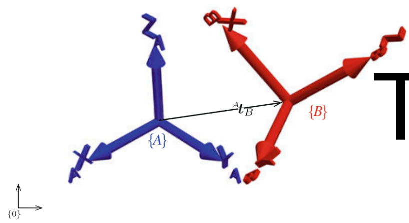

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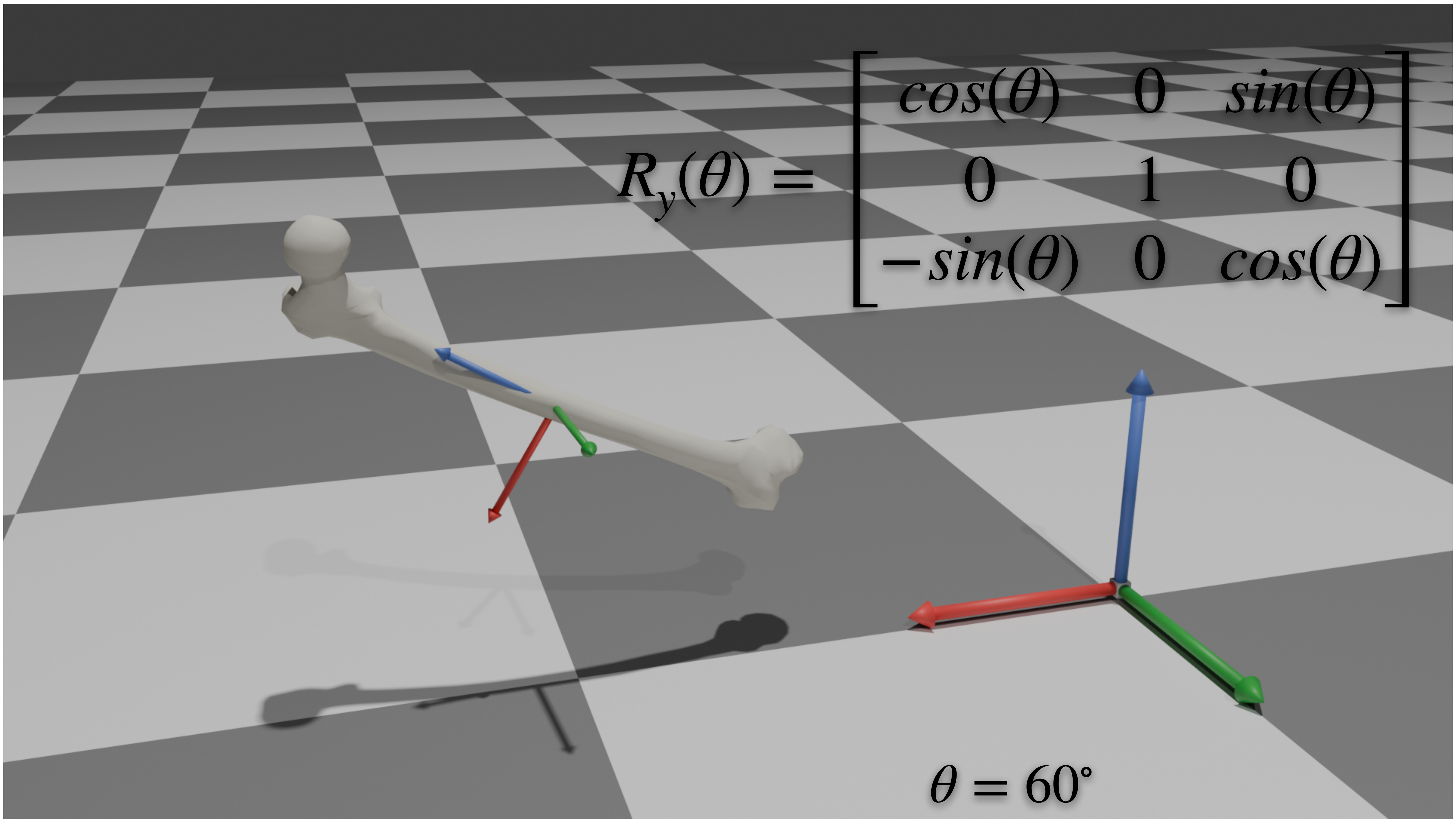
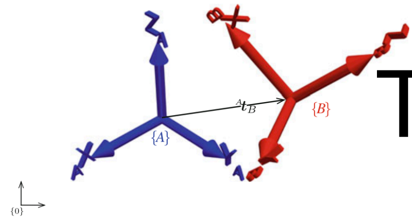




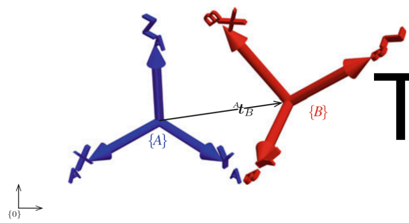
Transformation Matrix

yaw	pitch	roll

$$R_z(\alpha) = \begin{bmatrix} cos(\alpha) & -sin(\alpha) & 0 \\ sin(\alpha) & cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_y(\beta) = \begin{bmatrix} cos(\beta) & 0 & sin(\beta) \\ 0 & 1 & 0 \\ -sin(\beta) & 0 & cos(\beta) \end{bmatrix} \quad R_x(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\gamma) & -sin(\gamma) \\ 0 & sin(\gamma) & cos(\gamma) \end{bmatrix}$$

$$R = R_z(\alpha) \times R_y(\beta) \times R_x(\gamma)$$

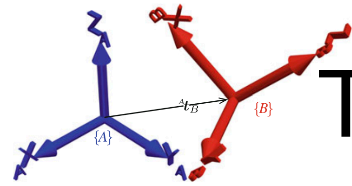
3D rotation matrices can be obtained from these three using matrix multiplication



Transformation Matrix

$$R = \begin{bmatrix} cos(\alpha)cos(\beta) & cos(\alpha)sin(\beta)sin(\gamma) - sin(\alpha)cos(\gamma) & cos(\alpha)sin(\beta)cos(\gamma) + sin(\alpha)sin(\gamma) \\ sin(\alpha)cos(\beta) & sin(\alpha)sin(\beta)sin(\gamma) + cos(\alpha)cos(\gamma) & sin(\alpha)sin(\beta)cos(\gamma) - cos(\alpha)sin(\gamma) \\ -sin(\beta) & cos(\beta)sin(\gamma) & cos(\beta)cos(\gamma) \end{bmatrix}$$

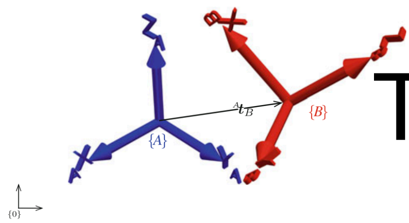
Represents a rotation whose yaw, pitch and roll angels are α, β ang γ, respectively.



Transformation Matrix

Tait–Bryan angles are also called **Cardan angles**

Proper Euler angles	Tait–Bryan angles
$X_\alpha Z_\beta X_\gamma = \begin{bmatrix} c_\beta & -c_\gamma s_\beta & s_\beta s_\gamma \\ c_\alpha s_\beta & c_\alpha c_\beta c_\gamma - s_\alpha s_\gamma & -c_\gamma s_\alpha - c_\alpha c_\beta s_\gamma \\ s_\alpha s_\beta & c_\alpha s_\gamma + c_\beta c_\gamma s_\alpha & c_\alpha c_\gamma - c_\beta s_\alpha s_\gamma \end{bmatrix}$	$X_\alpha Z_\beta Y_\gamma = \begin{bmatrix} c_\beta c_\gamma & -s_\beta & c_\beta s_\gamma \\ s_\alpha s_\gamma + c_\alpha c_\gamma s_\beta & c_\alpha c_\beta & c_\alpha s_\beta s_\gamma - c_\gamma s_\alpha \\ c_\gamma s_\alpha s_\beta - c_\alpha s_\gamma & c_\beta s_\alpha & c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma \end{bmatrix}$
$X_\alpha Y_\beta X_\gamma = \begin{bmatrix} c_\beta & s_\beta s_\gamma & c_\gamma s_\beta \\ s_\alpha s_\beta & c_\alpha c_\gamma - c_\beta s_\alpha s_\gamma & -c_\alpha s_\gamma - c_\beta c_\gamma s_\alpha \\ -c_\alpha s_\beta & c_\gamma s_\alpha + c_\alpha c_\beta s_\gamma & c_\alpha c_\beta c_\gamma - s_\alpha s_\gamma \end{bmatrix}$	$X_\alpha Y_\beta Z_\gamma = \begin{bmatrix} c_\beta c_\gamma & -c_\beta s_\gamma & s_\beta \\ c_\alpha s_\gamma + c_\gamma s_\alpha s_\beta & c_\alpha c_\gamma - s_\alpha s_\beta s_\gamma & -c_\beta s_\alpha \\ s_\alpha s_\gamma - c_\alpha c_\gamma s_\beta & c_\gamma s_\alpha + c_\alpha s_\beta s_\gamma & c_\alpha c_\beta \end{bmatrix}$
$Y_\alpha X_\beta Y_\gamma = \begin{bmatrix} c_\alpha c_\gamma - c_\beta s_\alpha s_\gamma & s_\alpha s_\beta & c_\alpha s_\gamma + c_\beta c_\gamma s_\alpha \\ s_\beta s_\gamma & c_\beta & -c_\gamma s_\beta \\ -c_\gamma s_\alpha - c_\alpha c_\beta s_\gamma & c_\alpha s_\beta & c_\alpha c_\beta c_\gamma - s_\alpha s_\gamma \end{bmatrix}$	$Y_\alpha X_\beta Z_\gamma = \begin{bmatrix} c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma & c_\gamma s_\alpha s_\beta - c_\alpha s_\gamma & c_\beta s_\alpha \\ c_\beta s_\gamma & c_\beta c_\gamma & -s_\beta \\ c_\alpha s_\beta s_\gamma - c_\gamma s_\alpha & c_\alpha c_\gamma s_\beta + s_\alpha s_\gamma & c_\alpha c_\beta \end{bmatrix}$
$Y_\alpha Z_\beta Y_\gamma = \begin{bmatrix} c_\alpha c_\beta c_\gamma - s_\alpha s_\gamma & -c_\alpha s_\beta & c_\gamma s_\alpha + c_\alpha c_\beta s_\gamma \\ c_\gamma s_\beta & c_\beta & s_\beta s_\gamma \\ -c_\alpha s_\gamma - c_\beta c_\gamma s_\alpha & s_\alpha s_\beta & c_\alpha c_\gamma - c_\beta s_\alpha s_\gamma \end{bmatrix}$	$Y_\alpha Z_\beta X_\gamma = \begin{bmatrix} c_\alpha c_\beta & s_\alpha s_\gamma - c_\alpha c_\gamma s_\beta & c_\gamma s_\alpha + c_\alpha s_\beta s_\gamma \\ s_\beta & c_\beta c_\gamma & -c_\beta s_\gamma \\ -c_\beta s_\alpha & c_\alpha s_\gamma + c_\gamma s_\alpha s_\beta & c_\alpha c_\gamma - s_\alpha s_\beta s_\gamma \end{bmatrix}$
$Z_\alpha Y_\beta Z_\gamma = \begin{bmatrix} c_\alpha c_\beta c_\gamma - s_\alpha s_\gamma & -c_\gamma s_\alpha - c_\alpha c_\beta s_\gamma & c_\alpha s_\beta \\ c_\alpha s_\gamma + c_\beta c_\gamma s_\alpha & c_\alpha c_\gamma - c_\beta s_\alpha s_\gamma & s_\alpha s_\beta \\ -c_\gamma s_\beta & s_\beta s_\gamma & c_\beta \end{bmatrix}$	$\boxed{Z_\alpha Y_\beta X_\gamma = \begin{bmatrix} c_\alpha c_\beta & c_\alpha s_\beta s_\gamma - c_\gamma s_\alpha & s_\alpha s_\gamma + c_\alpha c_\gamma s_\beta \\ c_\beta s_\alpha & c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma & c_\gamma s_\alpha s_\beta - c_\alpha s_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix}}$
$Z_\alpha X_\beta Z_\gamma = \begin{bmatrix} c_\alpha c_\gamma - c_\beta s_\alpha s_\gamma & -c_\alpha s_\gamma - c_\beta c_\gamma s_\alpha & s_\alpha s_\beta \\ c_\gamma s_\alpha + c_\alpha c_\beta s_\gamma & c_\alpha c_\beta c_\gamma - s_\alpha s_\gamma & -c_\alpha s_\beta \\ s_\beta s_\gamma & c_\gamma s_\beta & c_\beta \end{bmatrix}$	$Z_\alpha X_\beta Y_\gamma = \begin{bmatrix} c_\alpha c_\gamma - s_\alpha s_\beta s_\gamma & -c_\beta s_\alpha & c_\alpha s_\gamma + c_\gamma s_\alpha s_\beta \\ c_\gamma s_\alpha + c_\alpha s_\beta s_\gamma & c_\alpha c_\beta & s_\alpha s_\gamma - c_\alpha c_\gamma s_\beta \\ -c_\beta s_\gamma & s_\beta & c_\beta c_\gamma \end{bmatrix}$



Transformation Matrix

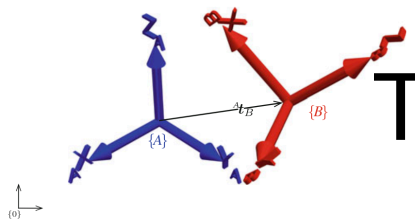
The table contains formulas for angles α, β and γ from elements of a rotation matrix R.

Without considering the possibility of using two different conventions for the definition of the rotation axes (intrinsic or extrinsic), there exist twelve possible sequences of rotation axes, divided in two groups:

Proper Euler angles (*z-x-z*, *x-y-x*, *y-z-y*, *z-y-z*, *x-z-x*, *y-x-y*)

Tait–Bryan angles (*x-y-z*, *y-z-x*, *z-x-y*, *x-z-y*, *z-y-x*, *y-x-z*).

Proper Euler angles		Tait–Bryan angles	
$X_\alpha Z_\beta X_\gamma$	$\alpha = \arctan\left(\frac{R_{31}}{R_{21}}\right)$ $\beta = \arccos(R_{11})$ $\gamma = \arctan\left(\frac{R_{13}}{-R_{12}}\right)$	$X_\alpha Z_\beta Y_\gamma$	$\alpha = \arctan\left(\frac{R_{32}}{R_{22}}\right)$ $\beta = \arcsin(-R_{12})$ $\gamma = \arctan\left(\frac{R_{13}}{R_{11}}\right)$
$X_\alpha Y_\beta X_\gamma$	$\alpha = \arctan\left(\frac{R_{21}}{-R_{31}}\right)$ $\beta = \arccos(R_{11})$ $\gamma = \arctan\left(\frac{R_{12}}{R_{13}}\right)$	$X_\alpha Y_\beta Z_\gamma$	$\alpha = \arctan\left(\frac{-R_{23}}{R_{33}}\right)$ $\beta = \arcsin(R_{13})$ $\gamma = \arctan\left(\frac{-R_{12}}{R_{11}}\right)$
$Y_\alpha X_\beta Y_\gamma$	$\alpha = \arctan\left(\frac{R_{12}}{R_{32}}\right)$ $\beta = \arccos(R_{22})$ $\gamma = \arctan\left(\frac{R_{21}}{-R_{23}}\right)$	$Y_\alpha X_\beta Z_\gamma$	$\alpha = \arctan\left(\frac{R_{13}}{R_{33}}\right)$ $\beta = \arcsin(-R_{23})$ $\gamma = \arctan\left(\frac{R_{21}}{R_{22}}\right)$
$Y_\alpha Z_\beta Y_\gamma$	$\alpha = \arctan\left(\frac{R_{32}}{-R_{12}}\right)$ $\beta = \arccos(R_{22})$ $\gamma = \arctan\left(\frac{R_{23}}{R_{21}}\right)$	$Y_\alpha Z_\beta X_\gamma$	$\alpha = \arctan\left(\frac{-R_{31}}{R_{11}}\right)$ $\beta = \arcsin(R_{21})$ $\gamma = \arctan\left(\frac{-R_{23}}{R_{22}}\right)$
$Z_\alpha Y_\beta Z_\gamma$	$\alpha = \arctan\left(\frac{R_{23}}{R_{13}}\right)$ $\beta = \arctan\left(\frac{\sqrt{1-R_{33}^2}}{R_{33}}\right)$ $\gamma = \arctan\left(\frac{R_{32}}{-R_{31}}\right)$	$Z_\alpha Y_\beta X_\gamma$	$\alpha = \arctan\left(\frac{R_{21}}{R_{11}}\right)$ $\beta = \arcsin(-R_{31})$ $\gamma = \arctan\left(\frac{R_{32}}{R_{33}}\right)$
$Z_\alpha X_\beta Z_\gamma$	$\alpha = \arctan\left(\frac{R_{13}}{-R_{23}}\right)$ $\beta = \arccos(R_{33})$ $\gamma = \arctan\left(\frac{R_{31}}{R_{32}}\right)$	$Z_\alpha X_\beta Y_\gamma$	$\alpha = \arctan\left(\frac{-R_{12}}{R_{22}}\right)$ $\beta = \arcsin(R_{32})$ $\gamma = \arctan\left(\frac{-R_{31}}{R_{33}}\right)$



Transformation Matrix

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Letter to the editor

ISB recommendation on definitions of joint coordinate system of various joints for the reporting of human joint motion—part I: ankle, hip, and spine

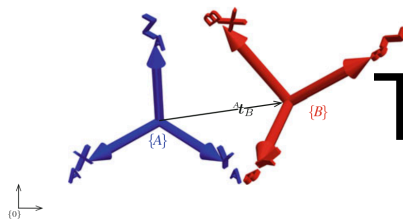
Abstract

The Standardization and Terminology Committee (STC) of the International Society of Biomechanics (ISB) proposes a general reporting standard for joint kinematics based on the Joint Coordinate System (JCS), first proposed by Grood and Suntay for the knee joint in 1983 (J. Biomech. Eng. 105 (1983) 136). There is currently a lack of standard for reporting joint motion in the field of biomechanics for human movement, and the JCS as proposed by Grood and Suntay has the advantage of reporting joint motions in clinically relevant terms.

In this communication, the STC proposes definitions of JCS for the ankle, hip, and spine. Definitions for other joints (such as shoulder, elbow, hand and wrist, temporomandibular joint (TMJ), and whole body) will be reported in later parts of the series. The STC is publishing these recommendations so as to encourage their use, to stimulate feedback and discussion, and to facilitate further revisions.

For each joint, a standard for the local axis system in each articulating bone is generated. These axes then standardize the JCS. Adopting these standards will lead to better communication among researchers and clinicians.

HAB718 Spor Biyomekaniğinde Hareket Analizi



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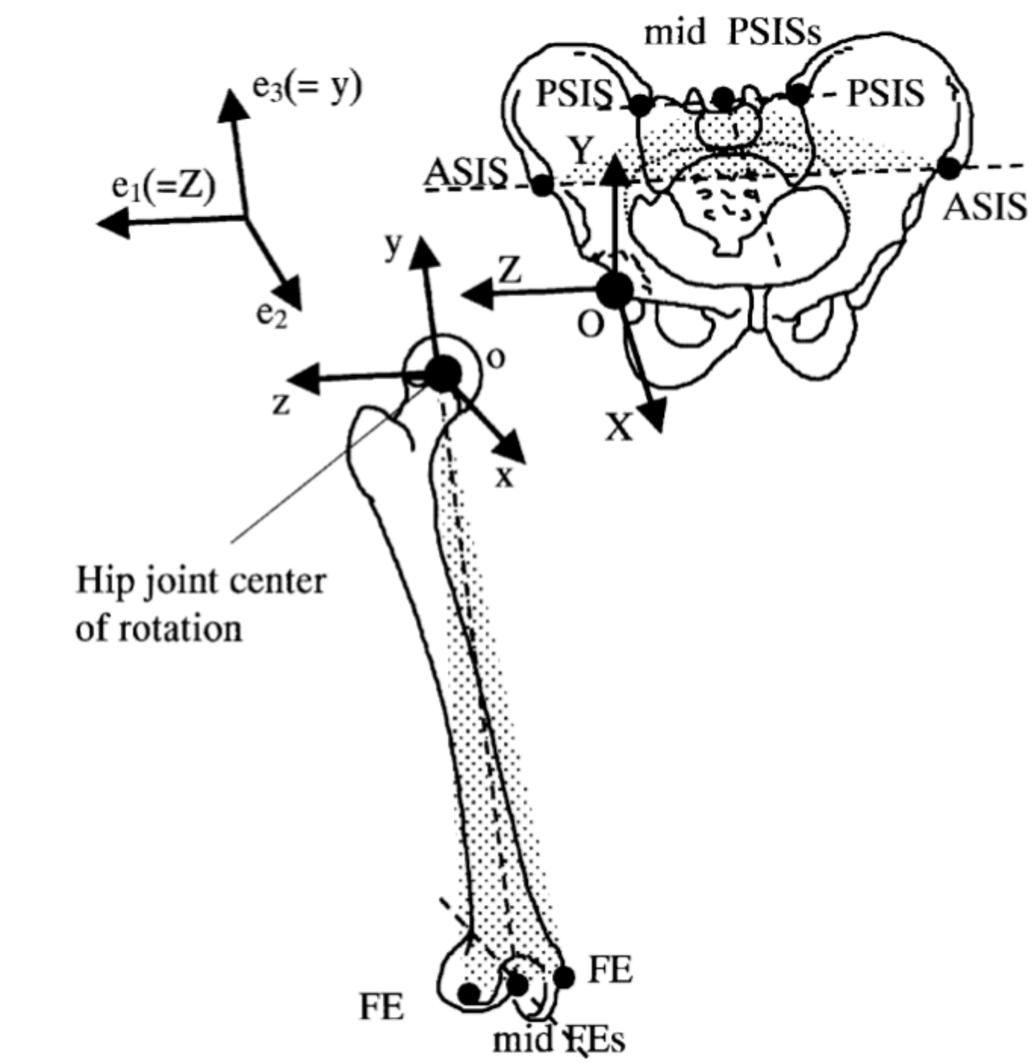


Fig. 3. Illustration of the pelvic coordinate system (*XYZ*), femoral coordinate system (*xyz*), and the JCS for the right hip joint.

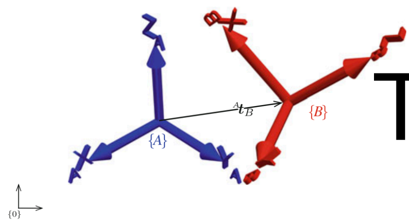
4.4. *Femoral coordinate system—xyz (Fig. 3)*

o: The origin coincident with the right (or left) hip center of rotation, coincident with that of the pelvic coordinate system (O) in the neutral configuration.

y: The line joining the midpoint between the medial and lateral FEs and the origin, and pointing cranially.

z: The line perpendicular to the *y*-axis, lying in the plane defined by the origin and the two FEs, pointing to the right.

x: The line perpendicular to both *y*- and *z*-axis, pointing anteriorly (Cappozzo et al., 1995).



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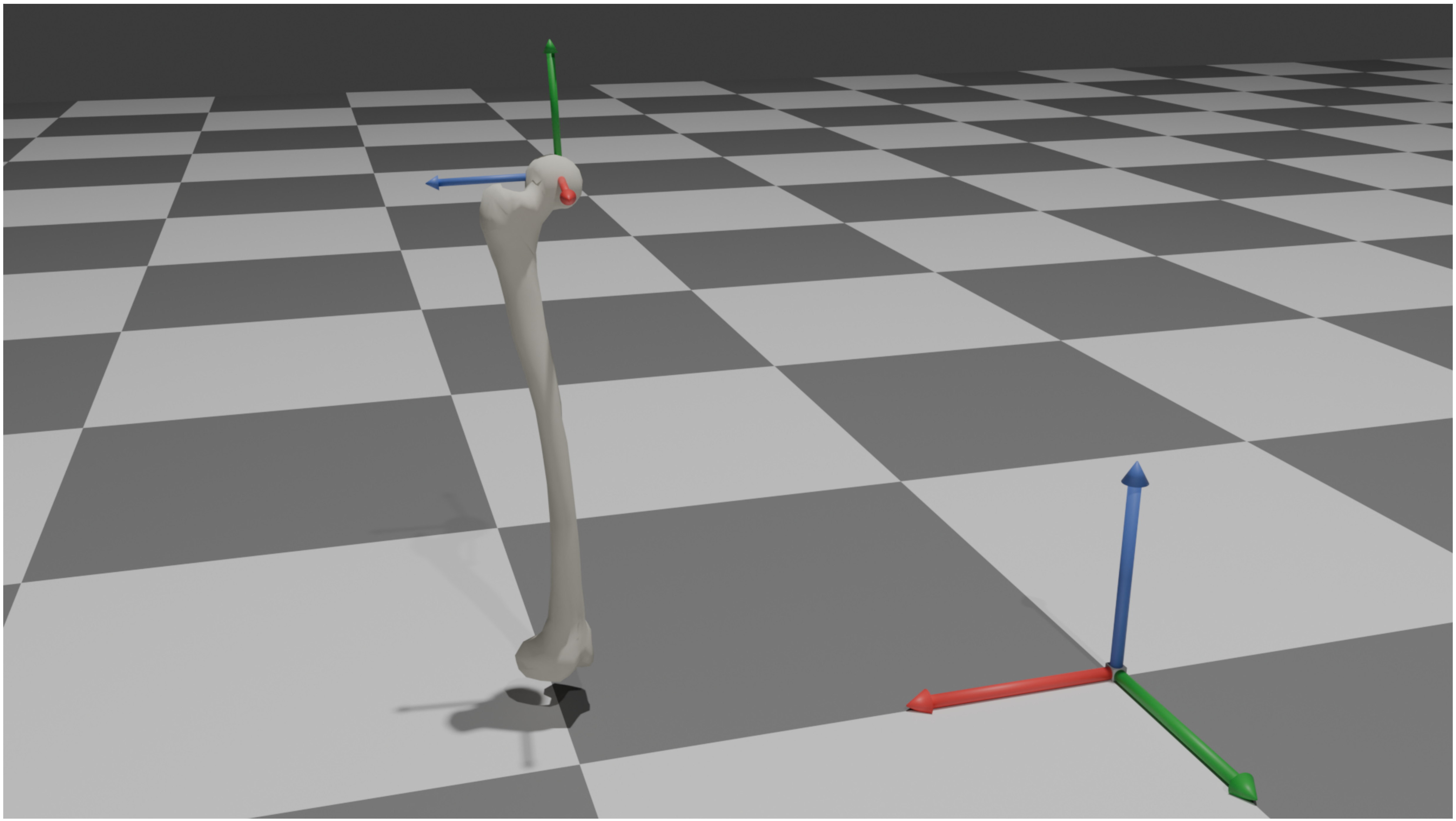

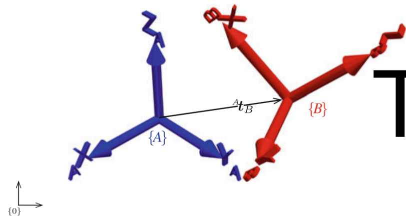

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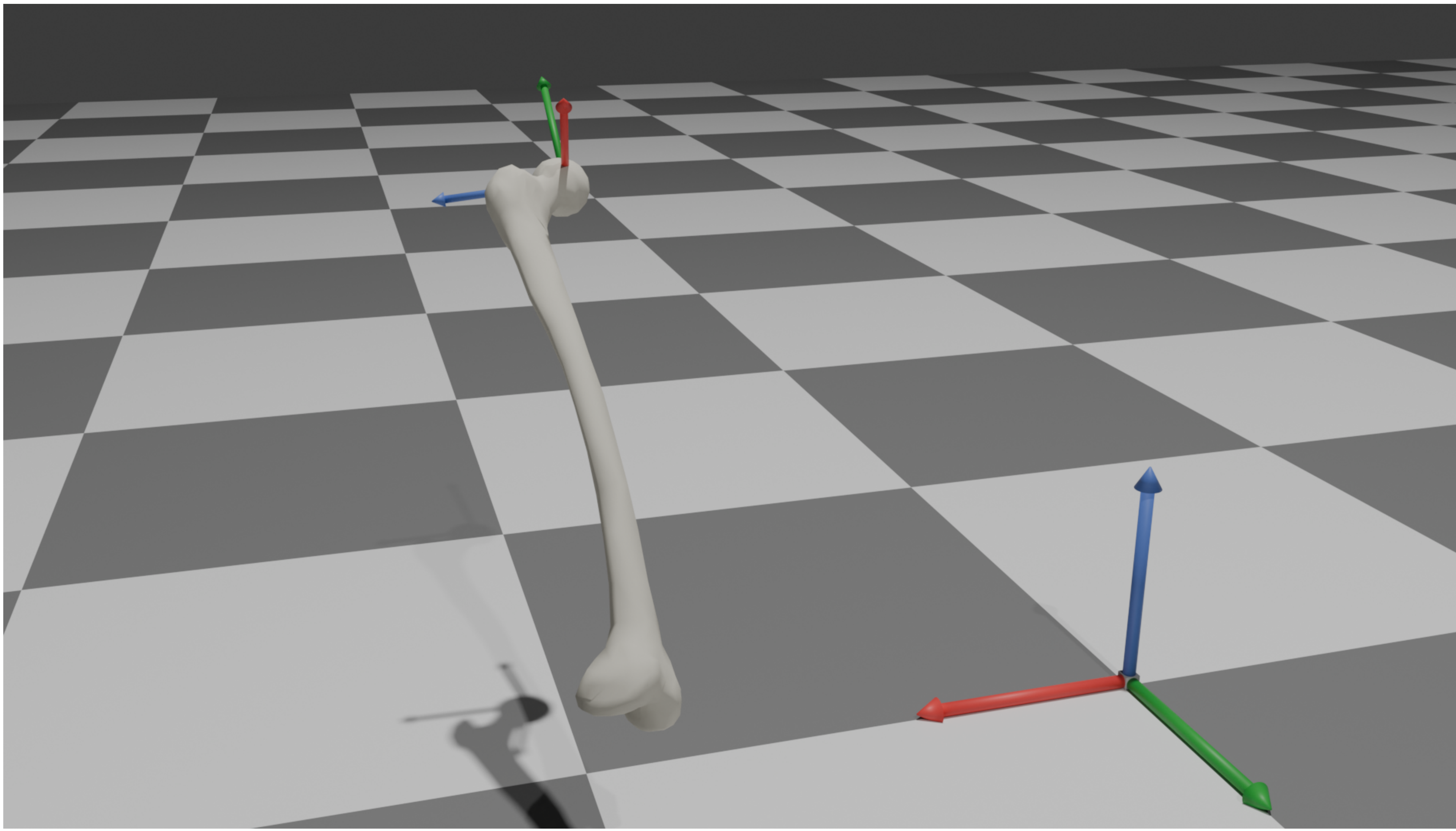

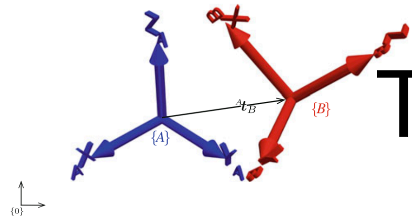

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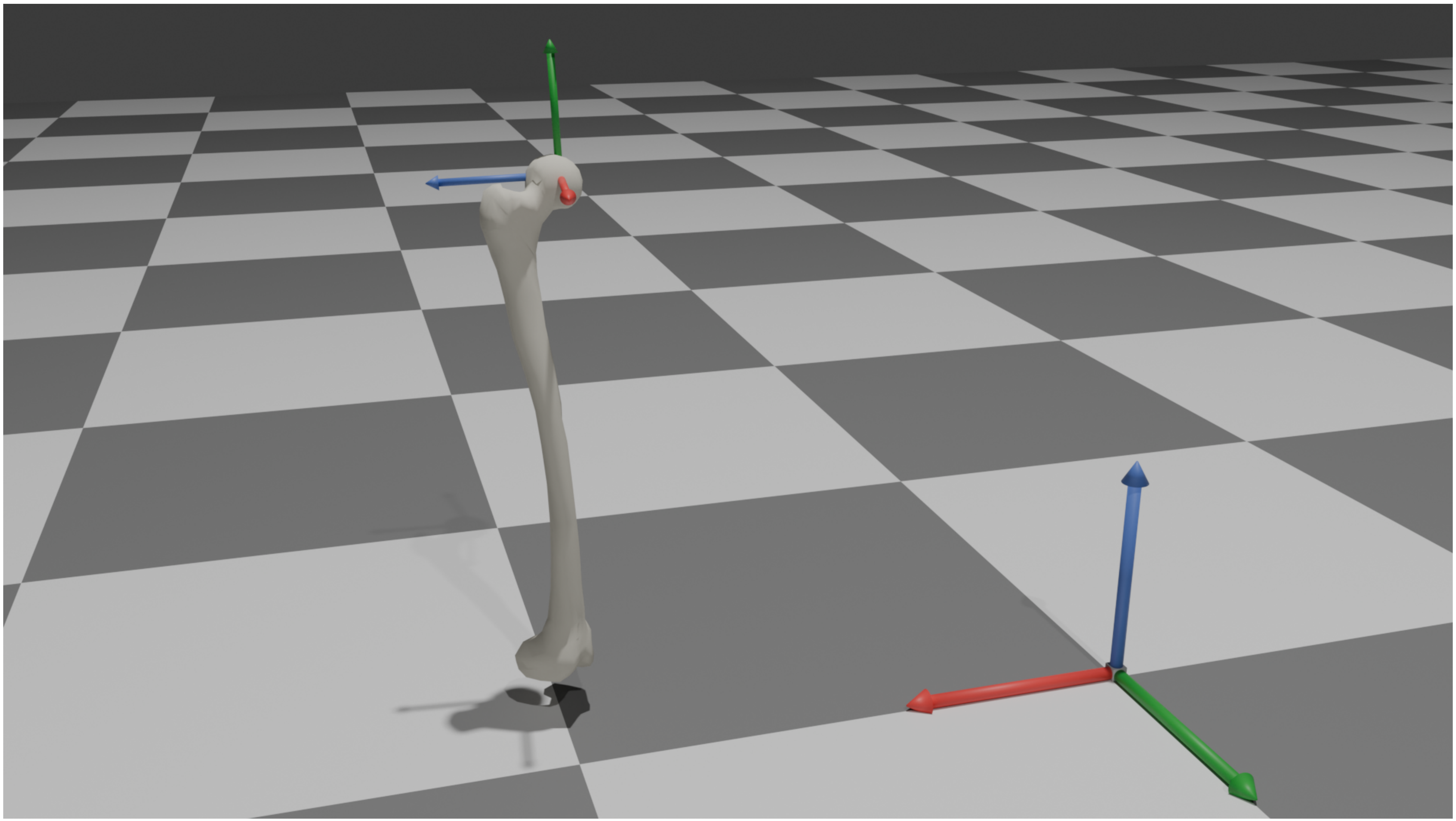

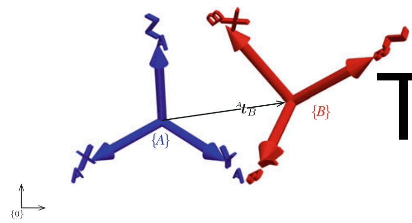

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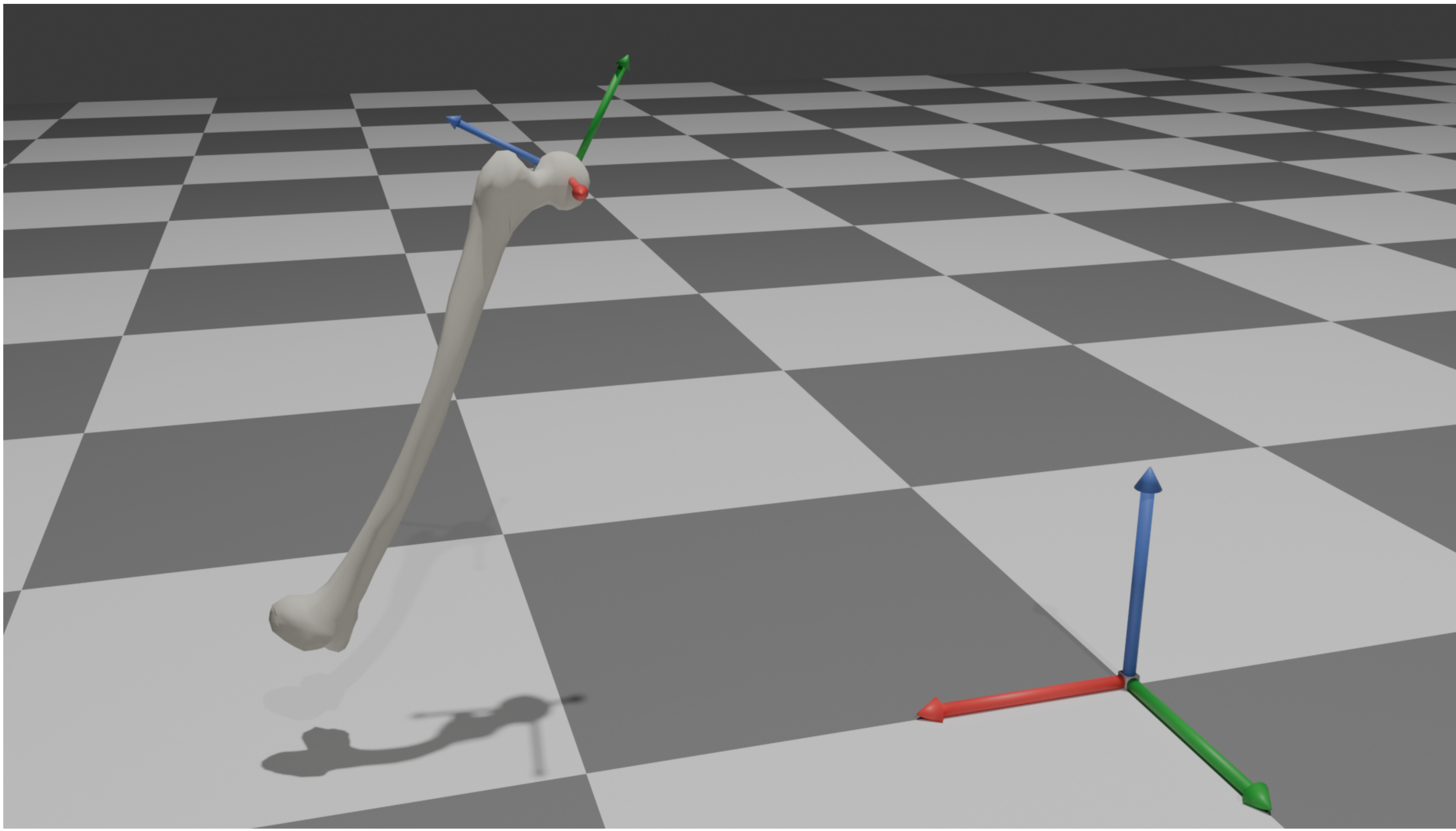

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ISB RECOMMENDATIONS FOR STANDARDIZATION IN THE REPORTING OF KINEMATIC DATA

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Letter to the editor

ISB recommendation on definition of joint coordinate systems for the reporting of human joint motion—part I: ankle, hip and spine

consistent results, only one choice of angles for each joint gives results that can be reconciled with conventional clinical terminology. Even this approach depends