HAB718 Spor Biyomekaniğinde Hareket Analizi

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- Transformation Matrix
- Three-Dimensional Kinematics

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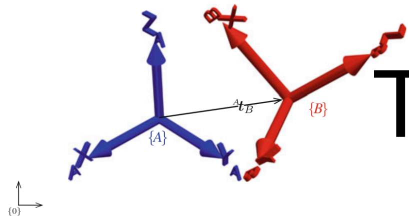
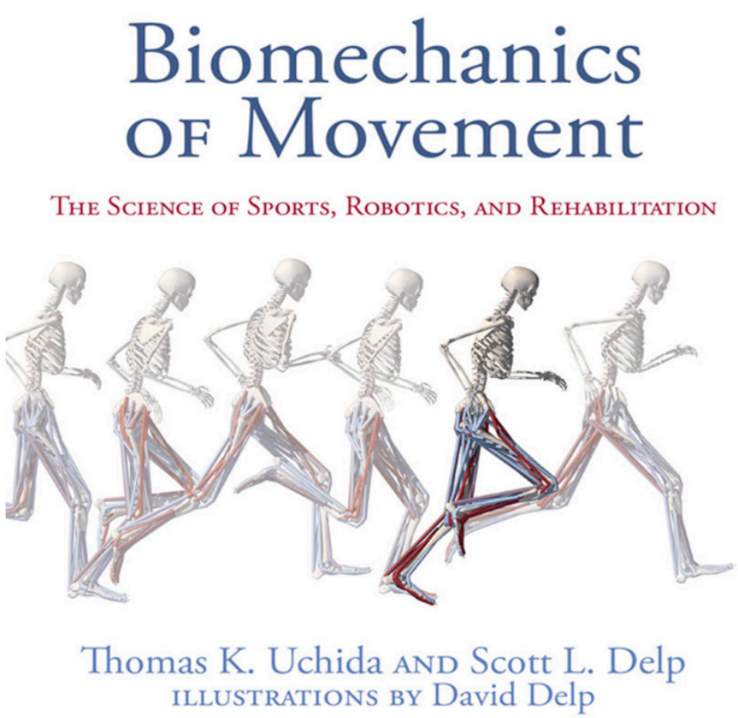


Transformation Matrix



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- Language : English
- Hardcover : 400 pages
- ISBN-10 : 0253330580
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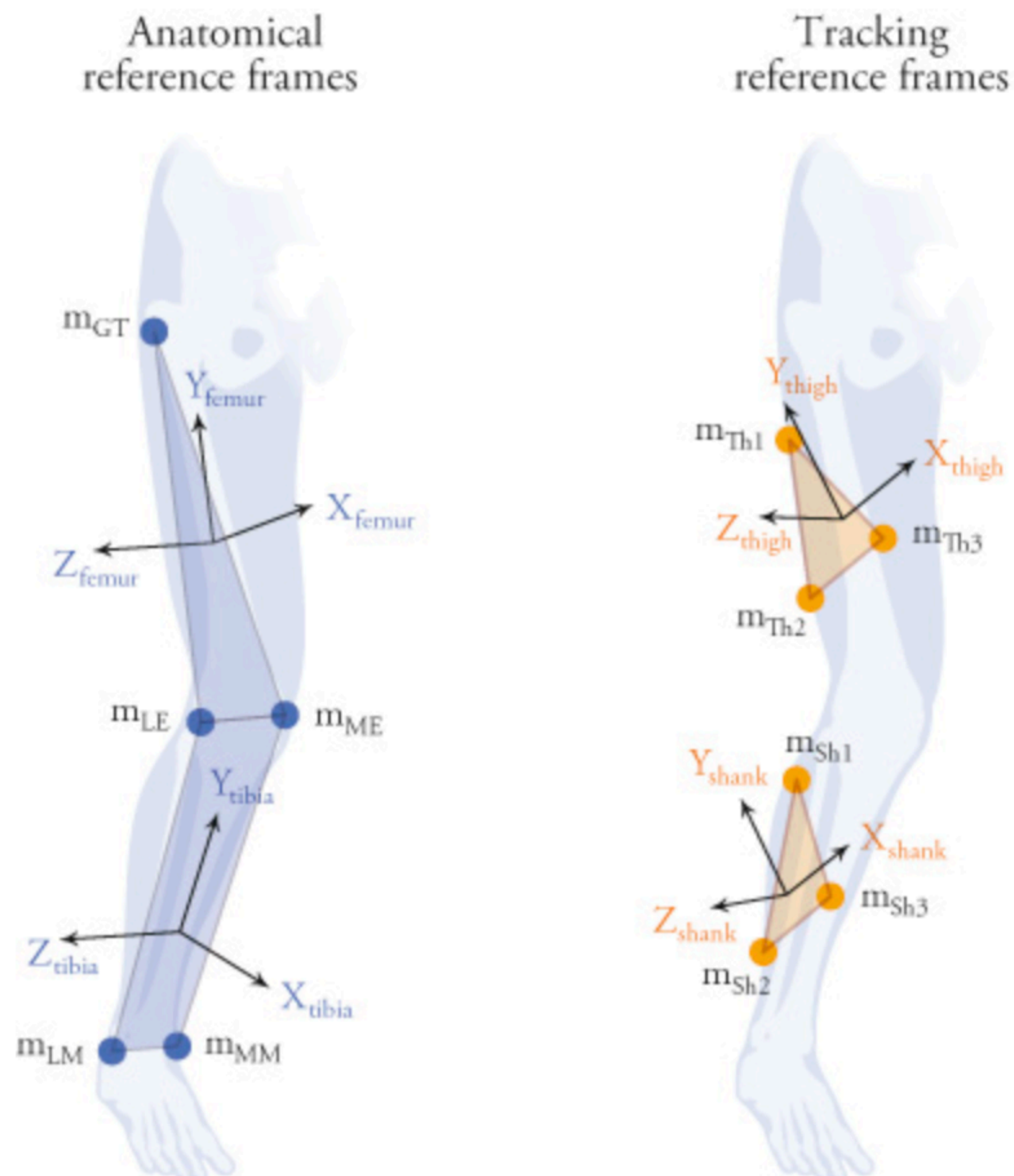
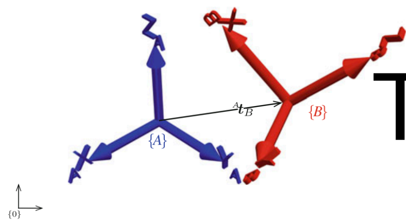


Figure 7.11 Body-fixed reference frames determined from markers mounted on anatomical landmarks (anatomical reference frames) and markers mounted on body segments (tracking reference frames); m = marker, GT = greater trochanter, ME/LE = medial/lateral femoral epicondyle, MM/LM = medial/lateral malleolus, Th = thigh, Sh = shank.



Transformation Matrix

The number and locations of markers can vary depending on the study objectives, **the motion being tracked, and the software used for analysis**. It is common to use <u>two sets of markers</u>, one to define anatomical reference frames and one to define tracking reference frames (Figure 7.11). Anatomical reference frames represent the underlying skeletal structure and are defined by placing markers on anatomical landmarks. For example, the anatomical reference frame fixed to the femur might be defined as follows:

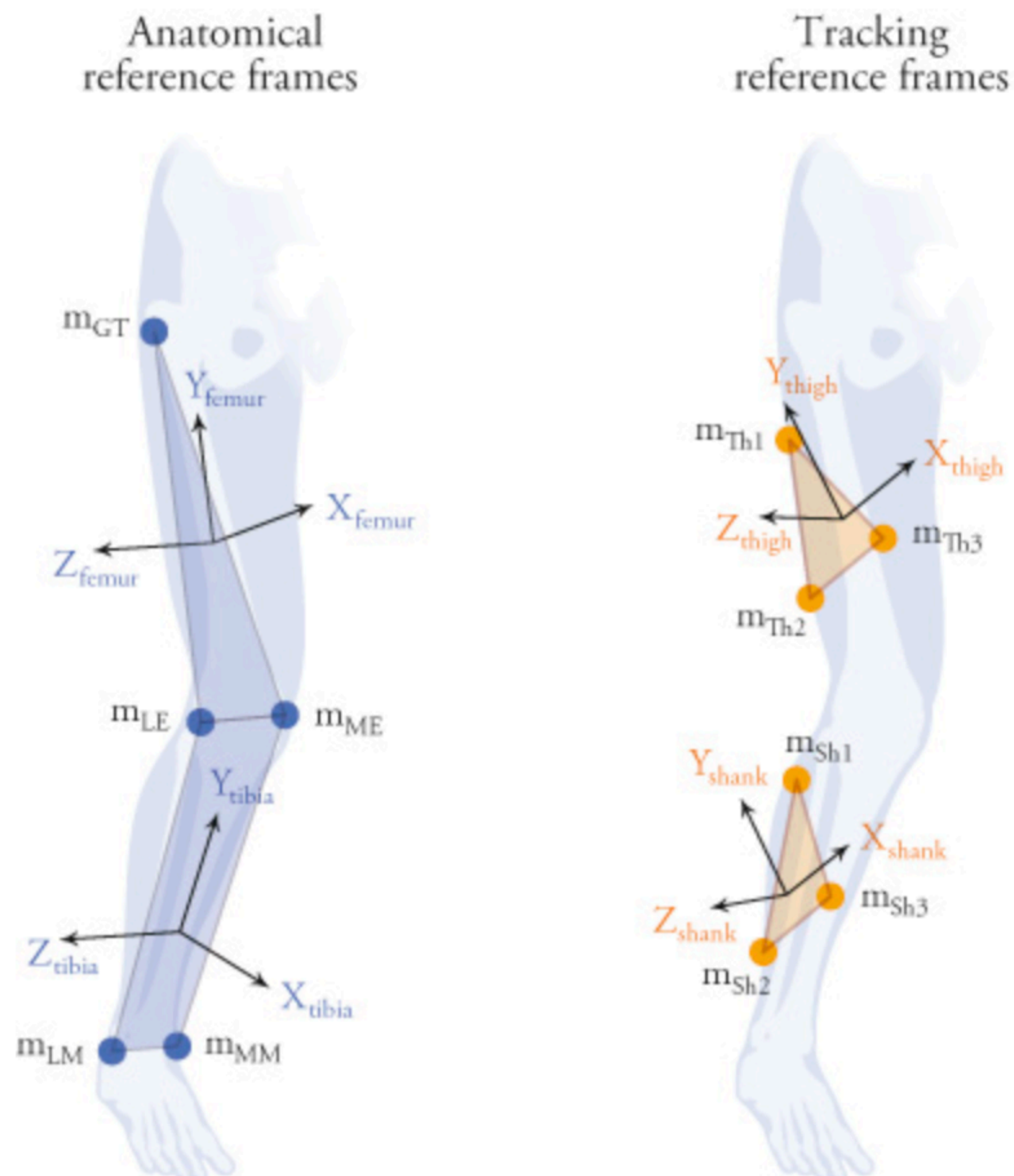
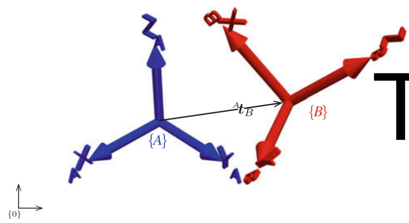


Figure 7.11 Body-fixed reference frames determined from markers mounted on anatomical landmarks (anatomical reference frames) and markers mounted on body segments (tracking reference frames); m = marker, GT = greater trochanter, ME/LE = medial/lateral femoral epicondyle, MM/LM = medial/lateral malleolus, Th = thigh, Sh = shank.



Transformation Matrix

1. The origin is midway between the greater trochanter marker (**mGT**) and the knee joint center (the midpoint between the medial and lateral femoral epicondyle markers, **mME** and **mLE**).

2. **Zfemur** is parallel to the knee joint axis, which is defined as the **vector from medial to lateral femoral epicondyle markers**, normalized to unit length.

3. **Xfemur** is the **cross product** of **Zfemur** and a vector from the greater trochanter marker to one of the femoral epicondyle markers, normalized to unit length

4. **Yfemur** is the **cross product** of **Zfemur** and **Xfemur**, which completes the right-handed reference frame.

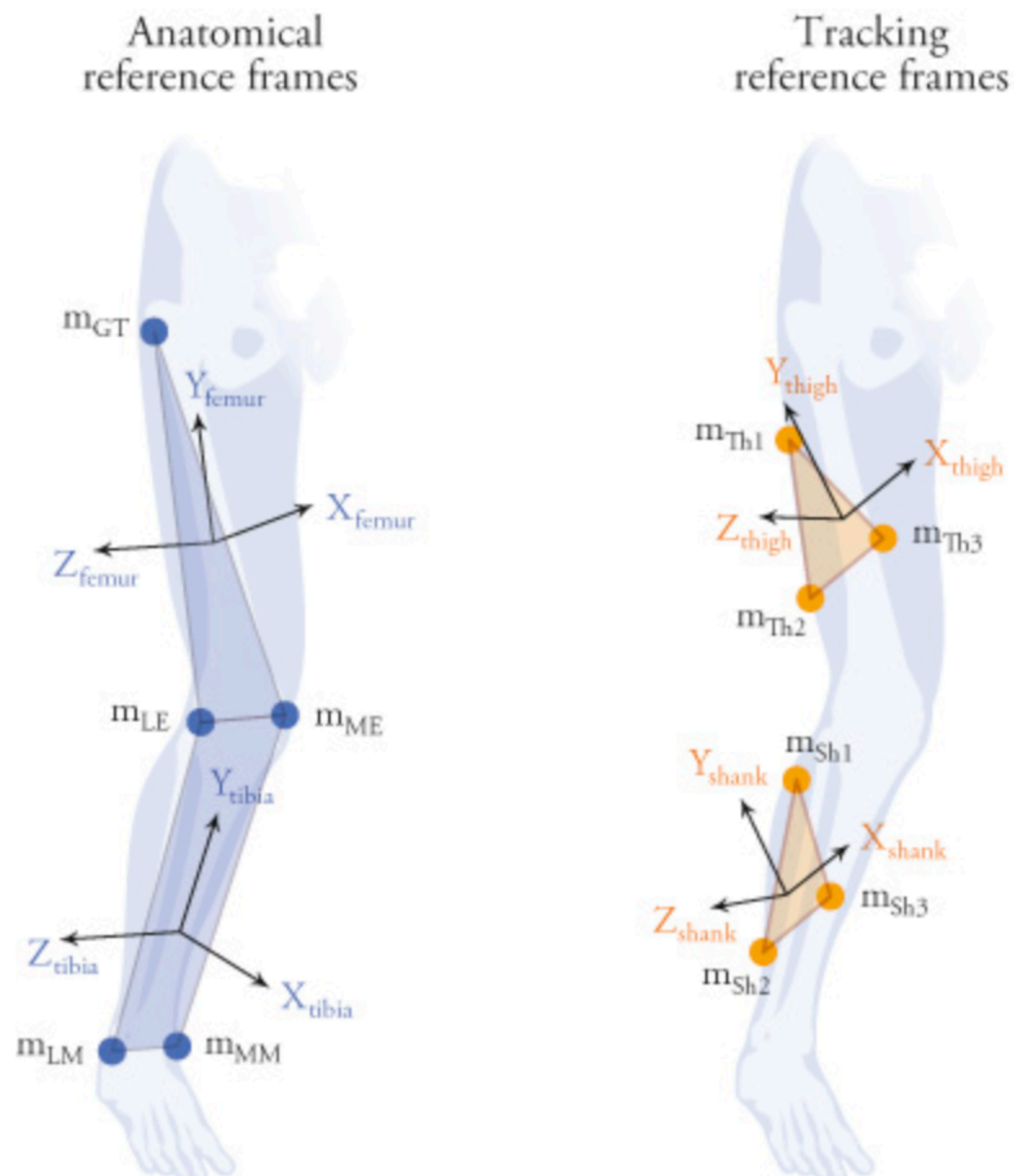


Figure 7.11 Body-fixed reference frames determined from markers mounted on anatomical landmarks (anatomical reference frames) and markers mounted on body segments (tracking reference frames); m = marker, GT = greater trochanter, ME/LE = medial/lateral femoral epicondyle, MM/LM = medial/lateral malleolus, Th = thigh, Sh = shank.

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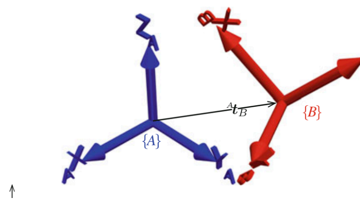
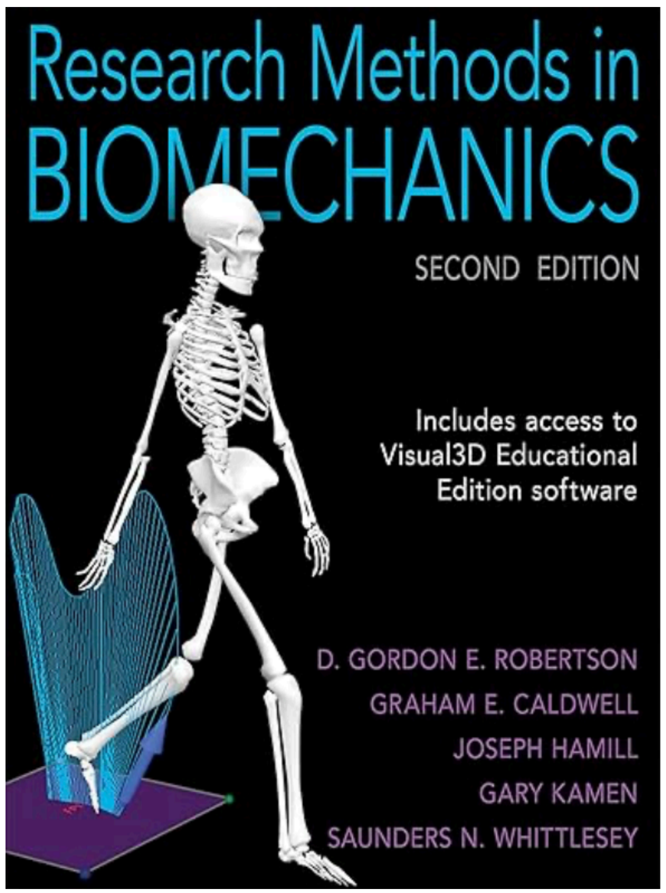


Transformation Matrix



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- **Hardcover : 440 pages**
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Pelvis Segment LCS

Markers are placed on the following palpable bony landmarks: right and left anterior-superior iliac spine ($\vec{P}_{RASIS}$, $\vec{P}_{LASIS}$) and right and left posterior-superior iliac spine ($\vec{P}_{RPSIS}$, $\vec{P}_{LPSIS}$) (figure 2.6). The origin of the LCS is midway between $\vec{P}_{RASIS}$ and $\vec{P}_{LASIS}$ and can be calculated as follows:

$$\vec{O}_{PELVIS} = 0.5 * (\vec{P}_{RASIS} + \vec{P}_{LASIS}) \tag{2.11}$$

To create the x-component (or lateral direction) of the pelvis, a unit vector $\hat{i}'$ is defined by subtracting $\vec{O}_{PELVIS}$ from $\vec{P}_{RASIS}$ and dividing by the norm of the vector:

$$\hat{i}' = \frac{\vec{P}_{RASIS} - \vec{O}_{PELVIS}}{\left|\vec{P}_{RASIS} - \vec{O}_{PELVIS}\right|} \tag{2.12}$$

Next we create a unit vector from the midpoint of $\vec{P}_{RPSIS}$ and $\vec{P}_{LPSIS}$ to $\vec{O}_{PELVIS}$:

$$\hat{v} = \frac{\vec{O}_{PELVIS} - 0.5 * (\vec{P}_{RPSIS} + \vec{P}_{LPSIS})}{\left|\vec{O}_{PELVIS} - 0.5 * (\vec{P}_{RPSIS} + \vec{P}_{LPSIS})\right|} \tag{2.13}$$

A unit vector normal to the plane (in the superior direction) containing $\hat{i}'$ and $\hat{v}$ is computed from a cross product:

$$\hat{k}' = \hat{i}' \times \hat{v} \tag{2.14}$$

Note that the order in which the vectors $\hat{i}'$ and $\hat{v}$ are crossed to produce a superiorly directed unit vector is determined by the right-hand rule. At this point we have defined the lateral direction and the superior direction. The anterior unit is created from the cross product

$$\hat{j}' = \hat{k}' \times \hat{i}' \tag{2.15}$$

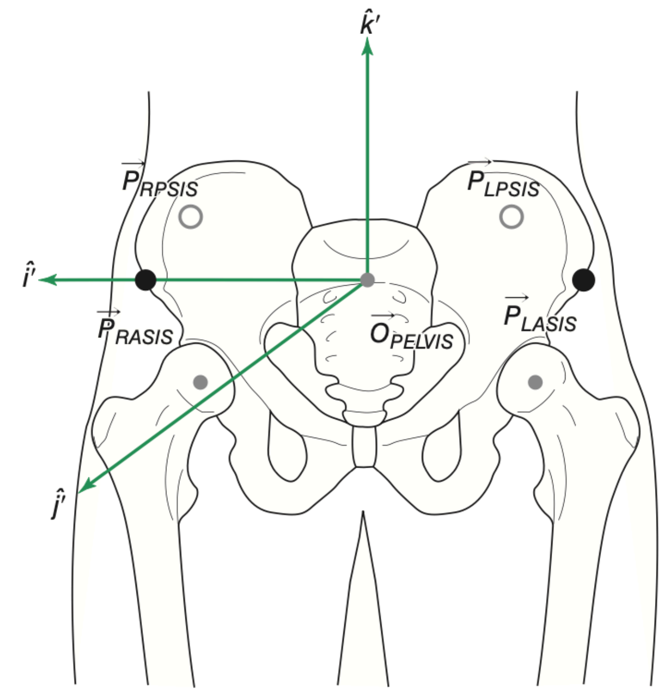

Figure 2.6 The origin of the pelvis LCS ($\vec{O}_{PELVIS}$) is midway between the right and left anterior-superior iliac spines. The right and left anterior-superior iliac spines ($\vec{P}_{RASIS}$ and $\vec{P}_{LASIS}$) and the posterior-superior iliac spines ($\vec{P}_{RPSIS}$ and $\vec{P}_{LPSIS}$) can be used to derive the pelvis LCS.

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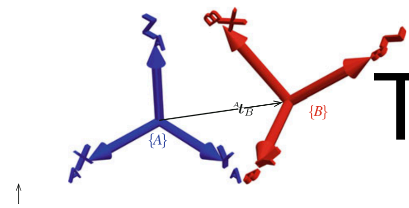


Transformation Matrix

We can transform the location of the hip joint from the pelvis LCS to the GCS as follows:

$$\vec{O}_{RTHIGH} = \vec{P}_{RHIP} = R'_{\text{PELVIS}} * \vec{P}'_{RHIP} + \vec{O}_{PELVIS} \qquad (2.18)$$

To develop the thigh LCS, a superior unit vector is created along an axis passing from the distal end (midpoint between the femoral epicondyles $\vec{P}_{RLK}$ and $\vec{P}_{RMK}$) to the origin ($\vec{O}_{RTHIGH}$) as follows:

$$\hat{k}' = \frac{\vec{O}_{RTHIGH} - 0.5 * (\vec{P}_{RLK} + \vec{P}_{RMK})}{\left|\vec{O}_{RTHIGH} - 0.5 * (\vec{P}_{RLK} + \vec{P}_{RMK})\right|} \qquad (2.19)$$

We then create a unit vector passing from the medial to the lateral femoral epicondyle:

$$\hat{v} = \frac{\overrightarrow{(\vec{P}_{RLK} - \vec{P}_{RMK})}}{\left|\vec{P}_{RLK} - \vec{P}_{RMK}\right|} \qquad (2.20)$$

The anterior unit vector is determined from the cross product of the $\hat{k}'$ and $\hat{v}$ vectors as follows:

$$\hat{j}' = \hat{k}' \times \hat{v} \qquad (2.21)$$

Care should be taken in the placement of the knee markers. The lateral marker is placed at the most lateral aspect of the femoral epicondyle. The medial marker should be located so that the lateral and medial knee markers and the hip joint define the frontal plane of the thigh.

Last, the unit vector in the lateral direction is formed from the cross product:

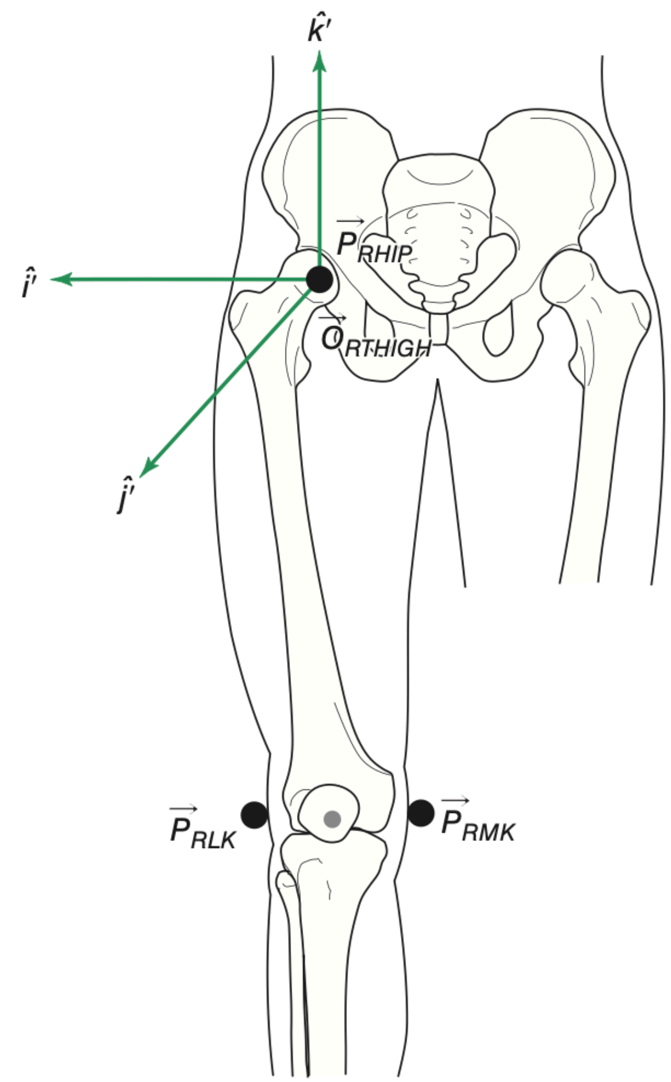
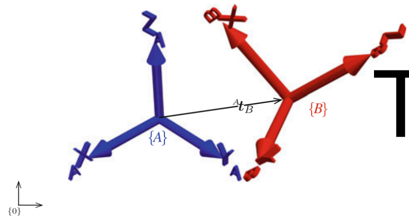


Figure 2.7 The origin of the thigh LCS ($\vec{O}_{RTHIGH}$) is at the hip joint center. The position of hip joint center ($\vec{P}_{RHIP}$) and the lateral and medial femoral epicondyles ($\vec{P}_{RLK}$ and $\vec{P}_{RMK}$) can be used to calculate the thigh LCS.

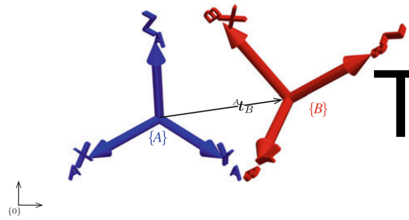


Transformation Matrix

$$
{}^{A}T_{B} = \left[\begin{array}{c:c} \textit{rotation} & \textit{position} \\ \textit{matrix} & \textit{vector} \\ \hdashline 0 & 1 \end{array}\right]
$$

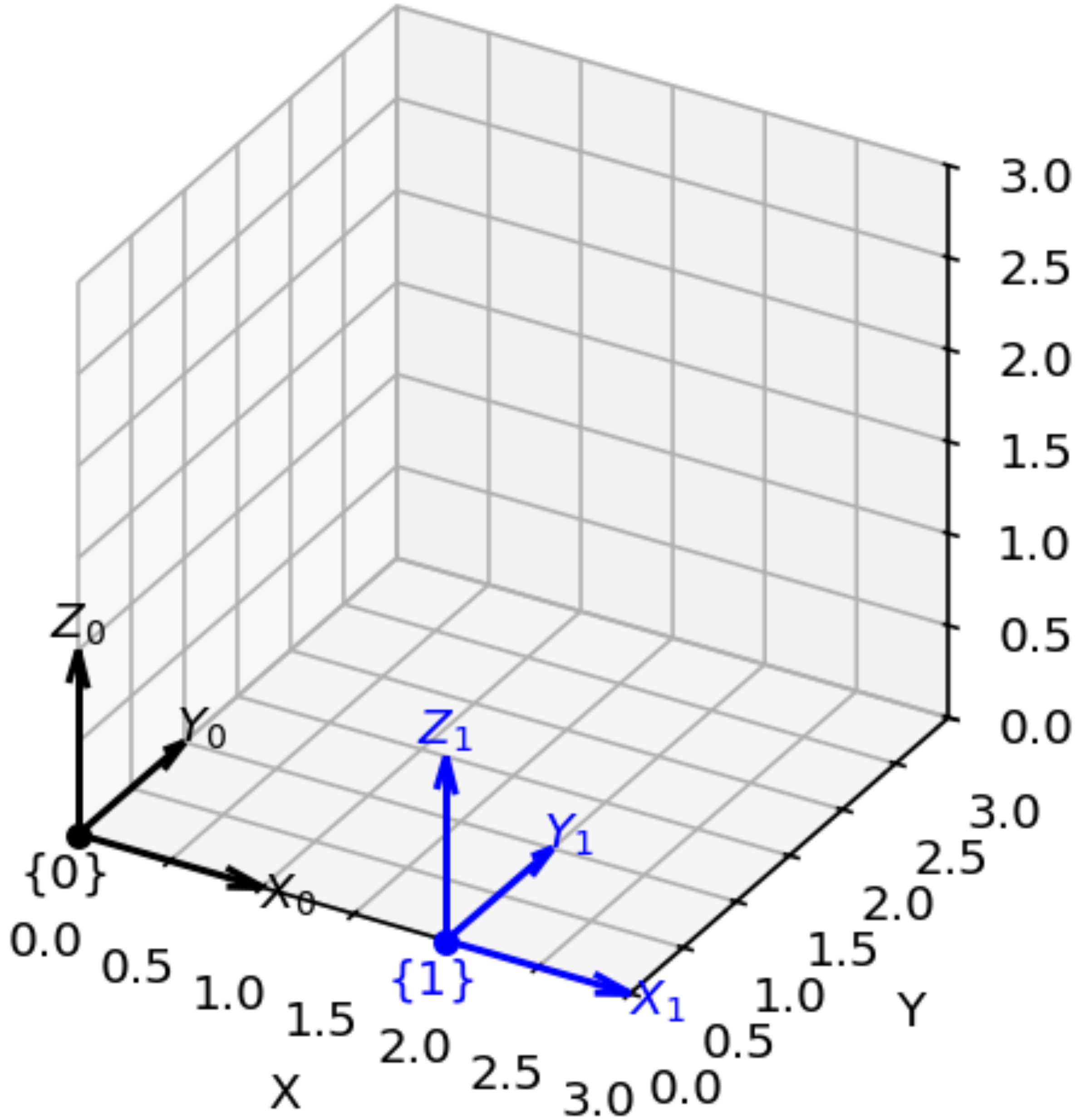
This ${}^{A}T_{B}$ matrix is called the **Homogenous Transformation Matrix** and is read as the transformation from B to A.

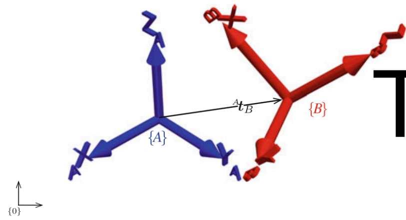
The transformation matrix is indeed a partitioned matrix where the upper left corner sub-matrix is the rotation matrix and the upper right vector is the position vector.



Transformation Matrix

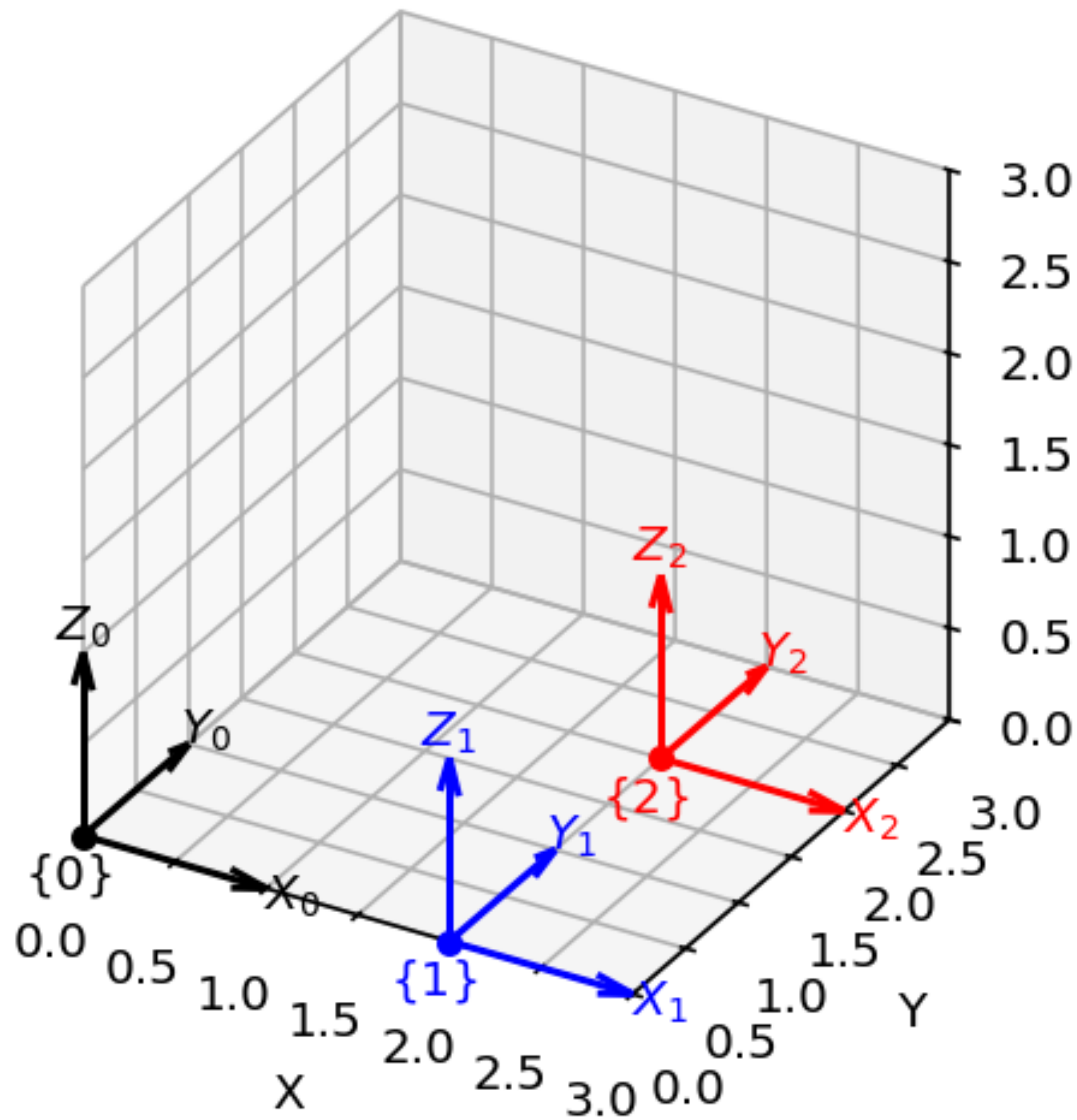
$$
{}^{0}T_{1} = = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]
$$

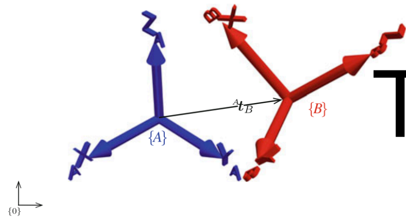




Transformation Matrix

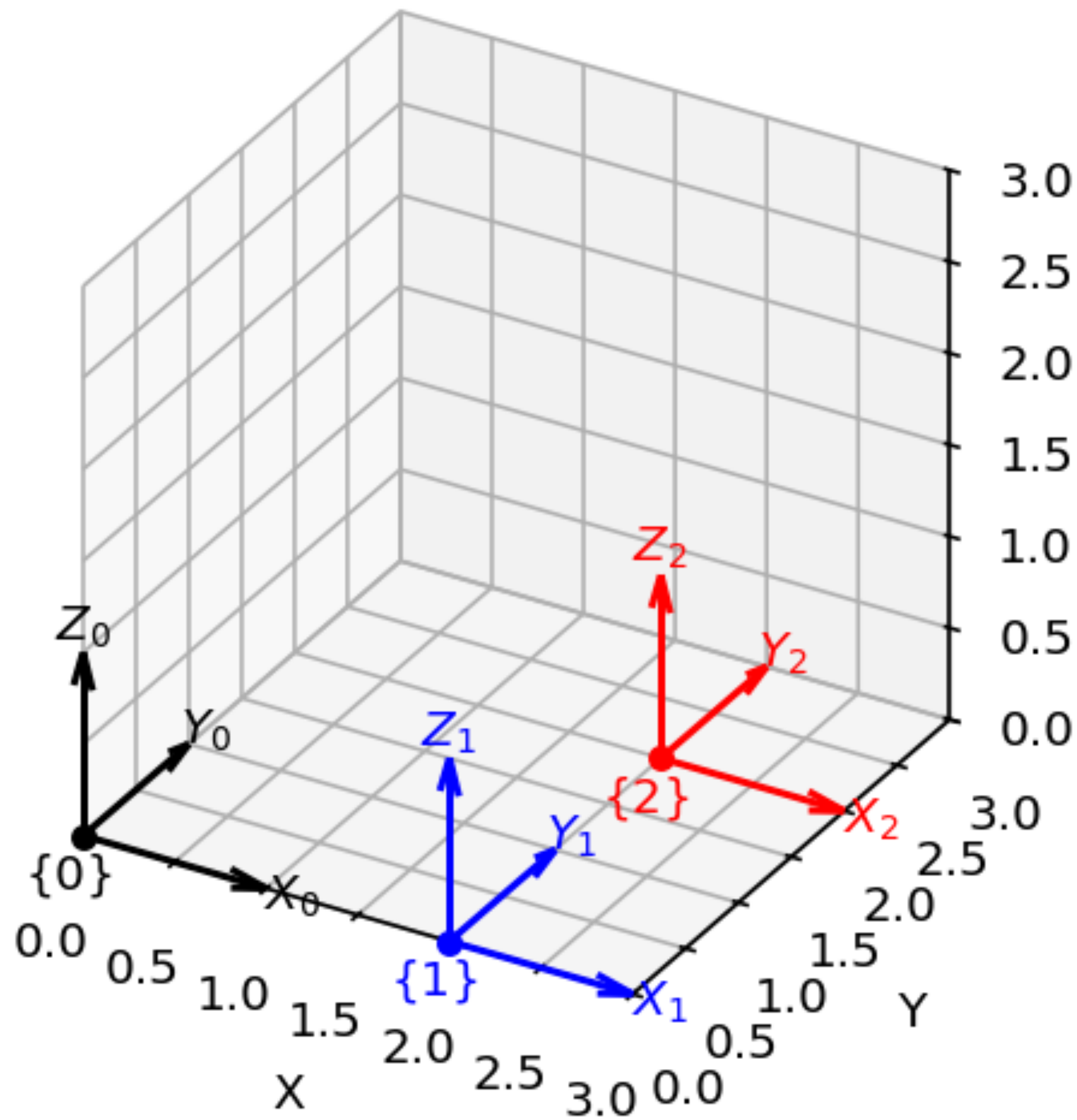
$$
{}^{1}T_{2} = = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$





Transformation Matrix

$$
{}^{0}T_{2} = = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]
$$



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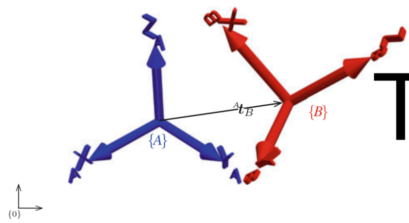
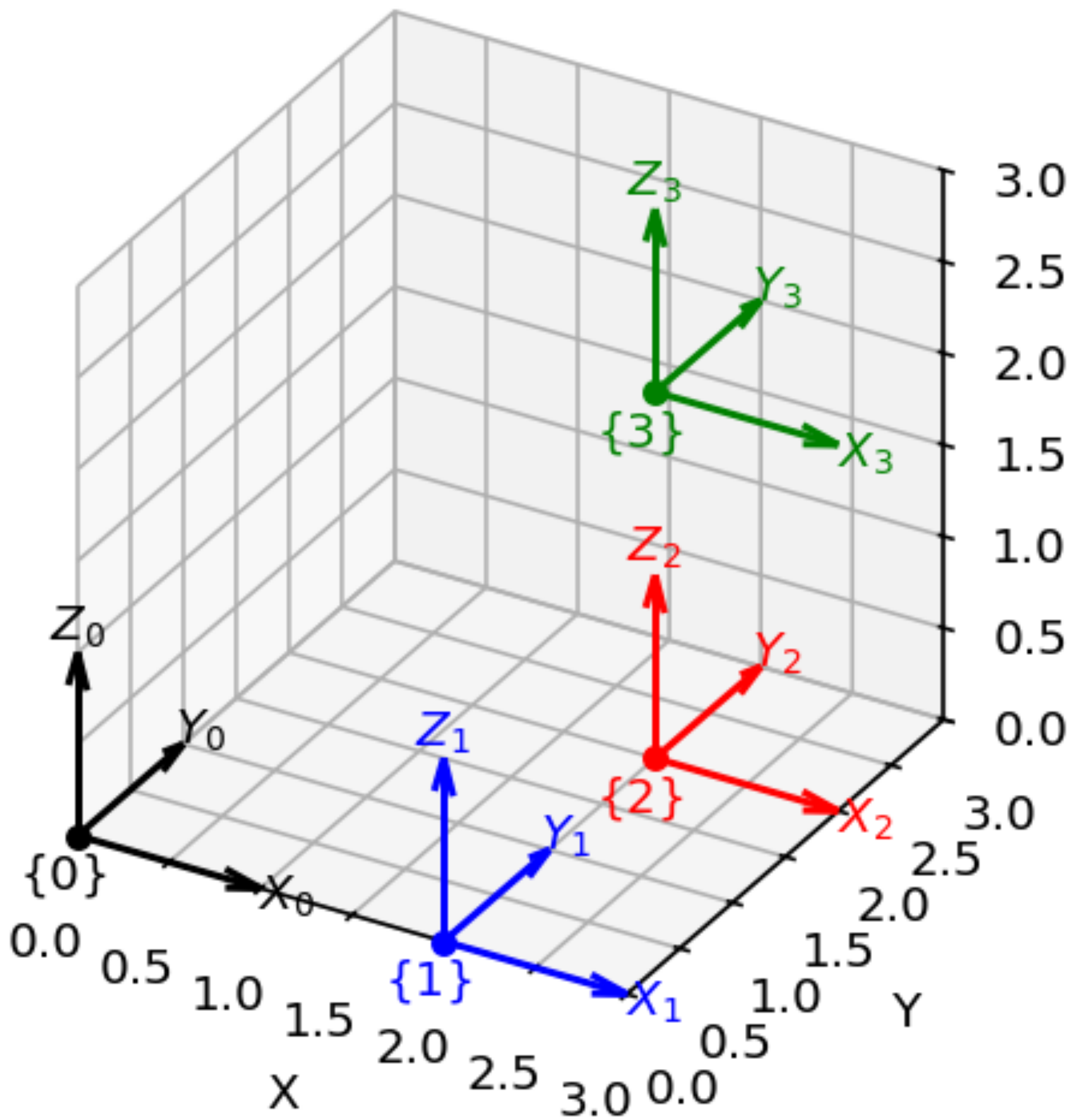
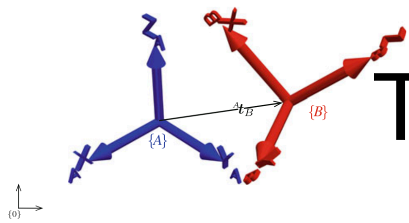


Transformation Matrix

$$
{}^{0}T_{3} = = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]
$$

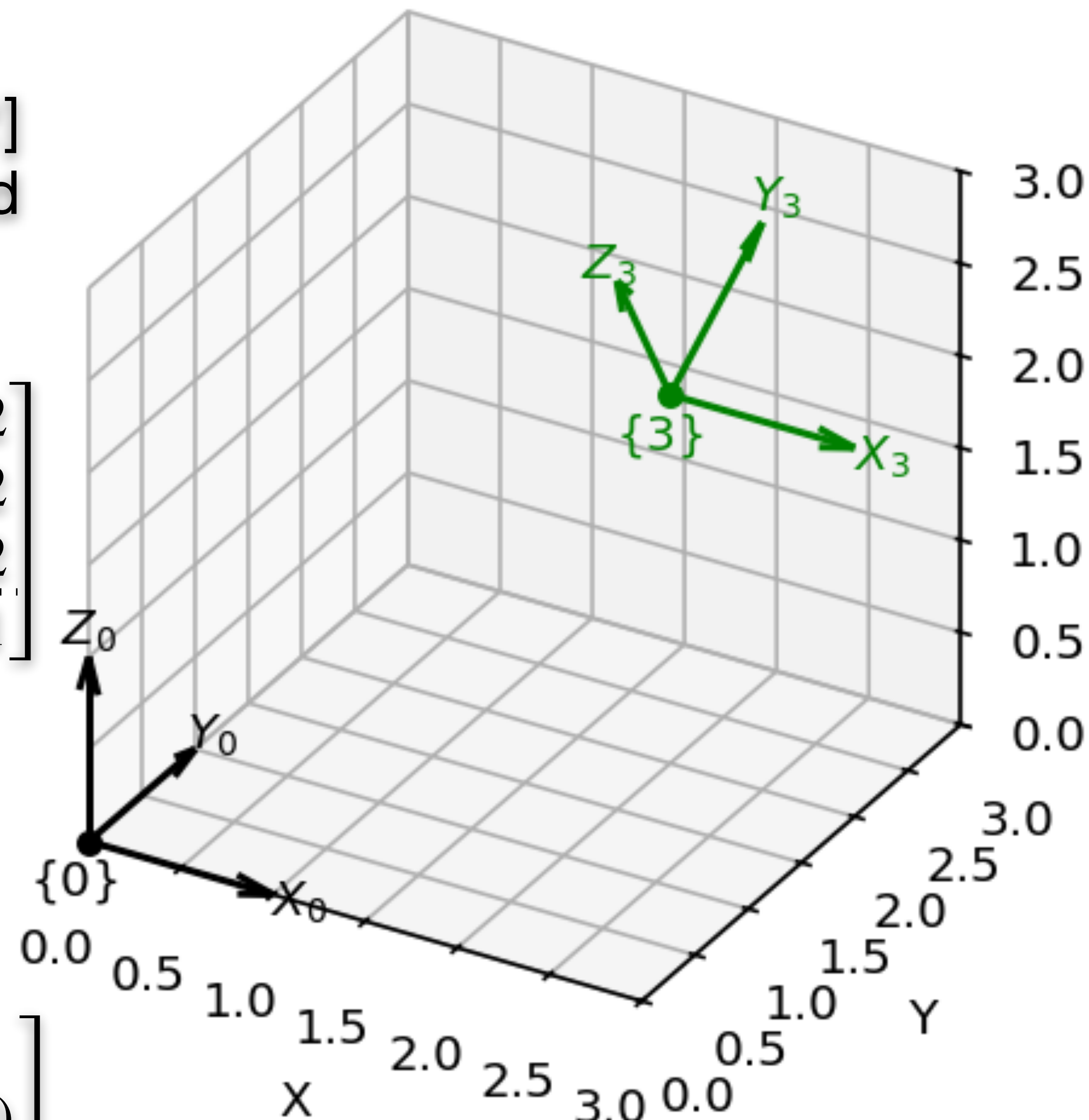




Transformation Matrix

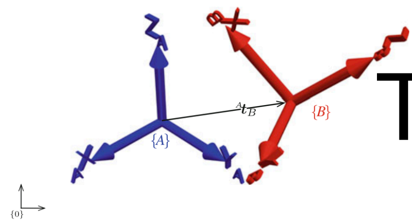
Transform to [2,2,2] then rotate 30° around the x-axis

$$^{0}T_{3} = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 0.866 & -0.5 & 2 \\ 0 & 0.5 & 0.866 & 2 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$



$$R_x(30) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(30) & -sin(30) \\ 0 & sin(30) & cos(30) \end{bmatrix}$$

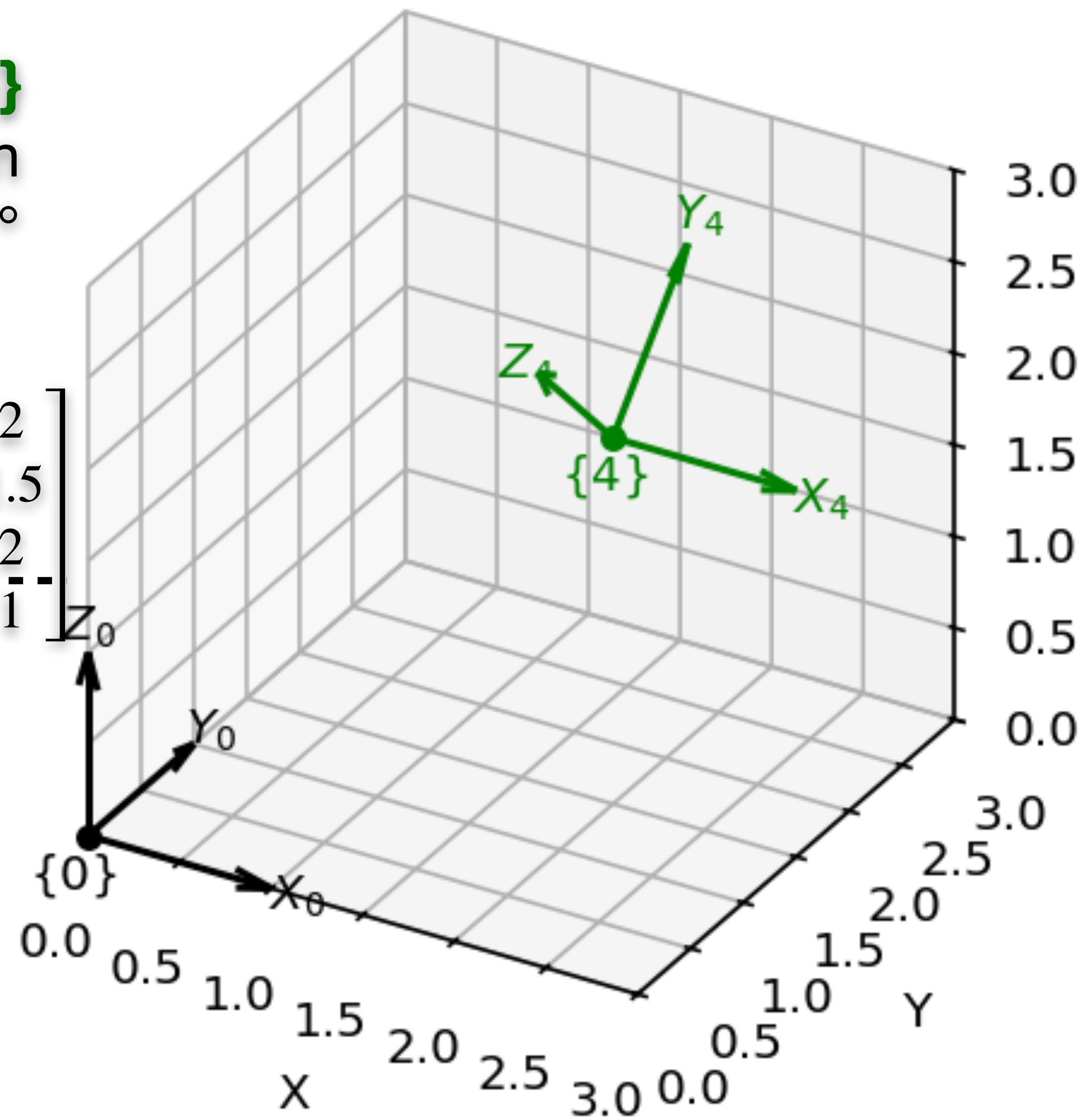
Look at the lecture 3 page 25

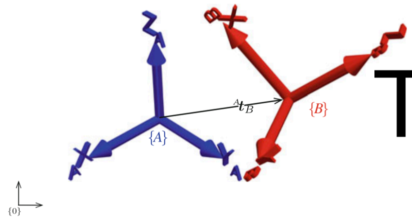


Transformation Matrix

Respect to **Pose{3}** move -0.5 in y-direction while rotating 15° around the x-axis

$$
{}^{0}T_{4} = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 0.7071 & -0.7071 & 1.5 \\ 0 & 0.7071 & 0.7071 & 2 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]
$$





Transformation Matrix

Distance between rotations

Let $^{0}\boldsymbol{R}_{3}$ and $^{0}\boldsymbol{R}_{4}$ be orthogonal matrices representing two **rotations** in **the same basis.**

Let $(\_\boldsymbol{R}\_)^{T}$ denote the *matrix transpose*

The **Difference Rotation Matrix** that represent the difference rotation is defined as

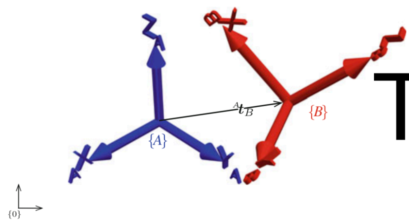
$\boldsymbol{R}_{diff} = (^{0}\boldsymbol{T}_{3})^{T} \times {^{0}\boldsymbol{T}_{4}}$

We can retrieve the angle of the difference rotation from the trace of $\boldsymbol{R}$

$$\mathrm{tr}\ \boldsymbol{R} = 1 + 2\cos\boldsymbol{\theta}$$

again using *arccos*

$$\theta = arccos\frac{tr\ R - 1}{2}$$



Transformation Matrix

$$
{}^{0}R_{3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8666 & -0.5000 \\ 0 & 0.5000 & 0.8666 \end{bmatrix} \qquad {}^{0}R_{4} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.7071 & -0.7071 \\ 0 & 0.7071 & 0.7071 \end{bmatrix}
$$

$$
R_{dif} = ({}^{0}R_{3})^{T} \times {}^{0}R_{4}
$$

$$
\theta = arccos\frac{trace\,(R_{dif}) - 1}{2} = 15
$$

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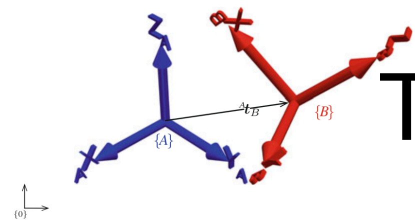
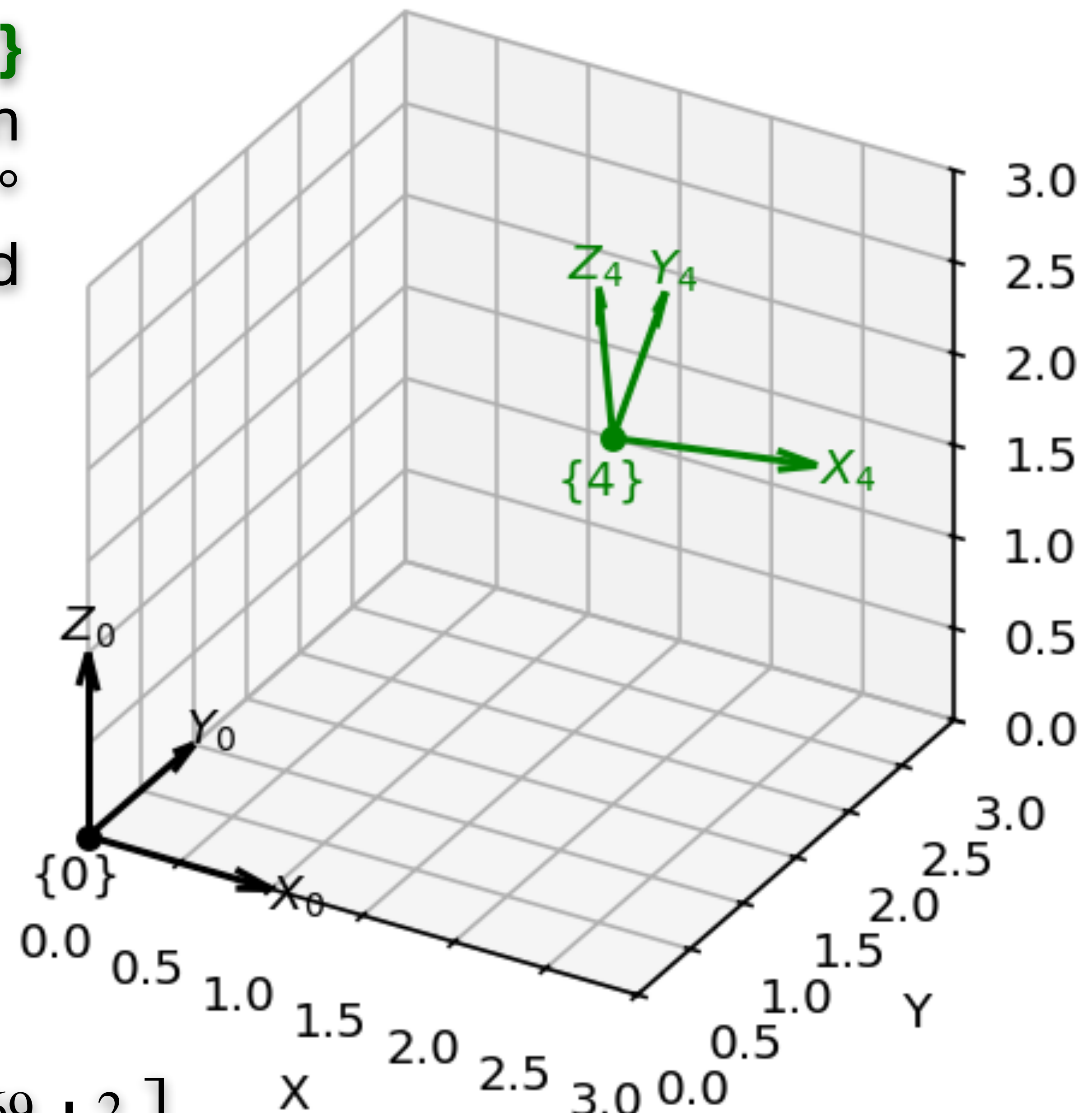
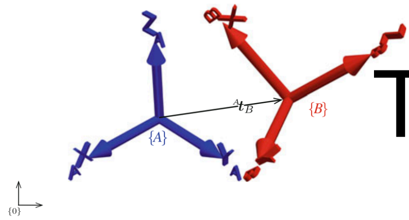


Transformation Matrix

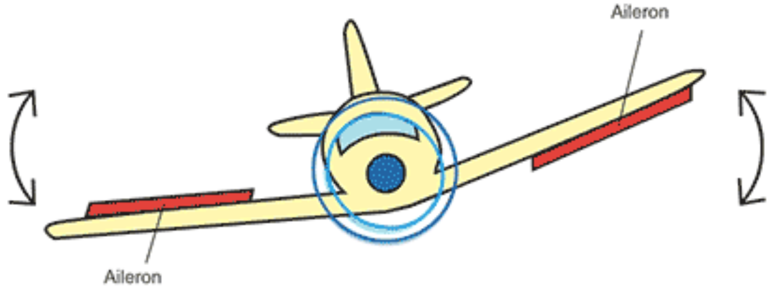
Respect to **Pose{3}** move -0.5 in y-direction while rotating 15° around the x-axis and z-axis



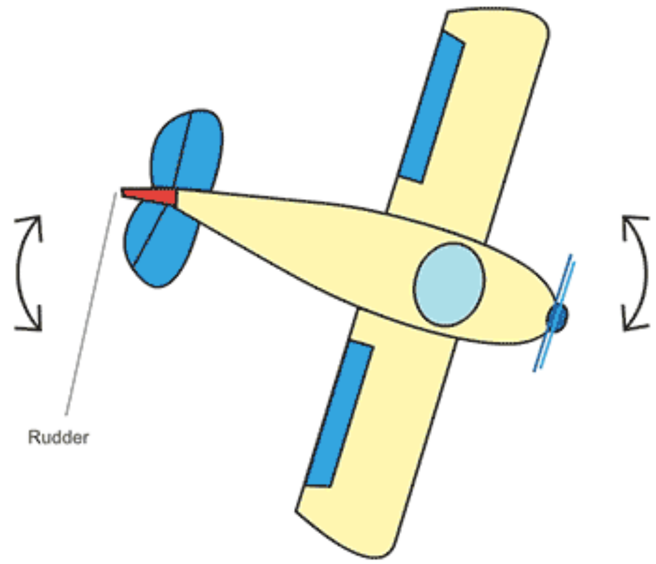
$$
{}^{0}T_{4} = \left[\begin{array}{ccc|c} 0.9659 & -0.25 & 0.0669 & 2 \\ 0.2588 & 0.9333 & -0.25 & 1.5 \\ 0 & 0.2588 & 0.9659 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$



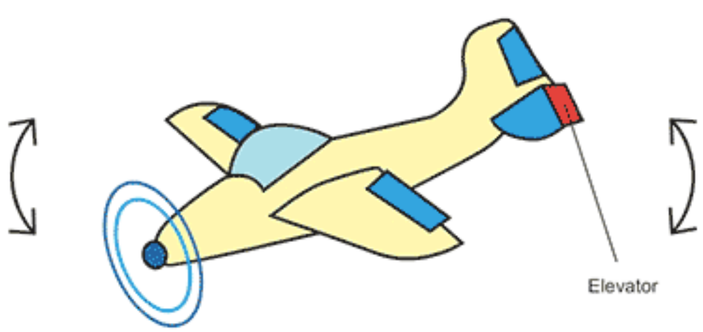
Transformation Matrix



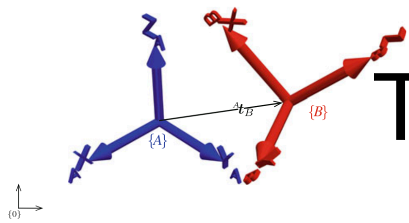
Use the ailerons to control
Roll



Use the rudder to control
Yaw



Use the elevator to control
Pitch



Transformation Matrix

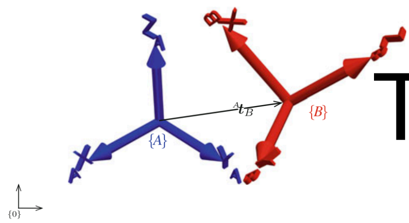
The table contains formulas for angles α, β and γ from elements of a rotation matrix R.

Without considering the possibility of using two different conventions for the definition of the rotation axes (intrinsic or extrinsic), there exist twelve possible sequences of rotation axes, divided in two groups:

Proper Euler angles (*z-x-z*, *x-y-x*, *y-z-y*, *z-y-z*, *x-z-x*, *y-x-y*)

Tait–Bryan angles (*x-y-z*, *y-z-x*, *z-x-y*, *x-z-y*, *z-y-x*, *y-x-z*).

Proper Euler angles		Tait–Bryan angles	
$X_\alpha Z_\beta X_\gamma$	$\alpha = \arctan\left(\frac{R_{31}}{R_{21}}\right)$ $\beta = \arccos(R_{11})$ $\gamma = \arctan\left(\frac{R_{13}}{-R_{12}}\right)$	$X_\alpha Z_\beta Y_\gamma$	$\alpha = \arctan\left(\frac{R_{32}}{R_{22}}\right)$ $\beta = \arcsin(-R_{12})$ $\gamma = \arctan\left(\frac{R_{13}}{R_{11}}\right)$
$X_\alpha Y_\beta X_\gamma$	$\alpha = \arctan\left(\frac{R_{21}}{-R_{31}}\right)$ $\beta = \arccos(R_{11})$ $\gamma = \arctan\left(\frac{R_{12}}{R_{13}}\right)$	$X_\alpha Y_\beta Z_\gamma$	$\alpha = \arctan\left(\frac{-R_{23}}{R_{33}}\right)$ $\beta = \arcsin(R_{13})$ $\gamma = \arctan\left(\frac{-R_{12}}{R_{11}}\right)$
$Y_\alpha X_\beta Y_\gamma$	$\alpha = \arctan\left(\frac{R_{12}}{R_{32}}\right)$ $\beta = \arccos(R_{22})$ $\gamma = \arctan\left(\frac{R_{21}}{-R_{23}}\right)$	$Y_\alpha X_\beta Z_\gamma$	$\alpha = \arctan\left(\frac{R_{13}}{R_{33}}\right)$ $\beta = \arcsin(-R_{23})$ $\gamma = \arctan\left(\frac{R_{21}}{R_{22}}\right)$
$Y_\alpha Z_\beta Y_\gamma$	$\alpha = \arctan\left(\frac{R_{32}}{-R_{12}}\right)$ $\beta = \arccos(R_{22})$ $\gamma = \arctan\left(\frac{R_{23}}{R_{21}}\right)$	$Y_\alpha Z_\beta X_\gamma$	$\alpha = \arctan\left(\frac{-R_{31}}{R_{11}}\right)$ $\beta = \arcsin(R_{21})$ $\gamma = \arctan\left(\frac{-R_{23}}{R_{22}}\right)$
$Z_\alpha Y_\beta Z_\gamma$	$\alpha = \arctan\left(\frac{R_{23}}{R_{13}}\right)$ $\beta = \arctan\left(\frac{\sqrt{1-R_{33}^2}}{R_{33}}\right)$ $\gamma = \arctan\left(\frac{R_{32}}{-R_{31}}\right)$	$Z_\alpha Y_\beta X_\gamma$	$\alpha = \arctan\left(\frac{R_{21}}{R_{11}}\right)$ $\beta = \arcsin(-R_{31})$ $\gamma = \arctan\left(\frac{R_{32}}{R_{33}}\right)$
$Z_\alpha X_\beta Z_\gamma$	$\alpha = \arctan\left(\frac{R_{13}}{-R_{23}}\right)$ $\beta = \arccos(R_{33})$ $\gamma = \arctan\left(\frac{R_{31}}{R_{32}}\right)$	$Z_\alpha X_\beta Y_\gamma$	$\alpha = \arctan\left(\frac{-R_{12}}{R_{22}}\right)$ $\beta = \arcsin(R_{32})$ $\gamma = \arctan\left(\frac{-R_{31}}{R_{33}}\right)$



Transformation Matrix

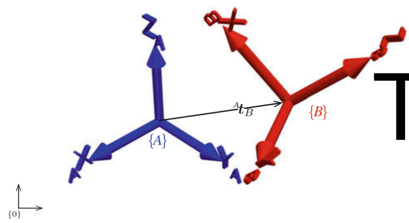
$$^{0}T_{4} = \left[\begin{array}{ccc|c} 0.9659 & -0.25 & 0.0669 & 2 \\ 0.2588 & 0.9333 & -0.25 & 1.5 \\ 0 & 0.2588 & 0.9659 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$R = R_z(\alpha) \times R_y(\beta) \times R_x(\gamma)$$

$$\alpha = arctan\frac{R_{21}}{R_{11}} = 15$$

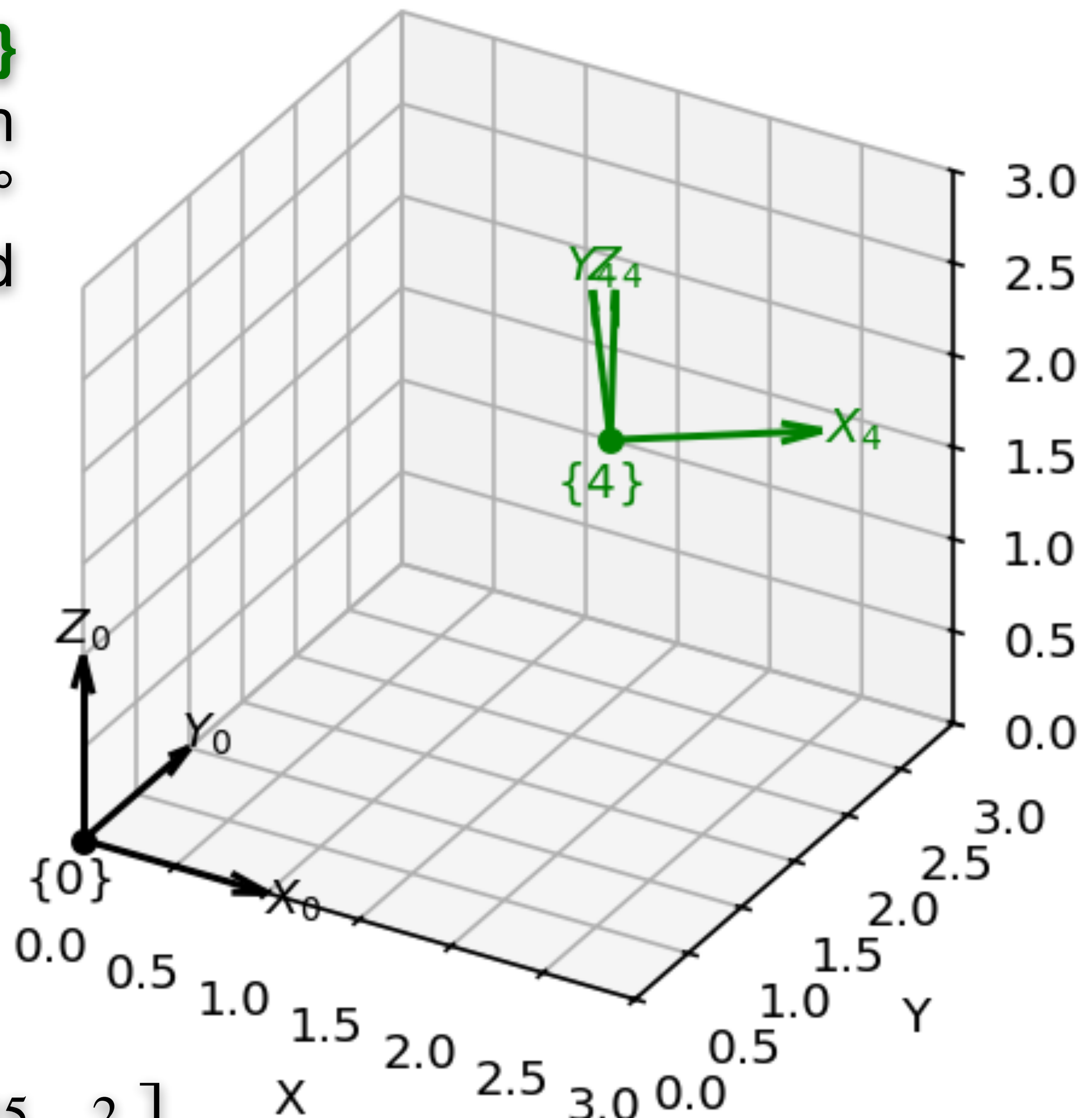
$$\beta = arcsin(-R_{31}) = 0$$

$$\gamma = arctan\frac{R_{32}}{R_{33}} = 14.99$$

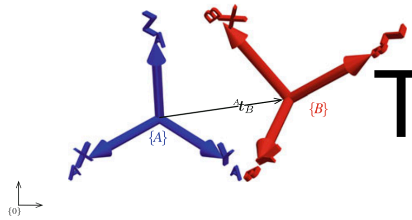


Transformation Matrix

Respect to **Pose{3}** move -0.5 in y-direction while rotating 15° around the x-axis and around the 35° z-axis



$$
{}^{0}T_4 = \begin{bmatrix} 0.8192 & -0.554 & 0.1485 & 2 \\ 0.5736 & 0.7912 & -0.212 & 1.5 \\ 0 & 0.2588 & 0.9659 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



Transformation Matrix

$$^{0}T_{4} = \left[\begin{array}{ccc:c} 0.8192 & -0.554 & 0.1485 & 2 \\ 0.5736 & 0.7912 & -0.212 & 1.5 \\ 0 & 0.2588 & 0.9659 & 2 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

$$R = R_z(\alpha) \times R_y(\beta) \times R_x(\gamma)$$

$$\alpha = arctan\frac{R_{21}}{R_{11}} = 34.99$$

$$\beta = arcsin(-R_{31}) = 0$$

$$\gamma = arctan\frac{R_{32}}{R_{33}} = 14.99$$

Homework

Read This Part

Plug-in Gait output angles

The output angles for all joints are calculated from the YXZ Cardan angles derived by comparing the relative orientations of the segments proximal (parent) and distal (child) to the joint.