



HAB718 Spor Biyomekaniğinde Hareket Analizi



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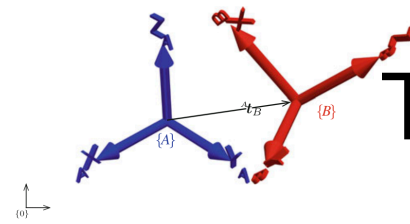
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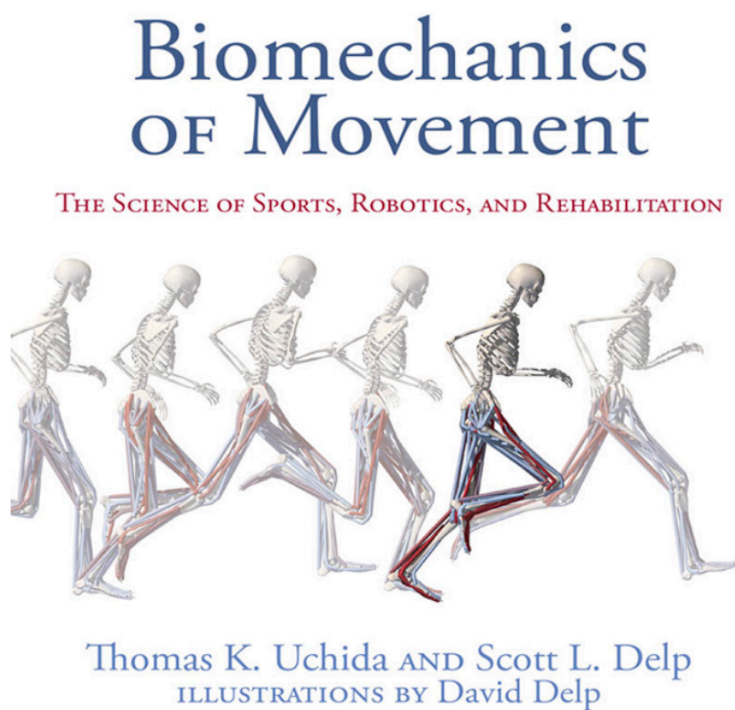
- Three-Dimensional Kinematics



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Transformation Matrix



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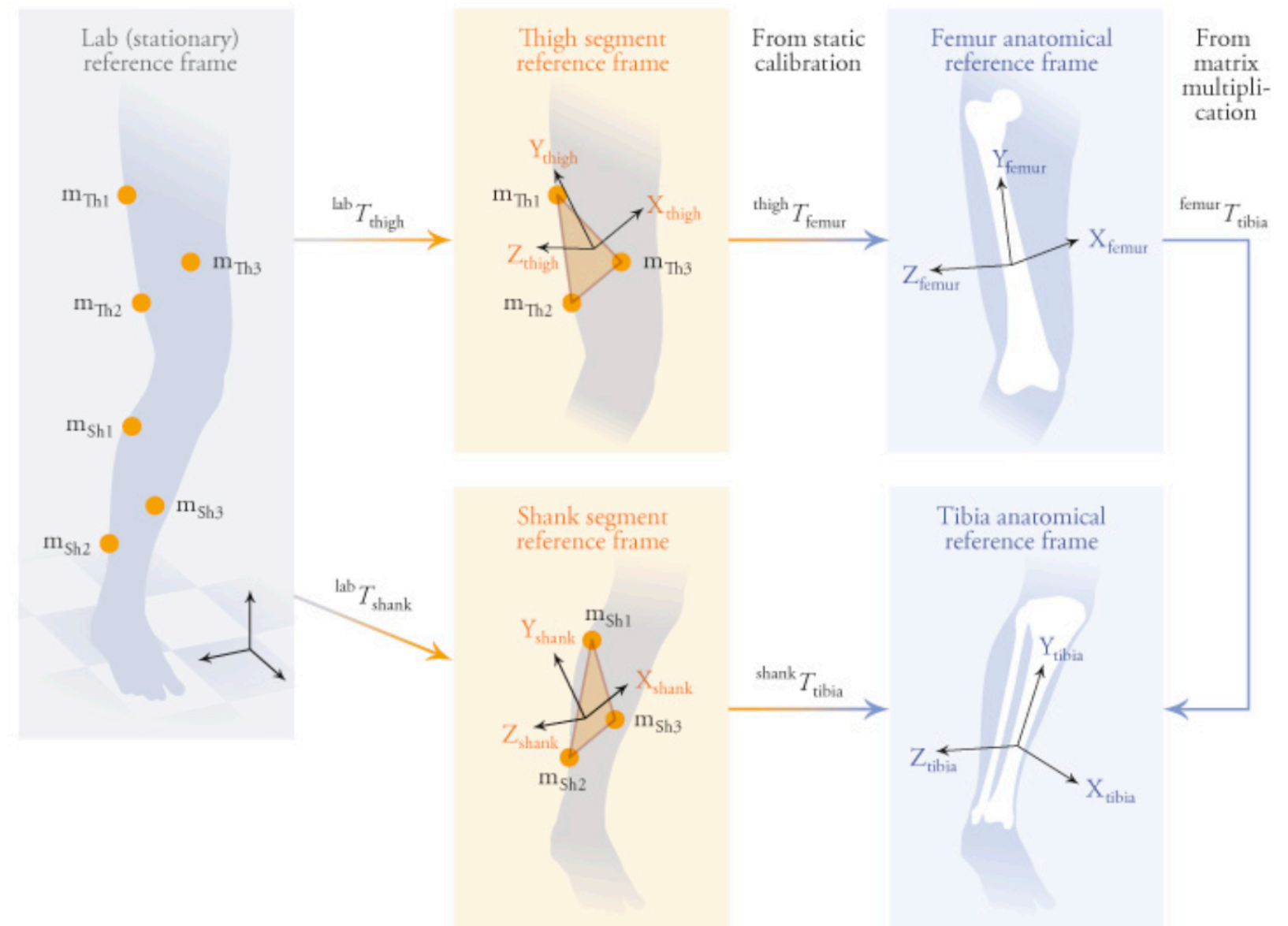
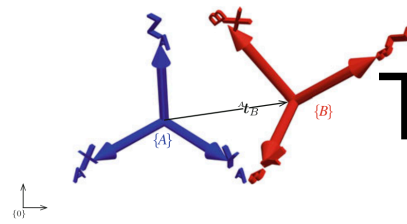


Figure 7.13 The relative position and orientation of any two reference frames can be described by a 4×4 transformation matrix T , which facilitates calculation of joint angles from marker locations measured relative to a global reference frame.



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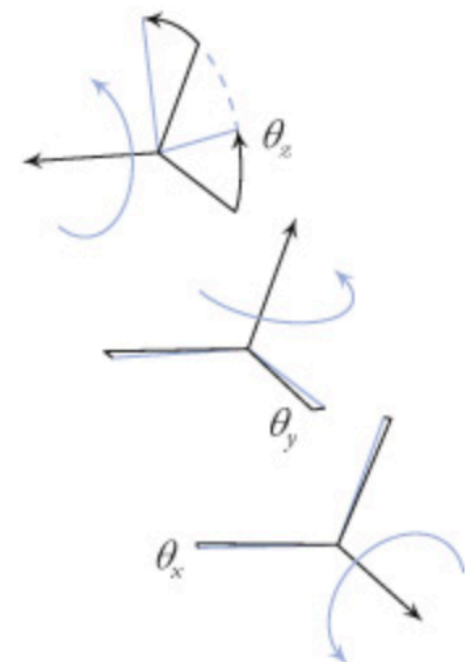
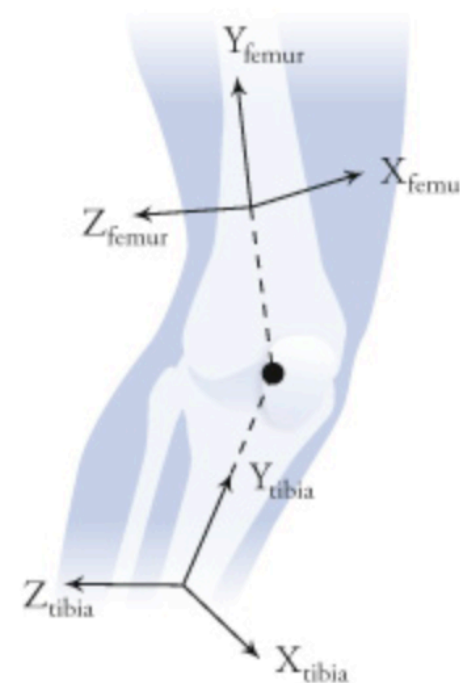
Transformation Matrix

Joint angles can be calculated by comparing the orientations of reference frames fixed to adjacent body segments. Knee flexion can be computed by inspection in a two-dimensional analysis (left), but a more formal approach is useful when computing the relative orientation between two frames in three dimensions (right).

Two-dimensional

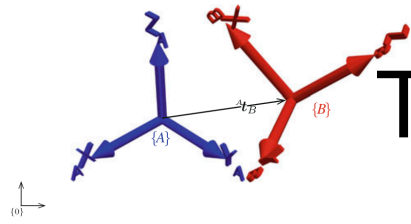


Three-dimensional





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Transformation Matrix

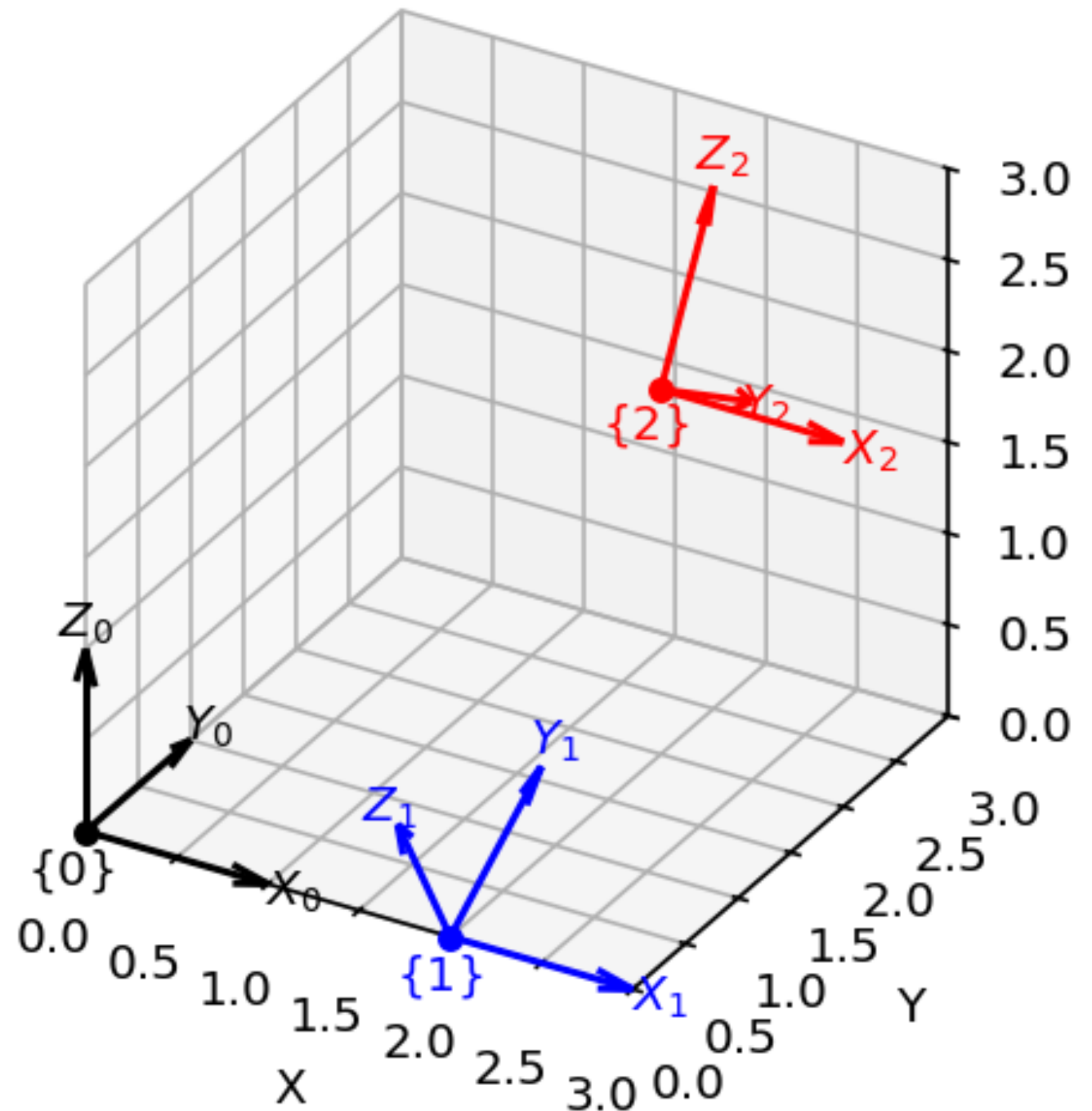
$${}^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0.866 & -0.5 & 0 \\ 0 & 0.5 & 0.866 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_2 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0.866 & 0.5 & 2 \\ 0 & -0.5 & 0.866 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & -0.5 \\ 0 & 0.5 & 0.866 \end{bmatrix}$$

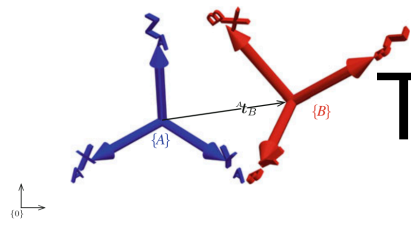
$$R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & 0.5 \\ 0 & -0.5 & 0.866 \end{bmatrix}$$

$${}^1R_2 = R_1^T \times R_2$$





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Transformation Matrix

$${}^1R_2 = R_1^T \times R_2$$

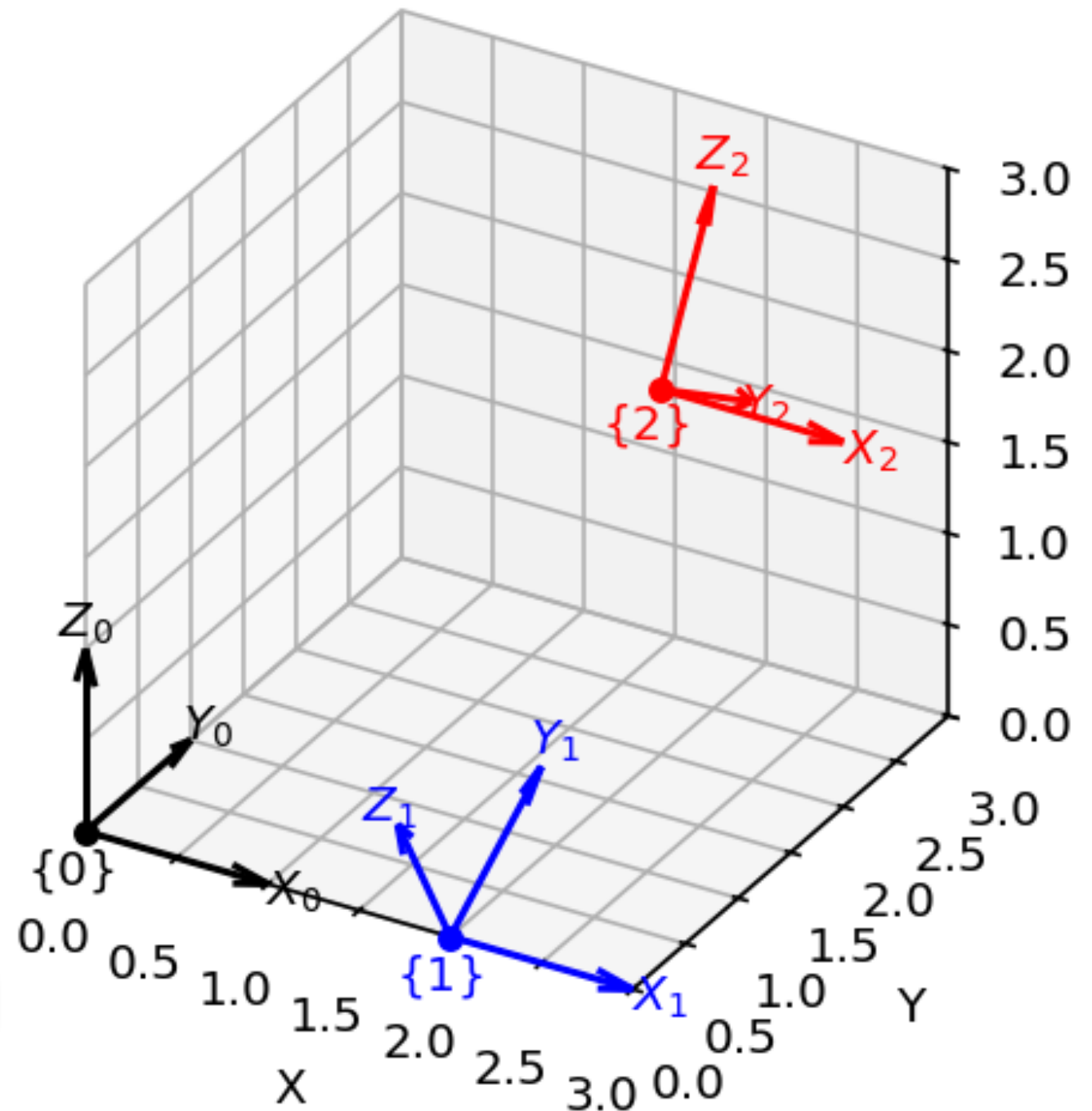
$${}^1R_2 = R_1^{-1} \times R_2$$

$${}^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.866 \\ 0 & -0.866 & 0.5 \end{bmatrix}$$

$${}^2R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -0.866 \\ 0 & 0.866 & 0.5 \end{bmatrix}$$

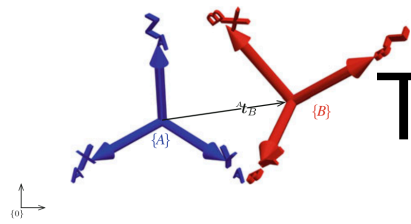
{1}to{2} : Roll Pitch Yaw [-60. -0. 0.]

{2}to{1} : Roll Pitch Yaw [60. -0. 0.]





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Transformation Matrix

$${}^1R_2 = R_1^T \times R_2$$

$${}^2R_1 = R_2^T \times R_1$$

$${}^1R_2 = \begin{bmatrix} 0.7431 & -0.6691 & 0 \\ 0.3345 & 0.3715 & 0.8660 \\ -0.5794 & -0.6435 & 0.5 \end{bmatrix}$$

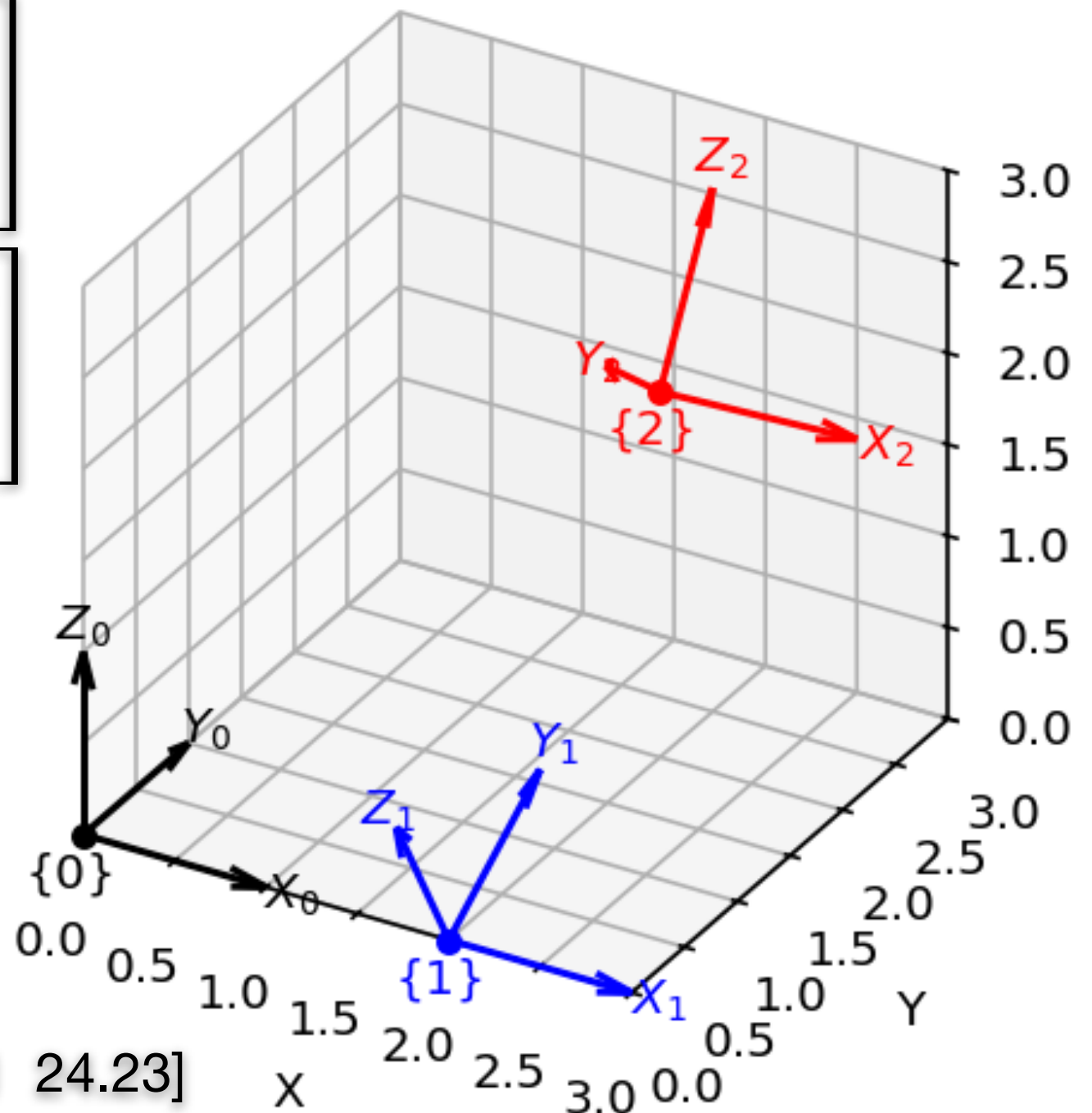
$${}^2R_1 = \begin{bmatrix} 0.7431 & 0.3345 & -0.5794 \\ -0.6691 & 0.3715 & -0.6435 \\ 0 & 0.8660 & 0.5 \end{bmatrix}$$

{0}to{1} : Roll Pitch Yaw [30. -0. 0.]

{0}to{2} : Roll Pitch Yaw [-23.22 19.54 37.94]

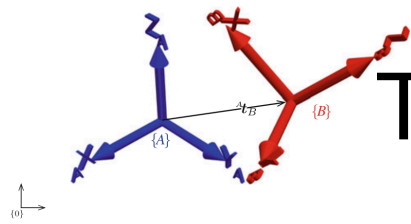
{1}to{2} : Roll Pitch Yaw [-52.15 35.41 24.23]

{2}to{1} : Roll Pitch Yaw [60. -0. -42.]





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Transformation Matrix

Reading a Rotation Matrix

the new frame has its **x-axis** in the old x-direction (1, 0, 0), its **y-axis** in the old z-direction (0,0,1), and the new **z-axis** in the old negative y-direction (0, -1, 0).

$$R_{old} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{new} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

In this case, the **x-axis** was unchanged, since this is the axis around which the rotation occurred.



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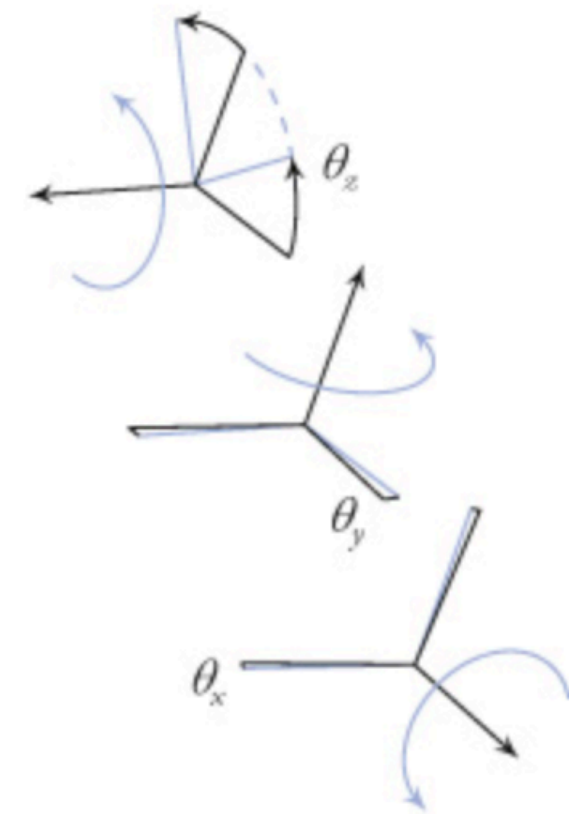
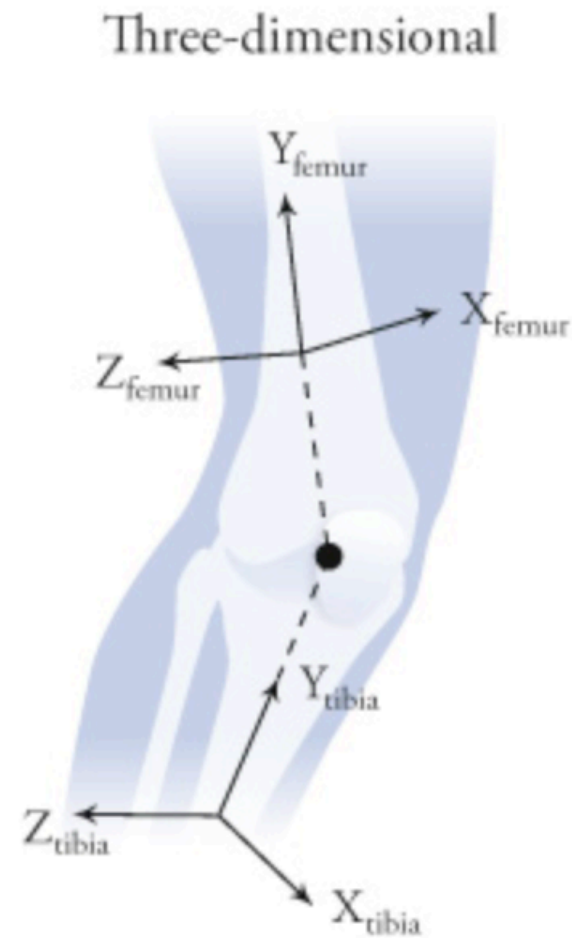
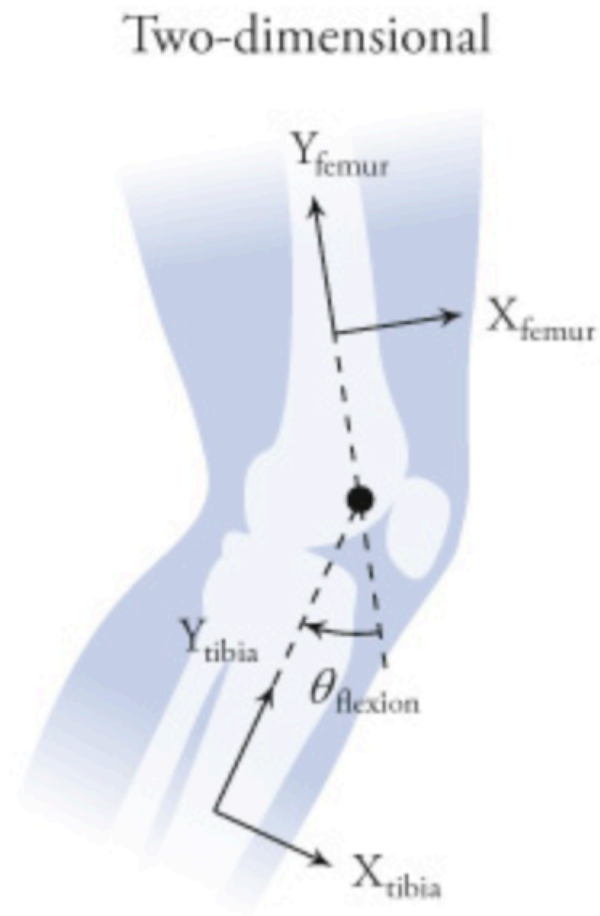
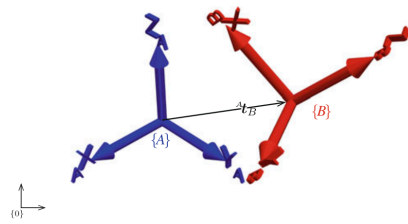


Figure 7.10 Joint angles can be calculated by comparing the orientations of reference frames fixed to adjacent body segments. Knee flexion can be computed by inspection in a two-dimensional analysis (left), but a more formal approach is useful when computing the relative orientation between two frames in three dimensions (right).



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Calculating joint angles with unconstrained inverse kinematics

The great advantage of transformation matrices is that they simplify navigation among multiple coordinate systems. In particular, they allow us to easily calculate anatomical joint angles from data collected in the laboratory frame. Recall that a mocap system records the positions of skin-mounted markers expressed relative to a reference frame fixed in the lab.

To calculate the knee flexion angle, for example, we can determine the transformation matrix relating the reference frames fixed to the femur and tibia:

$${}^{\text{femur}}T_{\text{tibia}} = {}^{\text{femur}}T_{\text{thigh}} {}^{\text{thigh}}T_{\text{lab}} {}^{\text{lab}}T_{\text{shank}} {}^{\text{shank}}T_{\text{tibia}} \quad (7.15)$$

Reading this formula from right to left describes how coordinates in the tibia frame are related to coordinates in the femur frame.



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Calculating joint angles with unconstrained inverse kinematics

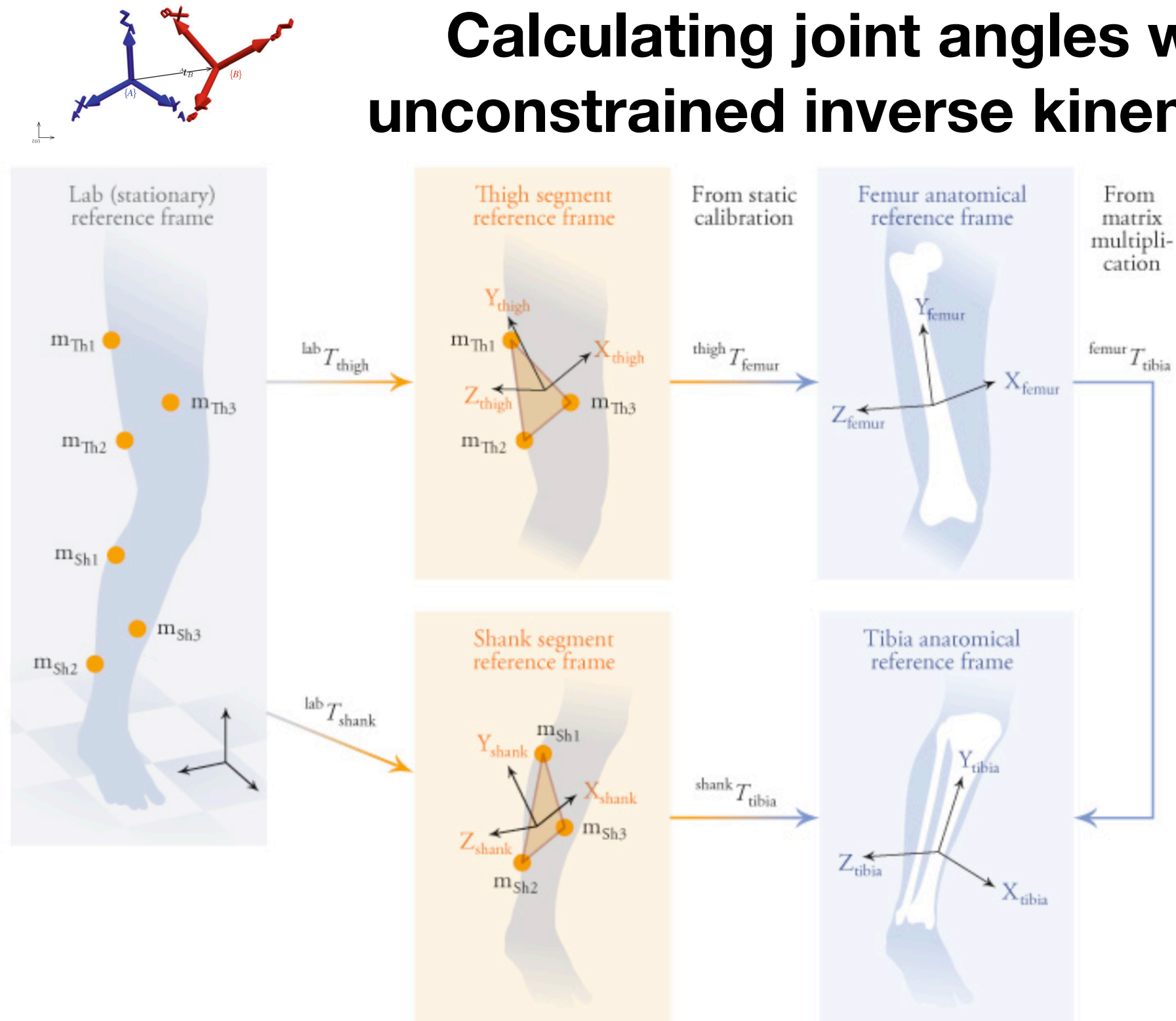
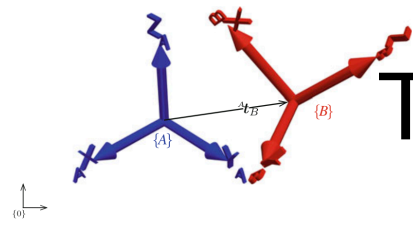


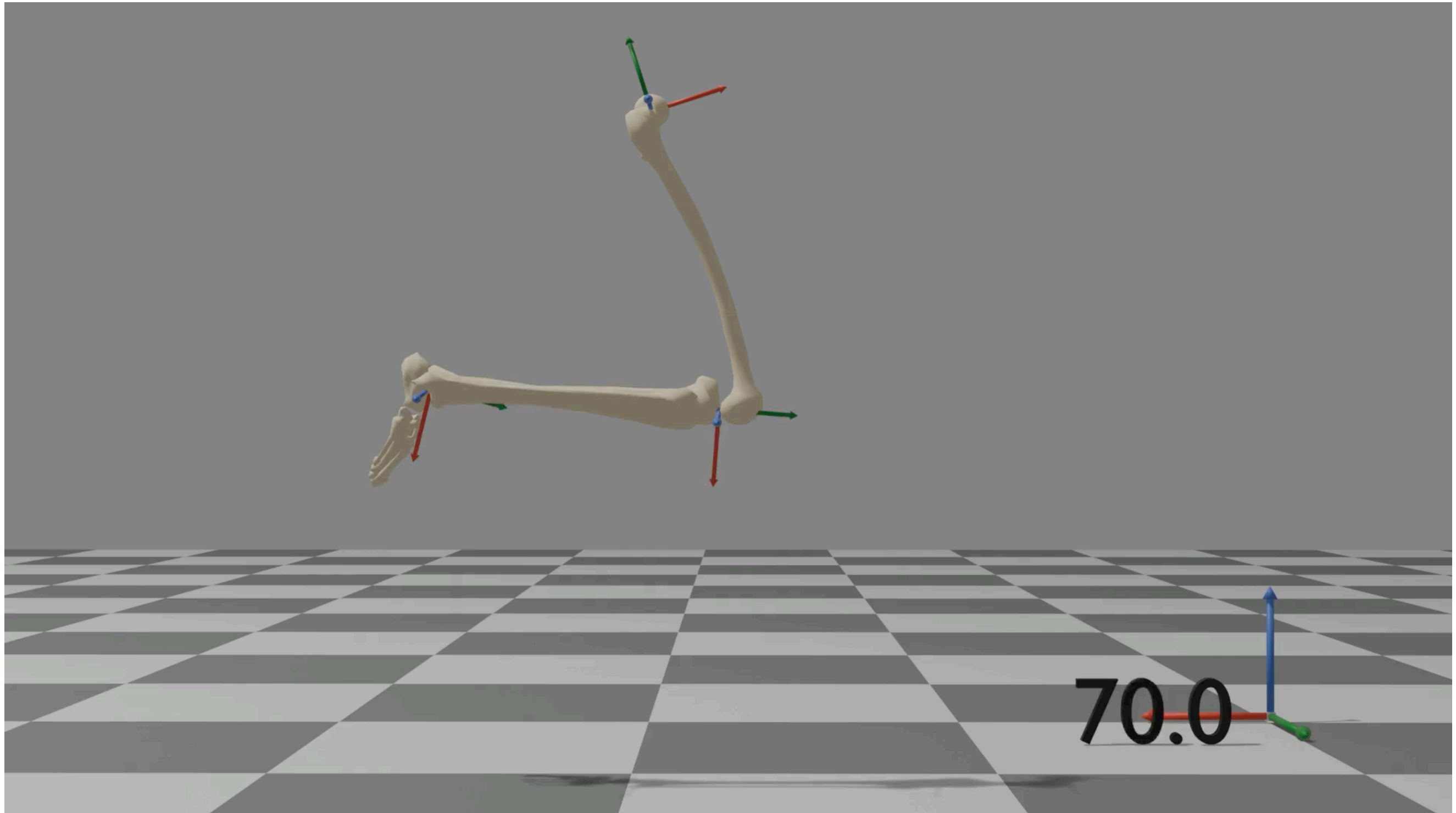
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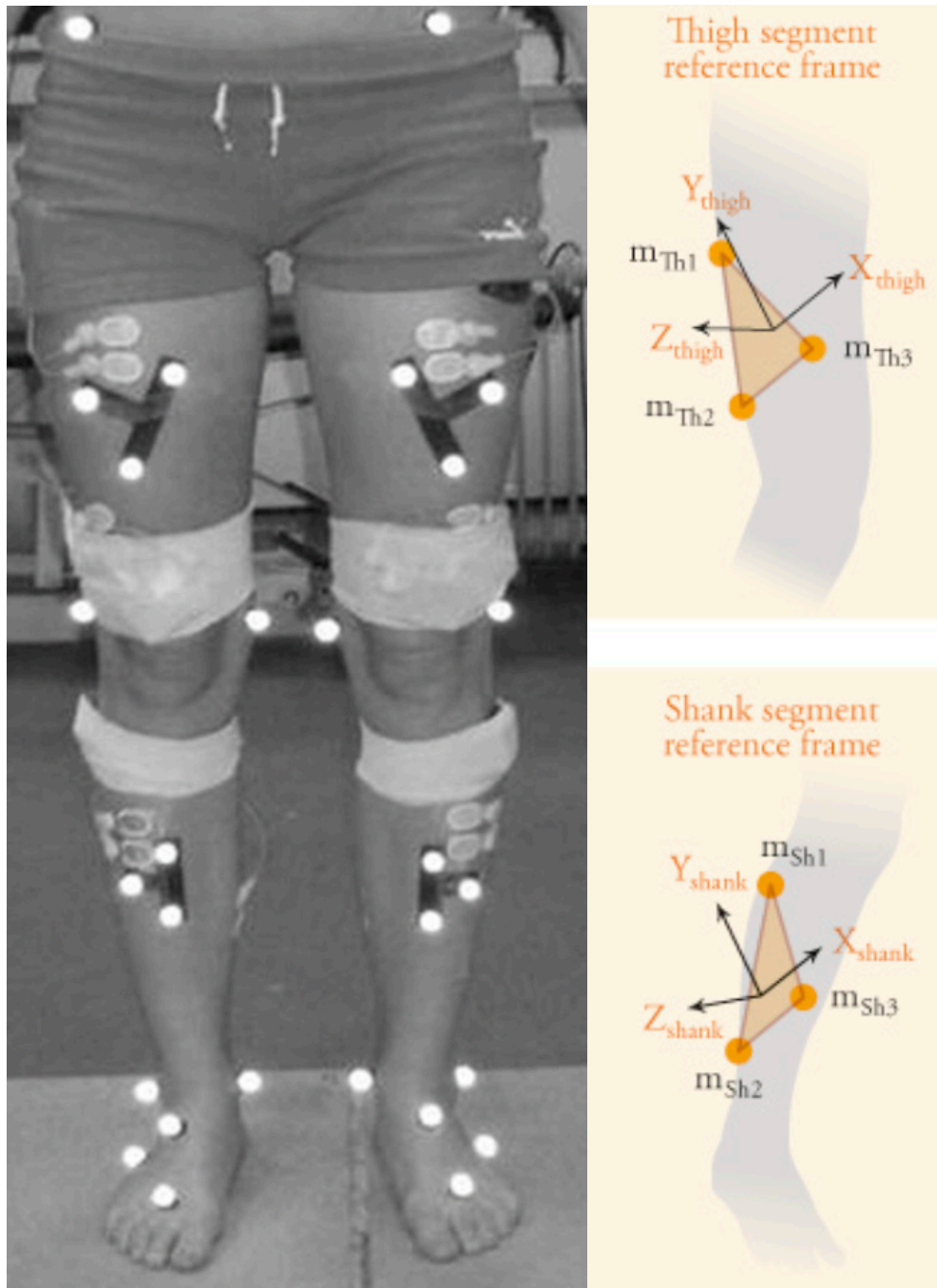
Transformation Matrix





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Lab Study: Joint Kinematics



Calculate knee angle from the cluster markers on thigh and tibia segment



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Lab Study: Joint Kinematics

***The error in global marker locations and reconstructed segment axes is referring to differences relative to the underlying anatomical (or true) segment axes that they represent and which we are trying to indirectly measure. Even when defining the anatomical axes in the subject calibration procedure the anatomical axes location contain error. Therefore, knowledge of sources of error from subject calibration, 3D marker reconstruction, post processing, skin movement artifact, number and placement of markers and mathematical approach need to be carefully considered. As well as the effect these errors have on axes misalignment relative to the anatomical (true) axes and the resulting non-linear errors in derived joint rotations.**

The statement that the benefit of clusters is that they are not placed on a specific anatomical location is not strictly correct, as critical thought is needed when deciding how many and where segment markers will be placed.

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