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SURVEY ARTICLE

POWER EQUATIONS IN ENDURANCE SPORTS

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Abstract—This paper attempts to clarify the formulation of power equations applicable to a variety of endurance activities. An accurate accounting of the relationship between the metabolic power input and the mechanical power output is still elusive, due to such issues as storage and recovery of strain energy and the differing energy costs of concentric and eccentric muscle actions. Nevertheless, an instantaneous approach is presented which is based upon the application of conventional Newtonian mechanics to a rigid segment model of the body, and does not contain assumptions regarding the exact nature of segmental interactions—such as energy transfer, etc. The application of the equation to running, cycling, speed skating, swimming and rowing is discussed and definitions of power, efficiency, and economy are presented.

NOMENCLATURE

A	mean work per cycle
a	acceleration
α	an angle
a_x	component of acceleration along x-axis
ΔS	rate of increase of entropy
Δs	displacement of point of force application
dv/dt	acceleration of the athlete
E	metabolic energy
$\dot{E}$	rate of expenditure of metabolic energy
E_{cg}	potential and kinetic energy of total body center of gravity
E_f	mechanical energy of foot (and skate) segment
e_g	gross efficiency (P_u/P_i)
E_h	energy degraded to heat
E_k	translational kinetic energy
E_l	mechanical energy of lower leg segment
E_m	metabolic energy liberated
e_m	muscle efficiency
E_p	potential energy
e_p	propelling efficiency
E_r	rotational kinetic energy
E_s	strain energy
ΣE_{seg}	sum of segmental energies
F	a force or force component
f	cyclic frequency ($1/T_0$)
$F_{a,i}$	reaction force at joint a on proximal (2) or distal segment (1) of the joint
F_{air}	net force due to air resistance
$F_{air,i}$	force of air friction at the center of mass of segment i
F_D	drag force
F_L	lift force
F_e	external force
F_f	frictional force
F_{ice}	force of ice friction
F_p	propulsive force
F_z	force component along z-axis
g	acceleration due to gravity
$\dot{H}$	chemical power (rate of decrease of enthalpy of foodstuff)
I	mass moment of inertia
m	mass
M_a	moment about the ankle joint
$M_{a,i}$	reaction moment at joint a on proximal (2) or distal segment (1) of the joint
M_e	moment applied to a segment by the environment
M_j	moment at joint j
M_k	moment about the knee joint
m_k	mass of segment k
ω_a	angular velocity of the ankle joint
ω_e	angular velocity of a segment subject to a moment M_e
ω_f	angular velocity of the foot segment
ω_j	angular velocity of joint j
ω_k	angular velocity of the knee joint
ω_s	angular velocity of the shank segment
ω_t	angular velocity of the thigh segment
P	power
P'_u	power output of a system including subject and equipment
P_c	conservative power output component (used to do work against conservative forces)
P_e	power dissipated to the environment
P_g	power associated with overcoming gravity
P_i	power input (the usable fraction of $\dot{H}$)
P_k	power dissipated in giving kinetic energy to the water in swimming
P_m	power used for maintenance activities
P_{nc}	non-conservative power output component (degraded to heat)
P_o	power output (mechanical muscle power)
P_u	mean generated power
T	temperature (K)
t	time
T_i	time interval for integration
T_0	cycle time for cyclic movements
$v(t)$	velocity of the athlete as a function of time
v_{air}	velocity of the air relative to the subject
v_c	velocity of center of mass of segment c
v_e	velocity of the point of application of an external force F_e
v_i	velocity of point i
v_p	velocity of the point of force application
v_x	velocity component along x-axis
v_z	velocity component along z-axis
W	work done on a body
z_c	projection on z-axis of height of point c above a datum

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1. INTRODUCTION

In studies of the mechanics and energetics of human and animal locomotion, the metabolic energy liberated ($\dot{E}$) is often regarded as the main input variable of the moving system. Speed v and its time derivative (acceleration, dv/dt), on the other hand, are seen as the final results of a process where equations of mechanical power are used in attempts to deduce causal relationships between $\dot{E}$ and v. The approaches which are most frequently used in modelling mechanical power can roughly be divided into two categories, a kinematic approach and a kinetic approach.

In the first method, calculations of power from changes of mechanical energy of body links or of the center of mass of the moving system are made. This approach is mainly used in studies on running and walking.

The kinetic approach involves calculations of power based on a summation of the components of power used to overcome frictional losses in work against the environment. This approach is mainly used in studies of terrestrial or aquatic locomotion other than running or walking.

Though a number of studies pay attention to both types of power, most studies seem to be based on ambiguous analysis of possible power dissipation. As noted by Aleshinsky (1986a), studies do not seem to exist where the power calculations for the entire athlete are based on the definition of power, as can be found in textbooks on mechanics. For separate body segments, power equations have been deduced by Quanbury *et al.* (1975) and applied by, e.g., Winter and Robertson (1978).

The first purpose of this paper is to present a general power equation for the assembly of body segments applicable to all endurance sports.

The fact that most power calculations are based on a more or less arbitrary summation of the components of power, rather than on the application of mechanical equations to an assembly of explicitly defined free body diagrams, leads to an overwhelming number of definitions of external and internal power, and of quotients between power and $\dot{E}$ usually referred to as 'efficiency'.

In running and walking, the problems associated with obtaining accurate definitions of power output render the concept of 'efficiency' almost meaningless. In other activities—such as rowing or cycling—where power output against an external resistance can be determined with more precision, a useful—if generally underestimated—measure of efficiency can be determined. There is, however, a need for unambiguous definitions and the second purpose of this survey is to provide such definitions. An additional purpose is to review the various methods presently used in the calculation of power. These methods are discussed first.

Though a number of examples will be deduced from or applied to other activities and other animals, this survey will mainly be focused on human cycling, rowing, running, speed-skating and swimming. Since this survey might be of interest for a broad audience, including exercise physiologists and zoologists, only Newtonian mechanics are used and (planar) examples are given in order to clearly illustrate mechanical concepts which otherwise may be difficult to conceptualize.

2. SUMMARY OF THE LITERATURE AND STATEMENT OF THE PROBLEM

2.1. *Energy liberation and performance*

Many studies show a correlation between the liberation of metabolic power $\dot{E}$ and mean speed as the main performance determining factor in endurance sports (Zatsiorsky *et al.*, 1982). Such correlations were often reported for running (Farrell *et al.*, 1979; Holmer, 1978; Di Prampero, 1986; Daniels, 1985) but also for swimming (Charbonnier *et al.*, 1975; Holmer, 1974; Di Prampero, 1986), cycling (Pugh, 1974; Di Prampero, 1986) and speed-skating (Holmer, 1978; Di Prampero, 1986; de Groot *et al.*, 1988). These relations are likely to have a solid exercise physiological basis, since humans and animals appear to be able to deliver a certain amount of mean metabolic power which (unlike engines) is strongly dependent on the duration of the exercise. $\dot{E}$ further depends upon, e.g., the amount of muscle mass involved (Tesch *et al.*, 1976; de Groot *et al.*, 1988), technique, environmental and body temperature, and psychological factors (see Åstrand and Rodahl, 1977; Cavanagh and Kram, 1985b; Morgan, 1985). Inter-individual differences in $\dot{E}$ are associated with factors like natural ability and training background (Åstrand and Rodahl, 1977).

Synthesizing the approach of various previous investigators (e.g., Tucker, 1975; Ward-Smith, 1985a), Fig. 1 shows a schematic representation of the flow of energy during exercise. The chemical power $\dot{H}$ represents the rate of decrease of enthalpy of the original foodstuff. According to the second law of thermodynamics, only a part (the rate of change of the 'free energy') of this power can be used for biological functions (Åstrand and Rodahl, 1977; Whipp and Wasserman, 1969; Gibbs and Gibson, 1972; Woledge, 1971). This useful component of power will henceforth be called 'power input' P_i (Tucker, 1975; Daniel, 1983) and is essentially equal to the metabolic rate $\dot{E}$ mentioned above. The other part is equal to the product of the rate of increase of entropy ΔS and temperature T (K). In gross body movements, only a part of the power input is converted into mechanical muscle power P_o which in principle might be used to perform movements and to do work against the environment. The rest, the 'maintenance power' (Tucker, 1975), concerns mainly the work-rate of muscles which operate the cardio–respiratory system, and power used in tissues that do no ('useful') mechanical work (e.g., the

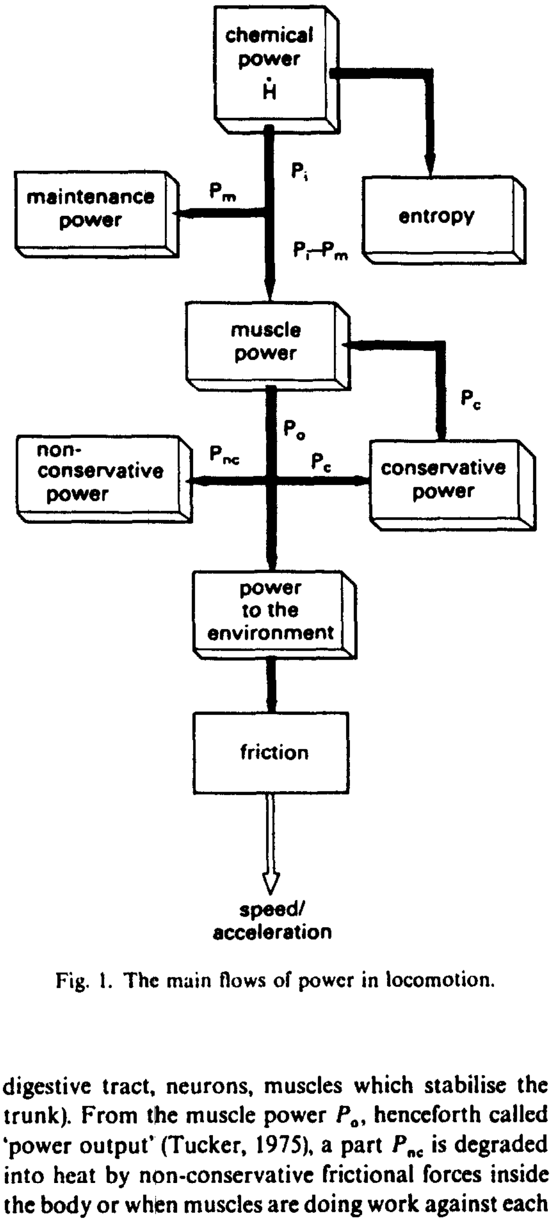

Fig. 1. The main flows of power in locomotion.

digestive tract, neurons, muscles which stabilise the trunk). From the muscle power P_o, henceforth called 'power output' (Tucker, 1975), a part P_{nc} is degraded into heat by non-conservative frictional forces inside the body or when muscles are doing work against each other (Alexander, 1980). A second part P_c is used to do work against conservative forces (e.g., gravity, stretching elastic elements) and can in principle be re-used as P_{nc} or P_e (see pointing arrows) in a later phase of the movement. P_e represents power dissipated to the environment (excluding power expended against gravity).

In running, most studies on mechanical power concern the modelling of P_{nc} and P_c, while in the other types of locomotion, the majority of studies is aimed at modelling P_e. A few attempts have been made to model a direct relation between $\dot{E}$ and speed $v(t)$ in running (Ward-Smith, 1985a; Morton, 1985) in order, for example, to predict the influence of wind on speed (Ward-Smith, 1985b). As discussed by van Ingen Schenau and Hollander (1987) and by Morton (1986), these models at present do not adequately describe the involved physiological processes. Other attempts to predict such influences as environmental conditions, technique, or body composition on speed are mainly based on equations of P_e. Examples of such predictions are the calculation of the influence of shielding (Kyle, 1979; van Ingen Schenau, 1982), technique (Sanderson and Martindale, 1986; Boer *et al.*, 1988) and altitude (Di Prampero *et al.*, 1979; Di Prampero, 1986; van Ingen Schenau, 1982) on speed. Though good agreement with actual performances can be obtained, these models of course have their limitations, since variables such as P_{nc}, P_c and $\dot{E}$ may be influenced by environmental conditions and technique as well. This dependency may also obscure the significance of changes in efficiency calculated on the basis of P_e and P_i.

2.2. *Mechanical power calculations*

2.2.1. *Running.* Since Fenn (1930a, b), many researchers have tried to deduce equations which might describe P_c and P_{nc}, and their interaction for walking and running. The calculation of this power (which is mainly associated with the need for accelerating, lifting and decelerating body segments) is largely based on the following methods (Zatsiorsky *et al.*, 1982):

(a) Calculations based on changes of segmental energy of all body segments (e.g., Winter, 1978; Norman *et al.*, 1976; Pierrynowski *et al.*, 1981; Alexander, 1980; Cavagna and Kaneko, 1977) or of the center of mass (e.g., Cavagna *et al.*, 1976).

(b) Calculations based on summation of joint powers defined as the product of joint moments and their angular velocities (e.g., Winter, 1983).

(c) A combination of these methods (Aleshinsky, 1986a, b; Quanbury *et al.*, 1975; Robertson and Winter, 1980).

In studies of running and walking, P_e is usually considered to represent such a small fraction of P_o that it is frequently neglected. Physiological studies (Pugh, 1971) and, more recently, wind tunnel studies (Kyle, 1979) have shown that P_e, even in distance running, can be at least 10% of P_o. Moreover, Webb *et al.* (1988) showed that the power associated with the deformation of the shoe can be even larger than 10% of P_o.

However if P_e is neglected, it will be clear that the three methods of power calculation lead to a zero mean power output in level running, if the cycle averaged value is calculated by a simple time integral of the instantaneous power. Since the instantaneous power $P_c + P_{nc}$ is not zero, and since the delivered positive and negative mechanical powers within a cycle are clearly associated with a certain P_i, many different assumptions are proposed in order to get an estimate for mean mechanical power in walking and running. These assumptions concern questions about how much energy might be transferred within and between segments [not necessary in method (b)], how much energy can be re-utilised from elastic structures where it was stored in an earlier phase of the cycle, and estimates for the mechanical efficiency of the conversion of P_i into P_e and P_{ne}, for both positive as well as for negative power. It is likely that almost all present methods overestimate the work done, and thus calculate 'efficiencies' which are unrealistically high. Since the methods and their problems are already extensively discussed in other papers (Cavagna and Kaneko, 1977; Williams and Cavanagh, 1983; Williams, 1985; Cavanagh and Kram, 1985a,b; Aleshinsky, 1986a–e), the reader is referred to these papers and their references; to prevent too much repetition, it is assumed in the remaining part of this paper that the reader is familiar with the content of those papers. It should be mentioned here that Aleshinsky (1986a) was the first author to present an analysis for the entire athlete based on a systematic and unambiguous application of basic mechanics. However, in his application of this method to running, he met with similar problems to those described above for other investigators.

2.2.2. *Other endurance sports.* As stated above, power measures for endurance sports other than running are mainly based on analysis of possible flows of energy to the environment. Di Prampero can be regarded as an important pioneer in this field. (Di Prampero *et al.*, 1971, 1974, 1976, 1979). He and most other authors (e.g., van Ingen Schenau, 1982, 1988) simply take the (scalar) product of opposing forces (mainly frictional forces with the environment) and the velocity of the athlete and his equipment (bicycle, boat), which is taken as equal to the velocity of the point of application of these forces. Though it may be true that the majority of P_o is used to overcome friction, and though this power calculation is based upon a reasonable application of the product of force and velocity, this modelling of P_e has nothing to do with a general and unambiguous application of classical mechanics on the entire system as advocated (and applied to running) by Aleshinsky (1986a–e). If the entire system of athlete and equipment (e.g., bike and rider or rower and boat) is taken as the free body for the purposes of analysis (which implicitly is the case in these referenced studies), all external forces (defined by the choice of the free body) should be accounted for, including the propulsive force(s). Moreover the question may arise how to define P_e as power causatively related to the metabolic rate P_i. If for example a skater suddenly stops pushing off, he can still deliver P_e for a certain time interval, but it is clear that P_e has no relation with P_i at the same time (so does the skater deliver external power or not?). Of course we know that in such a case the skater can overcome friction at the expense of his kinetic energy, but how should this power be defined? This example shows that here also is a need for an unambiguous approach in order to deduce proper definitions for power.

2.2.3. *Other approaches.* A number of authors have used the product of the propulsive force and the velocity of the center of mass of the system in order to get a measure for mechanical power (Fukunaga *et al.*, 1981; Sanderson and Martindale, 1986; Tucker, 1975; van Ingen Schenau *et al.*, 1985). Tucker (1975) states that this product is equal to the power needed to overcome drag. van Ingen Schenau *et al.* (1985) and Fukunaga *et al.* (1981) associate this power with the increase of mechanical energy of the athlete's center of mass. None of these studies (nor the many studies on jumping where the same method is used) provide convincing evidence for the applicability and validity of this product, which is not based on the definition of power since the velocity of the point of application of the propelling force is not equal to the velocity of the center of mass. In running (Fukunaga *et al.*, 1981) and jumping, the velocity of the point of application of this force is actually zero. The significance of this approach will be discussed below.

From the above it is apparent that at present a number of different methods are being used to calculate different types of power. It is not clear however how these various powers are related.

3. PROBLEMS IN APPLYING CLASSICAL MECHANICS

Since, apart from Aleshinsky, all referenced authors construct power equations for the entire athlete by adding up possible flows of energy, the question arises to what extent classical mechanics provides adequate tools for an approach based on mechanical laws and definitions.

When applying Newton's second law $\Sigma F = ma$, the result is unambiguous: after one has constructed a free body diagram, the internal and external forces are defined by the boundaries of the diagram, and the law provides a unique relation between the external forces on the one hand and the acceleration of the center of mass of the system on the other hand. This is also true for a system which is deformed under the influence of these external forces, with or without energy dissipation by internal forces. Internal forces do not play any role in this force equation. The same is true if the time integral of this force equation is taken. The (impulse–momentum) equation is also uniquely defined. In both equations, gravity does not play a specific role but is simply one of the external forces which should be accounted for.

Since power can be defined as a flow of energy, one might expect a comparable relationship between the flow of energy associated with the external forces and the rate of change of energy of the chosen free body. Such a power equation or its time integral called 'work–energy theorem' (Robertson and Winter, 1980), 'energy balance' (Aleshinsky, 1986b), or 'work–energy equation' (Meriam, 1975), however, cannot be constructed by the same unambiguous methodology as is the case for a force or a force–time integral equation. This has simply to do with the fact that the human body can liberate metabolic energy, degrade mechanical energy into heat, and store energy in elastic structures.

If we wish to link information concerning the force environment in a given biological problem with these factors, then the problem must be approached using a work–energy principle which states that the work done on a body is equal to the change in all these energies:

$$W = \Delta E_p + \Delta E_k + \Delta E_r + \Delta E_s + \Delta E_h + \Delta E_m \quad (1)$$

where E_p is potential energy, E_k is translational kinetic energy, E_r is rotational kinetic energy, E_s is strain energy, E_h is energy degraded to heat, E_m is liberated metabolic energy. If the time interval Δt is considered as $\Delta t \to 0$, the expression for power becomes:

$$P = \mathrm{d}E_p/\mathrm{d}t + \mathrm{d}E_k/\mathrm{d}t + \mathrm{d}E_r/\mathrm{d}t + \mathrm{d}E_s/\mathrm{d}t + \mathrm{d}E_h/\mathrm{d}t + \mathrm{d}E_m/\mathrm{d}t. \quad (2)$$

If the athlete is choosen as the free body, it will be clear that at present there are no reliable models available to calculate E_s and E_h. This means that an approach where equation (2) is applied to the entire athlete is not (yet) possible.

For a rigid body, however, the right-hand side of equation (2) can be fairly easily stated using the conventional expressions for potential and kinetic energies, since the last three terms are zero. The formulation of the left-hand side of the equation in terms of the forces acting, however, needs care in a gravitational environment, since the effect of gravity must be included either on the 'work' side of the equation or the 'energy' side of the equation, but not both (Meriam, 1975).

To illustrate this, consider a passive rigid body shown in Fig. 2 moving under the influence of the external force F_z in pure translation with a constant velocity v_z in the vertical direction. Its position above an arbitrary datum at time t is z. The only other external force acting is the force of gravity, mg.

Using equation (2) for this problem yields

$$\mathrm{d}(E_p)/\mathrm{d}t = \mathrm{d}(mgz)/\mathrm{d}t = mgv_z \quad (3)$$

since E_k is constant, E_r is set to zero and E_s is zero for this rigid body.

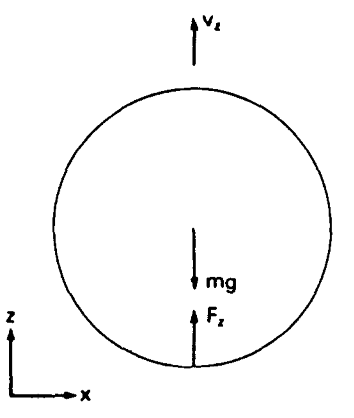

Fig. 2. If a body is moved under the influence of an external force F_z, one should include the effect of gravity either as a (potential) energy term or as a work term in the formulation of a work–energy equation for the body (see text).

The evaluation of P in terms of the forces acting requires the evaluation of the work done by the force during a displacement of its point of application as follows:

$$W = \mathbf{F} \cdot \Delta \mathbf{s}$$

where Δs is a displacement of the point of application. When evaluated in the time interval Δt as $\Delta t \to 0$, the expression for power becomes:

$$P = \mathbf{F} \cdot \mathbf{v}_z.$$

In the present example there are two forces acting on the body, the force F_z and gravity.

The power associated with the external force F_z is

$$P_e = \mathbf{F}_z \cdot \mathbf{v}_z$$

and the power associated with the force of gravity

$$P_g = -m\mathbf{g} \cdot \mathbf{v}_z$$

with the minus sign indicating that the force and the displacement are in opposite directions.

It would now be tempting to consider the total power as $P_e + P_g$ and substitute this in the left-hand side of equation (2), leading to the anomalous result:

$$\mathbf{v}_z \cdot (\mathbf{F}_z - m\mathbf{g}) = m\mathbf{g} \cdot \mathbf{v}_z.$$

This result is false since we know from Newton's second law that $F_z = mg$ and that the rate of change of potential energy, $m\mathbf{g} \cdot \mathbf{v}_z$, is not zero as the equation would predict.

Change in potential energy is simply the work done by or against gravity, and thus one or the other—but not both—should be included in the work–energy relationship.

Thus the correct formulation of equation (2) for this illustrative example would be

$$\mathbf{F}_z \cdot \mathbf{v}_z = m\mathbf{g} \cdot \mathbf{v}_z$$

which, since $F_z = mg$, is true. In the remainder of this paper, we shall choose to include the effects of gravity as an energy term on the right-hand side of the work–energy equation.

One final principle needs to be presented before examining the application of power equations to various problems in human movement. Clearly all external forces, other than gravity, should be included in the development of an adequate model. In endurance sports where forces such as friction provide the principal resistance to motion (including drag from air or water, rolling friction, etc.) there are no serious problems. Power associated with these forces can be calculated from the product of the forces and the velocities of their points of application. But the propelling force is somewhat more obscure.

In the simple passive system shown in Fig. 2, because the environment adds power to the system, the flow of energy could be derived according to the definition of power. The energy of a passive system (such as a discus or shot) can clearly only be increased at the expense of a source external to the system (such as the thrower). If we exclude paired activities (such as tandem cycling or crew) and environmental factors such as a strong back wind and compliant floors, such a situation does not exist in most locomotor activities. All mechanical power is delivered by the transformation of metabolic power into mechanical power (Zarrugh, 1981). The following definition of locomotor activities from the perspective of power therefore seems to hold true:

In locomotor activity there is no external force that adds positive power to the system.

Stated formally, the propulsive force $\mathbf{F}_p$ will always obey the relationship:

$$\mathbf{F}_p \cdot \mathbf{v}_p \leqslant 0 \tag{4}$$

where $\mathbf{v}_p$ is the velocity of the point of application of $\mathbf{F}_p$. As will be shown later, this relationship needs further discussion when power equations are applied to different types of locomotion.

The following conclusions may be drawn from the above:

(1) For an endurance athlete as a system (including equipment), a power equation based on the rate of change of mechanical energy of the system and the powers associated with the displacement of external forces should include internal power as dissipated into heat, stored in elastic structures and generated by the contractile elements of the muscles.

(2) For rigid bodies, equation (2) can unambiguously be applied provided that the special character of gravity is taken into account (Meriam, 1975). Such equations have previously been deduced from and applied to individual human limb segments by, e.g., Quanbury *et al.* (1975) and Winter and Robertson (1978).

4. AN INSTANTANEOUS POWER EQUATION

A power equation which includes the rate of change of all energies mentioned in equation (1) is difficult to construct (Norman and Komi, 1987). At the present state of knowledge there are no models available which predict reliable measures for positive and negative power from active elements of individual muscles and their efficiencies. This is not only due to the indeterminacy problem (Miller, 1979) since even the application of direct dynamics (e.g., Hatze, 1977, 1978) suffers from severe uncertainties. Apart from problems in evaluating the reliability of proposed cost functions, this is, e.g., a result of the lack of information about the compliance of the series elastic elements of the muscle tendon complexes involved. As can be concluded from the work of Cavagna *et al.* (1968, 1985), Alexander and Bennet-Clarke (1977), Morgan (1977), Morgan *et al.* (1978), Walmsley and Proske (1981), Jaric *et al.* (1985) and van Ingen Schenau (1984), the capacity for muscles and tendons to store elastic energy is often overestimated, while the effects of pre-stretch are, in part, not due to storage and re-utilisation of elastic energy but to a phenomenon usually referred to as muscle potentiation. Since Hill-type models (Hill, 1938) do not account for this potentiation, more reliable results might be expected in the future from Huxley-type models (Huxley, 1957; Zahalac, 1981, 1986). Other necessary information concerns the (instantaneous) total muscle lengths and the contractile properties of the individual muscles (Hof *et al.*, 1983; Bobbert *et al.*, 1986; Whiting *et al.*, 1984). For most muscles this information is not yet available.

All these (and other) uncertainties render the formulation of a generalised instantaneous power equation for locomotion based on equation (1) impossible for this time.

4.1. *A model consisting of rigid links*

Apart from movements involving the shoulder joint (which allows considerable translation), human movements can to a large extent be described as the sum of rotations performed at joints between the body segments. This means that the part of the work liberated in muscular action which is used for limb movement and to do work against the environmental forces is predominantly being reflected by these joint rotations. Many researchers have applied the method of analysis as proposed by Elftman (1939a, b) in order to assess the power associated with these joint rotations. In this method the body is modelled to consist of an interconnected chain of rigid links. The mechanical influence of segment i on the adjacent segment $i+1$ is calculated using inverse dynamics. In planar motions all these influences are summarised in a hypothetical reaction force $F_{a,i}$ and moment $M_{a,i}$. The analysis usually begins with a distal segment where the external forces and moments are known (e.g., ground reaction forces) and all intersegmental forces and moments are subsequently calculated (Winter, 1983; Zatsiorsky *et al.*, 1982; Ericson *et al.*, 1986). Note that rigid links do not generate power and do not degrade power into heat. This means that power is supplied to or absorbed from a segment by the hypothetical intersegmental forces and moments, and by environmental forces such as air resistance (Winter and Robertson, 1978; Robertson and Winter, 1980). As discussed above, equation (2) is applicable to each rigid segment. For running, Robertson and Winter (1980) showed that the flows of energy associated with the external forces and moments of a segment are indeed in balance with the rate of change of the mechanical energy of the segment. The only exceptions were found for the foot during foot strike and push-off where significant deformations of the foot occur.

As advocated by Zarrugh (1981), a power analysis for the entire athlete should include both the rate of change of segmental energies and the joint powers. In his original contributions to this journal, Aleshinsky (1986a, b) was the first to succeed in deducing such a power equation showing an unambiguous relation between the rate of change of segmental energies on the one hand and the sum of power from environmental sources and joint power on the other hand. Aleshinsky's deduction was based on a summation of the power equations constructed for each rigid link. These power equations were derived by differentiation of the segmental energy of the links (Aleshinsky, 1986a, b). To avoid the complicated mathematics necessary in that approach, we take here the left-hand side of equation (2) as a starting point. Figure 3 shows an example of three links with the external forces and moments per link (except gravity). The example concerns the leg of a skater but the deduction would essentially be the same if it concerned a skier, cyclist or a runner.

According to equation (2), the power equation for the foot is as follows:

$$\mathbf{F}_p \cdot \mathbf{v}_p + \mathbf{F}_{ice} \cdot \mathbf{v}_p + \mathbf{F}_{air,1} \cdot \mathbf{v}_2 + \mathbf{F}_{a,1} \cdot \mathbf{v}_3 + M_{a,1}\omega_f = \mathrm{d}E_f/\mathrm{d}t \qquad (5)$$

where $\mathbf{F}_p$ is the reaction force from the ice, $\mathbf{F}_{ice}$ and $\mathbf{F}_{air}$ the ice friction and air friction forces respectively, $\mathbf{F}_{a,1}$ and $M_{a,1}$ the joint force and moment. The forces are multiplied by the velocities of the points of force application and $M_{a,1}$ by the angular velocity ω_f of the foot. E_f is the mechanical energy of foot and skate:

$$E_f = \tfrac{1}{2} m_f v_c^2 + m_f g z_c + \tfrac{1}{2} I \omega_f^2 \qquad (6)$$

with m_f the mass of foot and skate, v_c the velocity of its center of gravity, z_c the vertical position of this segmental center of gravity and I the moment of inertia of foot and skate relative to the center of gravity. For the lower leg the power equation is:

$$M_{a,2}\omega_s + \mathbf{F}_{a,2} \cdot \mathbf{v}_3 + \mathbf{F}_{air,2} \cdot \mathbf{v}_4 + \mathbf{F}_{k,1} \cdot \mathbf{v}_5 + M_{k,1}\omega_s = \mathrm{d}E_s/\mathrm{d}t \qquad (7)$$

where the symbols used are comparable to those of the foot. These equations can be constructed for all other rigid links although not all external forces and moments per link have to be known. This may be the case, for example, in cycling where the trunk is supported by the saddle and where a reaction force and moment can act on the hands. When compared to the work presented by Quanbury *et al.* (1975), Winter and Robertson (1978), and Robertson and Winter (1980), the new step made here is that we now add the

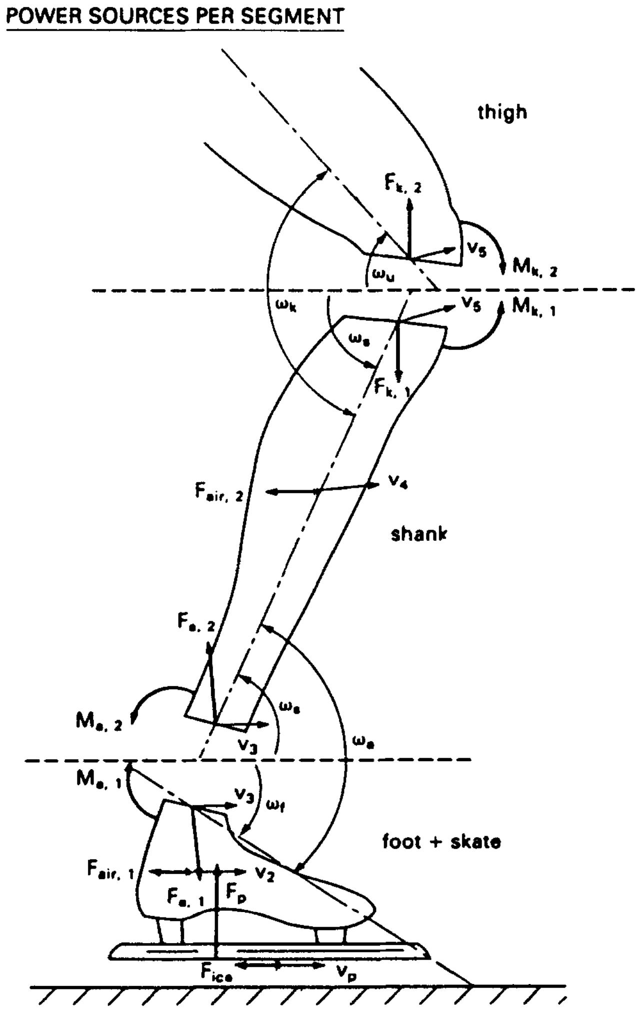

Fig. 3. If the body of a skater is modelled to consist of interconnected rigid links, the sources of power per link are the external moments and forces (except gravity) per link.

equations for the separate links in order to obtain one equation for the entire athlete. If one realises that $\mathbf{F}_{a,1} \cdot \mathbf{v}_3 + \mathbf{F}_{a,2} \cdot \mathbf{v}_3 = 0$ but that $M_{a,1}\omega_f + M_{a,2}\omega_s = M_a\omega_a$, it is clear that a summation of all these power equations yields the following power equation for the system:

$$\mathbf{F}_p \cdot \mathbf{v}_p + \mathbf{F}_{ice} \cdot v_p + \mathbf{F}_{air} \cdot \mathbf{v}_{air} + \Sigma M_j\omega_j = \mathrm{d}\Sigma E_{seg}/\mathrm{d}t \quad (8)$$

where M_j and ω_j are joint moment and angular velocity per joint respectively. $\mathbf{F}_{air} \cdot \mathbf{v}_{air}$, which represents the total power lost to air-friction, is approximately equal to the main air-friction force averaged over the segments times the mean velocity of the segments with respect to the air (v_{air}), and ΣE_{seg} is the sum of the segmental energies. If, as already discussed above, $\Sigma M_j\omega_j$ is to be judged as the origin of power in locomotion, all the other expressions can be judged as locations of power dissipation. To express the causal relation for this and other locomotor activities, all powers associated with external forces F_e are summarized and placed on the right-hand side, leading to the general instantaneous power equation:

$$\Sigma M_j\omega_j = \mathrm{d}\Sigma E_{seg}/\mathrm{d}t - \Sigma \mathbf{F}_e \cdot \mathbf{v}_e. \quad (9)$$

For the most general case, this equation can be extended with an expression $\Sigma M_e\omega_e$ reflecting power associated with environmental moments which might act on a segment which has an angular velocity ω_e (Aleshinsky, 1986a, b). Such contributions however are scarce since a pure moment requires a combination of pushing and pulling forces. This might occur in, e.g., the hands in cycling. However in that situation, ω_e will be negligible. It should be emphasised that in this power equation the contributions of the hypothetical joint forces are cancelled out. The meaning of this is that these joint forces cannot generate power; they can only redistribute power among the segments (see Winter and Robertson (1978) and Robertson and Winter (1980) for further discussion of this power concept). Although a different deduction is used here, equation (9) is essentially equal to the power equation deduced by Aleshinsky (see equation (20) in Aleshinsky, 1986b).

As stated before, each contribution $\mathbf{F}_e \cdot \mathbf{v}_e$ from the environment should be negative or zero. This means that the expression $-\Sigma \mathbf{F}_e \cdot \mathbf{v}_e$ can also be defined as the sum of positive powers which flow to the environment. Expressed in words, equation (9) says that, at each instant, the summed joint power is equal to the rate of change of the energy content of the segments plus the power which flows to the environment. As such, it shows the causal relation between the three main existing approaches mentioned above.

5. PROBLEMS IN APPLYING THE POWER EQUATION

5.1. *General remarks*

Though it appeared to be possible to unambiguously deduce an instantaneous power equation based on a linked segment model, this result should mainly be judged as a firm mechanical basis for existing approaches. Equation (9) does not solve most of the problems that were previously described for running. This is due to the (hypothetical) nature of the joint moments and joint power. Since joint moments are net moments (sum of all contributing structures), the origin of positive power and the ultimate location of destination of negative joint power cannot be assessed.

Positive power can originate from active as well as elastic elements of active muscles. Negative joint power can reflect power which is degraded into heat by friction in joints or other passive structures or by eccentric actions of active muscles, but it can also be stored in elastic structures and later be re-utilised as positive joint power. Positive joint power does not account for power which is simultaneously degraded into heat by friction or when agonists do work against their antagonists which cross the same joint. Bi-articular muscles may distribute power over the adjacent joints which are crossed by these muscles. As convincingly shown previously (van Ingen Schenau *et al.*, 1987; Bobbert and van Ingen Schenau, 1988; van Ingen Schenau, 1989) one cannot simply assign positive or negative joint power to the actions of muscles which cross a joint. For example, the bi-articular gastrocnemius can oppose the knee extensors in such a way that no, or even negative, power is generated in the knee joint, since the net moment is zero (or negative). In reality, the knee extensors may lift the calcaneus via the gastrocnemius muscle (Gregoire *et al.*, 1984; van Ingen Schenau *et al.*, 1987; Bobbert *et al.*, 1986). In other words, the joint power that appears computationally to have its source at the ankle can, to a large extent, be liberated in the knee extensor muscles. Since the extremities contain many important poly-articular muscles, such a redistribution of power over the joints will often occur. This does not mean that bi-articular muscles cause erroneous results when calculating the powers expressed by equation (9). Bi-articular muscles can cause an intercompensation of joint powers which, however, does not affect the summed joint power. Errors in the calculation of the joint powers will mainly be caused by the necessary assumptions concerning the rigidity of the links and the position of the (moving) axes of rotation.

It should be emphasized here that the redistribution of joint powers by action of poly-articular muscles reflects a process which is previously defined as a transport of energy (e.g., van Ingen Schenau, 1989). This expression should not be confused with the transport of energy between actual segments as described by, e.g., Quanbury *et al.* (1975), Winter and Robertson (1978), and Robertson and Winter (1980). If one is interested in the flow of energy to actual segments, one should use the separate equations [here, equations (5) or (7)]. After summation of these separate equations, the information about flows of energy between segments is lost.

On the other hand it should be stressed that the action of bi-articular muscles obviously also influences the net moments as used in the separate equations. This means that the warning concerning the actual origin of power as made with respect to equations (8) and (9) is also valid if one uses the separate equations as performed by Winter and Robertson (1978) and Robertson and Winter (1980).

Equation (9) is only useful when instantaneous flows of energy are to be studied. For many applications it is necessary to calculate a measure for mean power production. This means that equation (9) has to be integrated with respect to a time interval T_i and divided by T_i:

$$(1/T_i)\int_0^{T_i} M_j\omega_j \mathrm{d}t=(1/T_i)\int_0^{T_i} -\Sigma \mathbf{F}_e\cdot\mathbf{v}_e \mathrm{d}t + (1/T_i)\int_0^{T_i} (\mathrm{d}\Sigma E_{\mathrm{seg}}/\mathrm{d}t)\mathrm{d}t. \quad (10)$$

Since, in cyclic movements, the cycles are repeated each time interval T_0 with the cycle frequency $f=1/T_0$, equation (10) can be replaced by:

$$f\int_0^{T_0} \Sigma M_j\omega_j \mathrm{d}t=f\cdot\int_0^{T_0} -\Sigma \mathbf{F}_e\cdot\mathbf{v}_e \mathrm{d}t+f\cdot\Delta E_{\mathrm{cg}}. \quad (11)$$

Apart from the potential and kinetic energy of the body center of gravity, the mean change of segmental energies during a cycle is zero. This means that ΔE_{cg} reflects the change in potential and kinetic energy of the center of gravity of the athlete. The left hand side of equation (11) can be used as a measure of mean generated power P_u:

$$P_u=f\cdot A=f\int_0^{T_0}\Sigma M_j\omega_j \mathrm{d}t \quad (12)$$

where A is the mean work per cycle. This P_u is also equal to the mean power reflected by the right-hand side of equation (11). However, it is often convenient to take not only the athlete but also his equipment (bicycle, boat, ski) as a system. In that case one can use:

$$P'_u=f\int_0^{T_0}\Sigma-\mathbf{F}_e\cdot\mathbf{v}_e \mathrm{d}t+\Delta E_{\mathrm{cg}}\cdot f \quad (13)$$

where $P_u-P'_u$ equals the power lost in the transmission system (in friction with the oar pin, chain, gearing, etc.). $\mathbf{F}_e$ now reflects environmental forces which are external to the entire system (propelling force on the rear wheel, water friction on the boat etc.). The applicability of these equations is now discussed for a number of locomotor activities.

5.2. *Terrestrial locomotion*

5.2.1. *Running.* In running, equations (12) and (13) represent only a fraction of P_0 and are of little use except in sprinting situations where a significant fraction of P_0 is used to accelerate the body.

Equations (9) and (12), however, can be used to achieve insight into the order of magnitude of power lost against environmental forces. As can be deduced from Robertson and Winter (1980), a considerable amount of power may be dissipated by deformations of foot and shoe. For deformations of the shoe, the same conclusion was drawn by Webb *et al.* (1988) in a calorimetric study where the power input P_0 was compared with heat production. A considerable amount of power may also be associated with running on loose sand or running on a treadmill the belt speed of which is distinctly influenced by the runner. If the speed fluctuations of the belt are small (e.g., < 5%), the flow of energy to the treadmill can be neglected (Webb *et al.*, 1988) and no mechanical difference exists between level and overground running (van Ingen Schenau, 1980).

5.2.2. *Cycling.* In cycling, different applications of equations (12) and (13) are possible. When applying equation (12) to a segmental analysis of the lower extremity, the measurement of direction and magnitude of the force applied on the feet by the pedal (Soden and Adeyefa, 1979; Ericson *et al.*, 1986) should be included. A second (and relatively simple) approach possible in cycling is to use such a force measurement to calculate the power delivered to the pedal.

For the instantaneous power it should be noted that in ergometer cycling (no air friction), the flow of energy to the pedal should be equal to the summed joint power minus the rate of change of segmental energy [see equation (9)]. Recently this equality was validated by experimental data (van Ingen Schenau *et al.*, in press). Applying equation (13) to the entire system (athlete and bicycle) will lead to a slightly lower result since frictional losses in chain, gearing, etc. are not accounted for. A few notes should be made concerning this last application.

At first glance, the propelling force on the rear wheel seems not to obey equation (4). Since its point of force application is always at the same position relative to the bicycle frame and thus seems to have the same velocity as the frame, it seems as if the propelling force adds power to the system. This paradox however is due to the unique property of a wheel, and can easily be explained if it is realised that the velocity of the part of the tire which is in contact with the road is always zero (when slip does not occur). The particles of the tire which are in contact with particles of the road exert forces on each other with no relative velocity. A wheel thus can be imagined as a centipede with an infinite number of legs that consecutively push off against a fixed point of the road (Fig. 4). The main difference with running is that in contrast to running, the process of lifting these legs from the ground and bringing them to the next point of contact hardly requires any power. Given the other environmental forces, rolling and air resistance (Di Prampero *et al.*, 1979; Faria and Cavanagh, 1978; Raine, 1970; Zatsiorsky *et al.*, 1982) a complete power equation can be formulated for different situations (level, uphill or downhill, with or without wind).

Though the frictional forces are described sufficiently in the literature, a note should be made on the influence of (side-) wind. In the case of a wind velocity $\Delta\mathbf{v}$, which has a different direction from the velocity $\mathbf{v}$ of the cyclist, one should first determine the magnitude

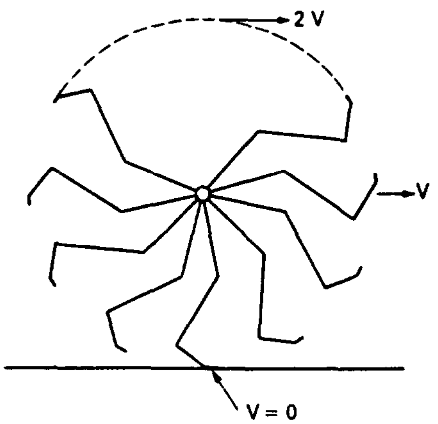

Fig. 4. A wheel acts as a centipede with an infinite number of legs that consecutively push off against a fixed point on the road.

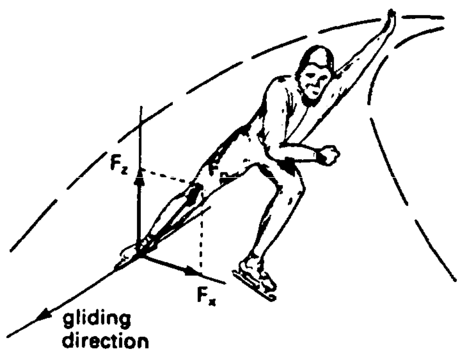

Fig. 5. Since the skate is gliding forwards during push-off, the propulsive force F_p of a speed skater can only lie in a plane which is at right angles to the gliding direction.

of the sum $(\mathbf{v} + \Delta\mathbf{v})$ before the air friction force $\mathbf{F}_{air}$ can be determined [proportional to $(\mathbf{v} + \Delta\mathbf{v})^2$]. The power required to overcome air friction then equals $\mathbf{F}_{air} \cdot \mathbf{v}$. A calculation based on 'components' of air friction force based on $\mathbf{v}$ and $\Delta\mathbf{v}$ respectively will lead to wrong results. If for example the wind $\Delta\mathbf{v}$ is at right angles to $\mathbf{v}$ (pure side-wind), the latter method will lead to a power estimate which is equal to cycling with the same velocity in the absence of wind. The correct approach however will show that a side-wind always requires more power.

5.2.3. *Speed skating.* In contrast to cycling where one might speak of a 'stepwise' displacement of the point of application of the push-off force, the proper gliding technique in speed skating requires a push-off while the skate continues to glide forwards (Haase, 1953; Djatschkow, 1977; van Ingen Schenau *et al.*, 1985; Boer *et al.*, 1986). The solution of this paradox is simple: it is essentially impossible for a forward gliding system to exert any force on the underlying surface in the direction opposite to this gliding direction. So skaters can (and do) push off only in a plane which lies at right angles to the gliding direction of the push off skate (Fig. 5). This results in the typical 'sine wave'-like trajectory of the body center of gravity (see van Ingen Schenau *et al.*, 1987 for further references). Though essentially different from cycling, skating thus also obeys equation (4), since the scalar product of force and velocity of its point of application is zero. The nature of the frictional forces is described in the referenced literature citations and is comparable to the frictional forces in cycling.

5.3. *Aquatic locomotion*

Though the inclusion of power associated with the propelling force might seem rather trivial in locomotion where the push-off takes place against the entire earth, the significance of this approach based on equation (9) may become more convincing when applied to aquatic locomotion. The overwhelming majority of power equations applied for analysis of human swimming show a summation of powers associated with the opposing (frictional) forces (see Toussaint *et al.*, 1983; de Groot and van Ingen Schenau, 1988, for references). We know, however, from experience, that during the strokes both in swimming and rowing the push-off force is displaced. This means that the power associated with this push-off force should be accounted for according to equation (9).

Since propulsion in water can only be achieved by giving water momentum or 'driving water backwards' (Alexander, 1975) the water acquires kinetic energy as a result of push-off. This phenomenon is well known from studies of fish swimming (Webb, 1971, 1975; Lighthill, 1975; Alexander and Goldspink, 1977). Even in fish with relatively large surfaces which are involved in generating propulsion, this amount of 'wasted power' (Alexander and Goldspink, 1977) is considerable. Expressed as fraction of P_e henceforth called 'propelling efficiency e_p' (Alexander and Goldspink, 1977; Lighthill, 1975), values for (fast) fish swimming are between 65 and 75% (Bone, 1975; Webb, 1975; Videler and Hess, 1984). The power associated with generating propulsive forces in human swimming would be relatively easy to determine if the limbs would simply move in a backwards direction using the drag on the limbs as the propulsive force. The propelling efficiency in such a technique, however, would result in a waste of power of more than 50% of P_e (de Groot and van Ingen Schenau, 1988). Since the work of Counsilman (1968), practitioners know that in particular the hands should not follow such a short trajectory, but they should make as many side and upwards excursions as possible. Schleihauf (1979), and Schleihauf *et al.* (1983) showed that this technique is associated not only with the need to push off against water which was not previously accelerated (Counsilman, 1968; Toussaint *et al.*, 1983), but also with the creation of lift forces such as those that occur in wings of birds and in hydrofoils (Lighthill, 1975; Alexander and Goldspink, 1977). The wasted power associated with generating lift forces on the limbs is considerably less than the power lost in generating drag forces (Lighthill, 1975; Alexander and Goldspink, 1977; de Groot and van Ingen Schenau, 1988). Lift, however, cannot be generated without drag. Much future research in human swimming must therefore be focused on the relation between technique and e_p. The exact analysis of this propulsion mechanism is complicated from a hydrodynamic point of view (see Cheng and Murillo, 1984, for a three-dimensional analysis). But even an experimental arrangement according to Schleihauf (1979) and Schleihauf *et al.* (1983), who experimentally determined the lift and drag forces on hand and arm, will not allow a simple calculation of power. At first glance, one would tend to take the scalar product of the resultant of both forces and the velocity of the hand. This however would ignore the so-called 'induced power', which is associated with the creation of lift forces (Alexander and Goldspink, 1977; Tucker, 1975; Weis-Fogh, 1975). One can envisage this effect by realising that the hydrodynamics involved in creating lift are also responsible for a deviation of the relative velocity of the water with respect to the hand in a direction opposite to the direction of the lift force (Fig. 6). This means that one cannot take the velocity of the hand relative to the undisturbed water as the velocity of the point of application of the resultant force on the water. Wardle and Videler (1980) show four different approaches which are used to model the hydrodynamic processes involved in creating propulsion in fish swimming. Apart from a few hypothetical examples (de Groot and van Ingen Schenau, 1988) based on hydrofoil theories, no studies have yet been performed which relate technique to the wasted power or to e_p in human swimming. The first indirectly determined values for e_p in freestyle swimming are reported to be approximately 65% (Toussaint *et al.*, 1988).

There is considerable debate over the nature of opposing forces in swimming. It has been suggested that frictional forces (in swimming mostly referred to as 'drag') experienced by swimmers moving their arms and legs may be different from the drag on a passively towed subject (e.g., Di Prampero *et al.*, 1974; Clarijs, 1978). Since movements may disturb a steady-state motion of the water relative to the body, this suggestion seems reasonable (Daniel, 1983, 1984). A number of authors have used indirect methods to determine this so called 'active drag' (Di Prampero *et al.*, 1974; Pendergast *et al.*, 1977; Rennie *et al.*, 1975; Clarijs, 1978). In a more direct approach by measuring propulsive forces Schleihauf (1979), Schleihauf *et al.* (1983) and Van der Vaart *et al.* (1987) found values which are not significantly different from drag on passively towed swimmers (see Van der Vaart *et al.* (1987) for further discussion).

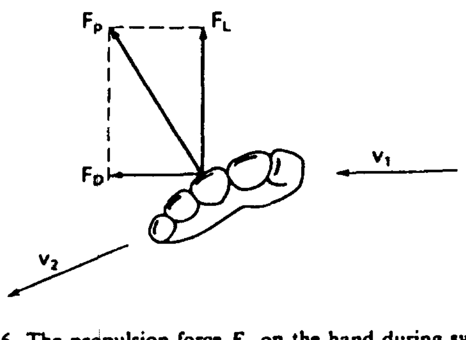

Fig. 6. The propulsion force F_p on the hand during swimming is generated by moving the hand through the water (relative velocity v_1). Depending on the shape of the hand and the angle between the hand and the direction of v_1, this relative flow does not only induce a drag force F_D but also a lift force F_L. The generation of this lift force causes a change of direction of the relative velocity of the water.

Sanderson and Martindale (1986) reported a value of 75% for e_p in rowing. Despite the absence of significant lift forces in rowing, this value is still higher than that calculated for swimming, probably because of the larger surface area of the oar blades. As with cycling, different parts of the overall system can be modelled with a free body diagram using equation (9), (12) or (13).

Taking the man–boat–oar system, only air friction (small), drag, and propulsive forces determine P_u. The net power output P_u of the rower can be measured from the power input of the oar (Sanderson and Martindale, 1986). One should take care, however, with force measurements on the oarlock. Celentano *et al.* (1974) used the force on the oarlock as the propulsive force of the boat. This force however is only an external force if the boat (excluding man and oar) is taken as a free body. In such an application, the force from the seat and the forces on the foot stretcher should be accounted for as well. Neglecting inertial effects of the oar, the force on the oarlock may, however, be used to calculate the power input to the oar (Fukunaga *et al.*, 1986). It should be noted that, particularly in rowing but essentially for all other endurance sports too, one should not take the mean drag force and the mean velocity of the boat (or athlete) when calculating the power to water and air friction. Instead, the time varying forces within the cycle should be used to calculate cycle averaged power, according to equation (13). The mean force and velocity can lead to a significant underestimation of the actual mean power (Nigg, 1983; Sanderson and Martindale, 1986).

When comparing the examples presented of terrestrial and aquatic locomotion, it is clear that the differences in friction may explain the differences in speed which are achieved in exercises of comparable duration (Di Prampero, 1986). Next to the (small) differences in power input which are due mainly to a difference in total muscle mass involved in producing P_o, the differences in speed in terrestrial locomotion are mainly due to a difference in P_{nc} which is associated with the creation of propulsive forces.

In running, as well as in the classical diagonal stride cross-country skiing, the push-off takes place against a fixed point on the earth. This means that the athlete's speed cannot exceed the maximal horizontal compon-

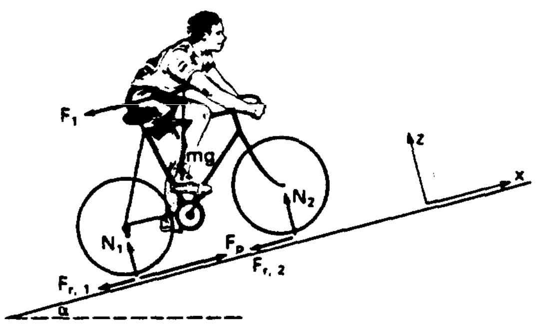

Fig. 7. External forces acting on a cyclist. Note that in the reference frame used the system has potential energy which depends on both coordinates.

ent of the velocity, which can be realised between the body center of gravity and the sole of the foot (Djatschkow, 1977). This velocity is, to a large extent, dependent on the extension velocity of the joints of the lower extremity. In cycling and speed skating, the speed of the athlete is not constrained by the extension velocity of the leg since the point of application of the propelling force is displaced during a leg extension. This phenomenon partly explains why competitive cross-country skiers use a skating-like technique nowadays.

At the end of this discussion of the application of power equations, one additional approach should be considered. Imagine a system (such as a cyclist) which moves uphill under the influence of a propulsive force F_p and is opposed by friction forces F_f (Fig. 7). An additional equation for P can be deduced if the rate of change of energy of the center of mass is calculated. In the coordinate system indicated in Fig. 7, the potential energy can be expressed as $E_p = mg \sin\alpha \cdot x + mg \cos\alpha \cdot z$, while the kinetic energy equals $\frac{1}{2}mv_x^2$ with m the mass of the system and v_x its velocity in the x-direction, if the velocity in the z-direction is zero ($v_z = 0$).

The time derivative of the total energy than equals:

$$\begin{aligned} dE/dt &= mg \sin\alpha\, v_x + mg\cos\alpha\, v_z + mv_x a_x \\ &= mg\sin\alpha\, v_x + (F_p - F_f - mg\sin\alpha)v_x \\ &= F_p v_x - F_f v_x. \end{aligned}$$

According to equation (9), the power is then:

$$P_u = \mathbf{F}_p \cdot \mathbf{v}_p - \mathbf{F}_f \cdot \mathbf{v}_x + F_f v_x - F_f v_x.$$

In terrestrial locomotion, $v_p = 0$. The magnitude of $-\mathbf{F}_f \cdot \mathbf{v}_x$ is $\mathbf{F}_f \cdot \mathbf{v}_x$, since $\mathbf{F}_f$ and $\mathbf{v}_x$ have opposite directions. Then

$$P_u = F_p v_x.$$

This means that the instantaneous power which is available to overcome friction and to increase the energy of the center of mass can simply be calculated from the propulsive force and the velocity of the mass center of gravity, which may serve as an additional tool in many applications. Note however that in aquatic locomotion this expression does not account for 'wasted' power at push-off.

6. DEFINITIONS

Based on the foregoing deduction of an unambiguous power equation, it is now possible to reconsider the various definitions that are commonly used.

6.1. *Power*

As indicated in the Introduction, there are a number of different definitions of power and efficiency in the literature of human and animal locomotion. The same power is expressed as internal by one author and external by another. In particular in running and walking there is no concensus about these expressions (e.g., Cavagna and Kaneko, 1977, vs Winter, 1978). Most authors, however, use reasonable arguments for their definitions. It does not seem possible to relate external and internal power simply to the free body diagrams which are used in the different approaches. Based on the free body diagrams for the rigid links used by Aleshinsky (1986a) and by the present authors, one might argue that all power comes from sources external to these links. But a definition based on such a number of different free body diagrams does not hold as soon as the derived equations are added in order to achieve the power equation for the entire athlete. Following the arguments of Winter (1979), only the second expression on the right-hand side of equation (9) deserves the designation of external power. The first expression should be defined as internal power. Though such a definition would be closely connected to the environmental (external) forces, it would change as soon as one replaces the rate of change of potential energy by the work done by gravity. The same is true for kinetic energy, which can be replaced by work done by (external) inertial forces. So it can be concluded that there are no decisive arguments against a definition of external power according to equation (9) if one intends to couple the expressions 'internal' and 'external' to a well-defined free body. It should further be noted that Winter's definition can lead to situations where the system does positive external work on the environment without simultaneous muscle actions. A cyclist who, for example, passively descends a hill, would deliver external work (against friction), and negative internal work. Though both work expressions would cancel out in Winter's proposed expression for total mechanical work (Winter, 1979), the same zero work output can be achieved by a more straightforward approach using equation (9), (12) or (13). As already discussed above, the summed joint power is the power which is most closely related to muscle power. So it is proposed here to define this power as external power. All power $P_o - P_u$ is then internal power. This, at least, is an unambiguous definition which fits the majority of previous definitions for activities other than walking and running. Note, however, that this power depends upon the chosen free body which at least includes the athlete but which may also be extended to the athlete and his equipment (bicycle, boat, skate, ski, etc.) using equation (13).

The majority of the mechanical power as calculated using different methods for level running and walking is thus part of the internal power. The best expression to use here is probably 'mechanical power'. Other expressions which are unambiguously defined in the literature and which are already used in this paper concern (Fig. 1):

(1) power input or metabolic power P_i;

(2) power output P_o or muscle power;

(3) maintenance power P_m;

(4) conservative power P_c;

(5) non-conservative power P_{nc} (e.g., Tucker, 1975).

Note that P_c can be part of both external and internal power, while P_{nc} is always part of the internal power. Other definitions which are recommended concern basically swimming:

(6) wasted power (e.g., Alexander and Goldspink, 1977) P_k (k: kinetic energy given to the water). This is all power associated with F_p;

(7) induced power (e.g., Lighthill, 1975), the power associated with lift forces in the generation of F_p (part of the wasted power);

(8) profile power (Tucker, 1975; Weis-Fogh, 1975), the power associated with drag forces in the generation of F_p (also part of the wasted power);

(9) parasitic power (Tucker, 1975; Weis-Fogh, 1975), the power associated with F_f, the opposing frictional forces. This definition is also recommended for terrestrial locomotion.

6.2. *Efficiency*

The many different definitions of efficiency that can be found in the literature are, on the one hand, associated with the different types of mechanical power and, on the other hand, with different estimates of metabolic power input. Again, there has been much discussion concerning the various definitions (e.g., Whipp and Wasserman, 1969; Donovan and Brooks, 1977; Stainsby *et al.*, 1980; Fish, 1984; Cavanagh and Kram, 1985a,b) and reasonable concensus has emerged concerning some of them. This is especially true for muscle efficiency e_m, which is defined as the product of the phosphorylative coupling efficiency and the contraction coupling efficiency. This muscle efficiency $e_m = P_0/(P_i - P_m)$ (see Fig. 1) reflects the efficiency of the contractile machinery and is estimated not to exceed the $e_m = 30\%$ (Åstrand and Rodahl, 1977). This estimation is based on thermodynamical considerations (Stainsby *et al.*, 1980; Åstrand and Rodahl, 1977). In contrast to measurements of efficiency on the level of total body movements, efficiency data on isolated muscles or muscle fibers are relatively scarce. Such data, however, are important, since clearly the efficiency of total body movements has to be lower than the efficiency measured on the level of the contractile machinery.

For various animal muscles, experiments on isolated muscles show efficiencies from 14 to 40% (Gibbs and Gibson, 1972; Wendt and Gibbs, 1973; Heglund and Cavagna, 1987; de Haan *et al.*, 1989).

However, the highest values are obtained by only accounting for the positive work done in the concentric phase of muscle action (e.g., Heglund and Cavagna, 1987). It should be noted that one might even find efficiencies up to 100% or higher if only positive work is taken into account. Imagine for example that the contractile element acts as a force generator (without detachment of cross-bridges) in a 'stretch–shortening cycle' of a muscle tendon complex with short muscle fibers and a long compliant tendon. In such actions the positive work (from re-utilisation of elastic energy) can be considerable with little metabolic cost. If, however, the efficiency is measured on the level of the contractile elements, the (actual) muscle efficiency seems not to exceed the 25% in actions starting from an isometric state as well as in stretch–shortening cycles (de Haan *et al.*, 1989).

For negative work one can also speak about efficiency, but it should be noted that here this expression has a totally different meaning. At the level of contractile machinery, an eccentric muscle action simply acts as a brake (Stainsby *et al.*, 1980) converting mechanical work into heat. This means that the expression efficiency now provides a measure for the cost to control the brake. It has even been suggested that during eccentric muscle actions, the cross-bridges can attach and develop tension without metabolic costs (Curtin and Davies, 1974), the costs of an eccentric action being mainly due to the process of muscle activation. The magnitude of this efficiency of negative work will (at equal metabolic costs for the activation process) be strongly dependent on the force level and range of lengthening of the contractile element, and can of course in principle be considerably larger than 100%.

It should be emphasized that one cannot relate phases of negative summed joint work to this negative muscle work and to the efficiency of negative work. Particularly during locomotion, much of this negative joint work will be stored as elastic energy in the series elastic elements of the muscles, and re-utilised as positive work.

At the level of separate joints one can also expect that negative work at one joint appears as positive work in another joint through actions of bi-articular muscles (van Ingen Schenau, 1989).

6.3. *Gross efficiency*

Expressed in terms of power components as proposed in this paper, this widely used expression may unambiguously be defined as

$$e_g = P_u/P_i.$$

The quotients between other possible measures for power discussed above and power input P_i might be defined by a more trivial expression 'mechanical efficiency', indicating that this efficiency is not associated with an explicitely defined free body (as can be done with contractile element(s) or the entire athlete).

Clearly, the gross efficiency will always be lower than the muscle efficiency discussed above. In this respect it should be realized that, particulary in not-highly-trained subjects, one can expect a contribution of anaerobic power already at relatively low levels of exercise, which is not accounted for in the denominator if P_i is estimated on the basis of oxygen consumption. This leads to an underestimation of P_i and an overestimation of the gross efficiency.

6.4. *Baseline subtractions*

It is obvious that the gross efficiency will always be (much) smaller than e_m. Values reported in the literature are about 22–23% for cycling (Pugh, 1974; Steinacker *et al.*, 1984), 17–19% for rowing (Steinacker *et al.*, 1984; Fukunaga *et al.*, 1986), 21% for cross-country skiing (Niinimaa *et al.*, 1978) and 19–21% for speed skating (de Groot *et al.*, 1988). For human swimming, much lower values are reported (e.g., Craig *et al.*, 1985; 6–8%). These values however do not include the wasted power P_k. If the total power P_e is accounted for, one may expect a gross efficiency in swimming close to or even higher than the values of about 15% as reported for armcranking (Powers *et al.*, 1984).

Given the differences in the involvement of muscle mass in generating P_o, in maintenance power P_m, and given the large differences in P_{nc} which may exist between the different movements, these variations in reported gross efficiency can be understood. Part of the variations will also be due to differences in e_m as a result of differences in, e.g., the force and velocity patterns of the contractile elements. Though the number of unknown variables is far in excess over the number of equations which can be deduced by measuring P_i at rest, in unloaded movements ($P_e = 0$) and in different loaded movements ($P_e > 0$), many physiologists still perform questionable attempts to identify e_m from measurements of P_i and P_e in gross body movements. These attempts are known in the literature as base-line subtractions (e.g., Gaesser and Brooks, 1975; Donovan and Brooks, 1977). The idea is that, e.g., the power input in unloaded cycling equals all power except the power input necessary to produce P_e in loaded cycling. So by subtracting P_i in unloaded cycling from the power input in cycling at an external power P_e, one attempts to deduce a measure for muscle efficiency.

Many arguments can be and are formulated against these approaches (e.g., Stainsby *et al.*, 1980; Cavanagh and Kram, 1985a). The main argument is that it is unlikely to be true that the maintenance power would be independent of the workload. Moreover the so-called work or apparent efficiencies $[=P_e/(P_{i,\text{loaded}} - P_{i,\text{unloaded}})]$ and delta efficiencies ($=\Delta P_e/\Delta P_i$) (Gaesser and Brooks, 1975) lead to values which are far in excess of the thermodynamically determined maximal muscle efficiency (see Asmussen and Bonde Petersen, 1974; Pugh, 1971; Zacks, 1973). Since the examples concern efficiencies of external power, these discrepancies cannot be explained by internal storage and re-utilisation of the elastic energy of muscles and tendons.

There appears to be enough evidence to completely reject the validity of any base-line subtraction. There is only one unambiguous definition possible for gross body movements at the present state of knowledge: the gross efficiency. This is the only quotient which is based on valid, well defined powers.

6.5. *Economy*

As argued by Cavanagh and Kram (1985a); Morgan (1985); Williams (1985) and Daniels (1985), the concept of gross efficiency or a mechanical efficiency is of little use in the study of the mechanics and energetics of walking and running (the gross efficiency is close to zero in level walking and running). Therefore these authors advocate the application of 'economy' defined for a particular activity (e.g., a certain speed) as the rate of submaximal oxygen uptake either in absolute units or relative to body weight (ml kg^{-1} min^{-1}). While this allows both different activities (such as uphill running) and different individuals performing the same activity to be compared, it can not make any statement with respect to efficiency. One individual may be less economical than another, but may actually be more efficient because he is actually performing considerably more work. However, economy is a fairly unambiguous concept which might be applied for other types of locomotion as well.

Note however that this expression is also used in muscle mechanics to represent the force-time integral divided by energy input over the integration time, mostly applied in isometric conditions (Goldspink, 1978).

7. CONCLUDING REMARKS

The literature reviewed here exhibits a considerable diversity in the approach to the formulation of a power equation for endurance activities. The method that we have suggested can be seen as somewhat of a bridge between the energy approach of Winter and his associates (Winter, 1979, 1983; Robertson and Winter, 1980) and the theoretical approach of Aleshinsky (1986a–e). The former method involves assumptions concerning energy exchange while the latter approach, although computationally rigorous, is fairly complex in form and lacks application to specific activities. The application of the present approach to a number of terrestrial and aquatic activities indicates the utility of the method. It should be realized that there is unlikely, in the near future, to be an entirely satisfactory solution to the problem of power output calculation in locomotor activities, since a number of intrinsic factors remain difficult to quantify. It is, however, hoped that the clarification of certain concepts related to force, power, efficiency and economy during movement presented in this review will assist in the further development of this sometimes ambiguous field of study.

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