HAB718 Spor Biyomekaniğinde Hareket Analizi

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- Constrained Inverse Kinematics

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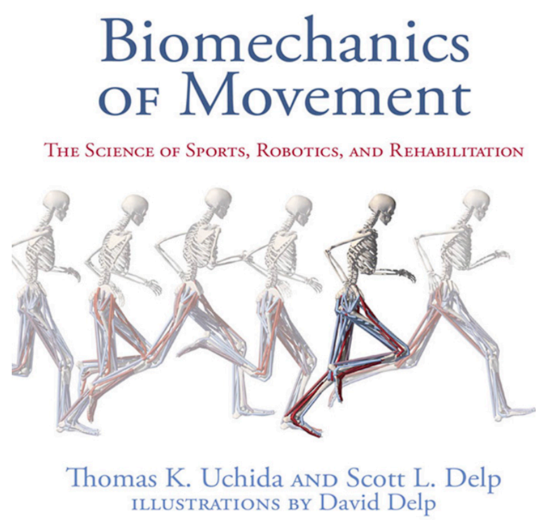
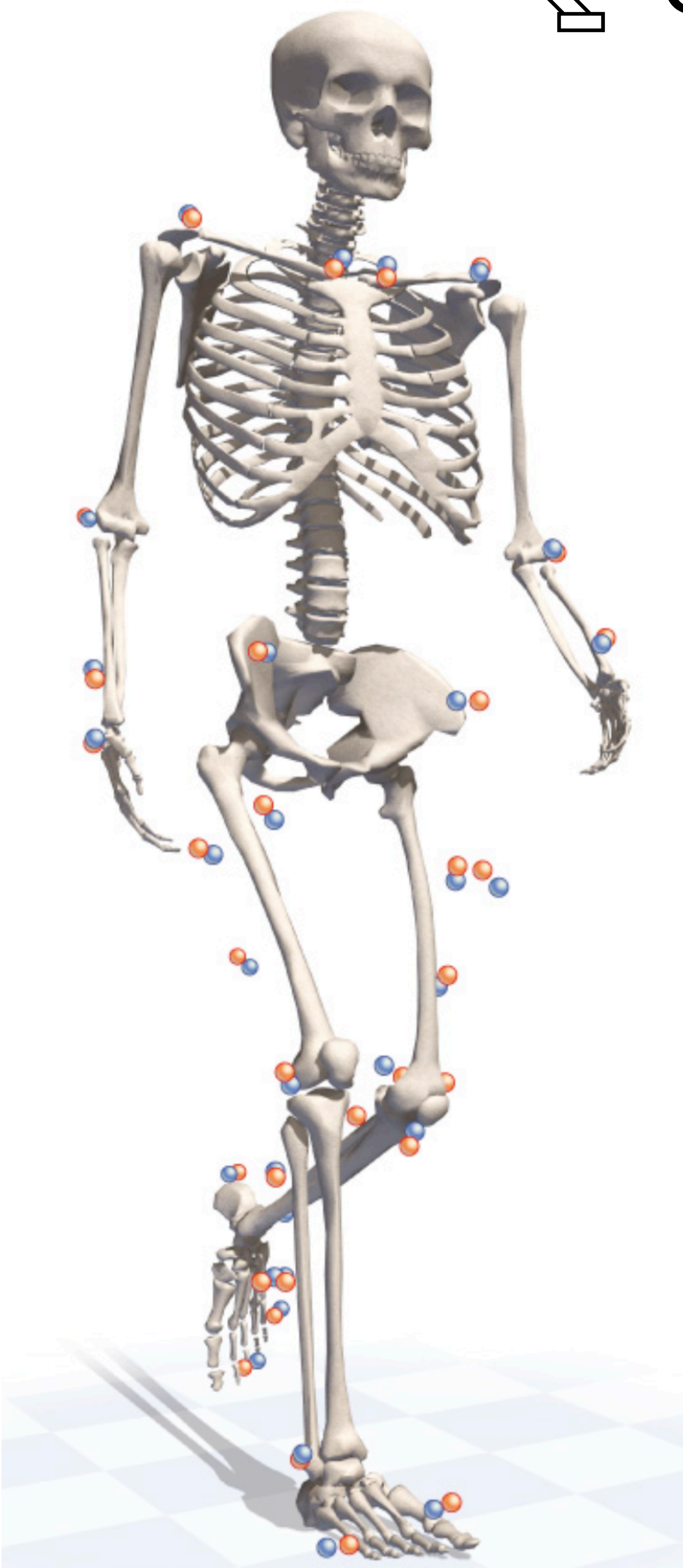
Constrained Inverse Kinematics



The constrained inverse kinematics approach uses optimization to minimize the distance between the locations of experimental markers affixed to the subject and the locations of corresponding markers affixed to a kinematic model of the subject's skeleton.

Constrained Inverse Kinematics



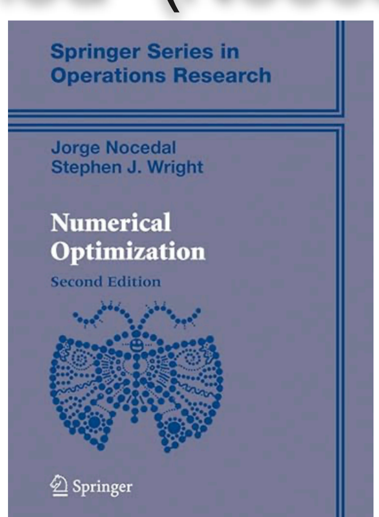
The model is composed of rigid bodies interconnected by anatomically accurate joints that permit only motions that can be reliably measured. The model is scaled to match the dimensions of the subject. Using a model that incorporates knowledge about the subject's geometry and mobility can produce more accurate estimates of joint angles than the unconstrained inverse kinematics approach. The constraints in such a model limit the set of possible solutions to those that represent anatomically feasible movements.

An inverse kinematics algorithm may produce more accurate estimates of joint angles if the solutions are constrained by an underlying skeletal model, where markers placed on the model (orange) track the trajectories of markers placed on the subject (blue).

Constrained Inverse Kinematics

You can think of each weight as the stiffness of a spring between the experimental marker and the marker in the model. The goal of the optimization is to minimize the energy stored in these imaginary springs.

This goal can be posed as a weighted **least-squares** problem and solved using a suitable numerical algorithm, such as the Broyden–Fletcher–Goldfarb–Shanno (BFGS) iterative method (Nocedal and Wright, 2006)



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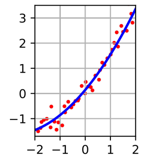
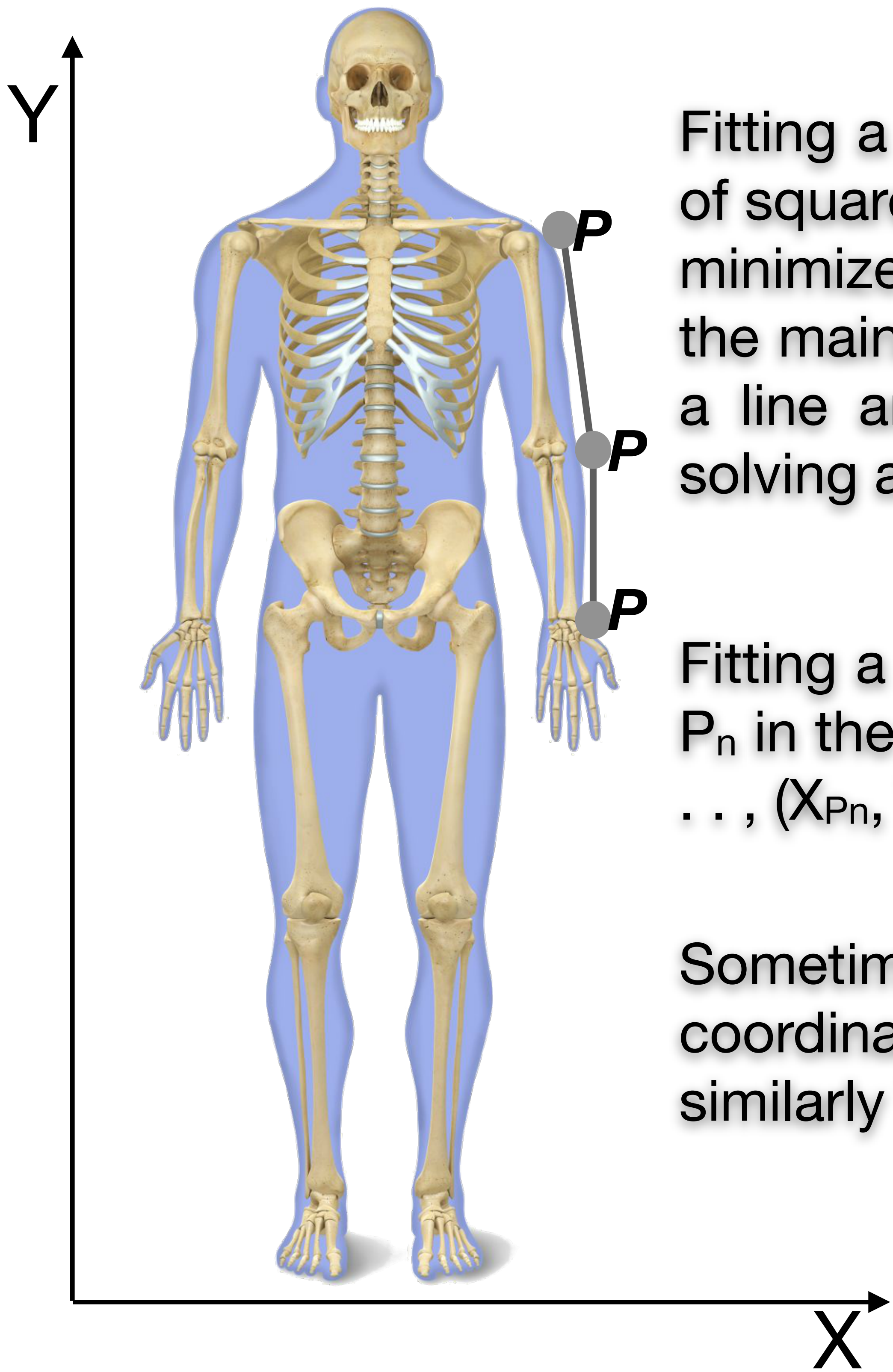


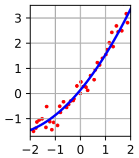
Constrained Least Squares



Fitting a line to a set of points in such a way that the sum of squares of the distances of the given points to the line is minimized, is known to be related to the computation of the main axes of an inertia tensor. In this fact is used to fit a line and a plane to given points in the 3d space by solving an **eigenvalue** problem for a **3 x 3** matrix.

Fitting a straight line to a set of given points P_1 , P_2 , • • • , P_n in the plane . Their coordinates with (X_{P1},Y_{P1}), (X_{P2}, Y_{P2}) . . . , (X_{Pn}, Y_{Pn}).

Sometimes it is useful to define the vectors of all X and Y coordinates. X_P for the vector (X_{P1}, X_{P2}, . . . , X_{Pn}) and similarly Y_P for the Y coordinates.



Constrained Least Squares

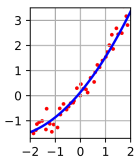
Linear regression means to fit the linear model ***y= ax + b*** to the given points, i.e. to determine the two parameters ***a*** and ***b*** such that the sum of squares of the residual is minimized.

$$\sum_{i=1}^{n} r_i^2 = min \qquad r_i = Y_{Pi} - aX_{Pi} - b$$

minimize (*the sum of squares of the distances of the points from the fitted straight line***)**

$$c + n_1 x + n_2 y = 0 \qquad n_1^2 + n_2^2 = 1$$

The unit vector (n_1, n_2) is orthogonal to the line.



Constrained Least Squares

$$r = c + n_1 x_p + n_2 y_p$$

$$||r|| = \sum_{i=1}^{n} r_i^2 = min$$

$$\begin{bmatrix} 1 & x_{p_1} & y_{p_1} \\ 1 & x_{p_2} & y_{p_2} \\ \vdots & \vdots & \vdots \\ 1 & x_{p_n} & y_{p_n} \end{bmatrix} \begin{bmatrix} c \\ n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} \quad \text{and} \quad n_1^2 + n_2^2 = 1$$

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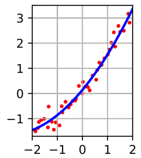
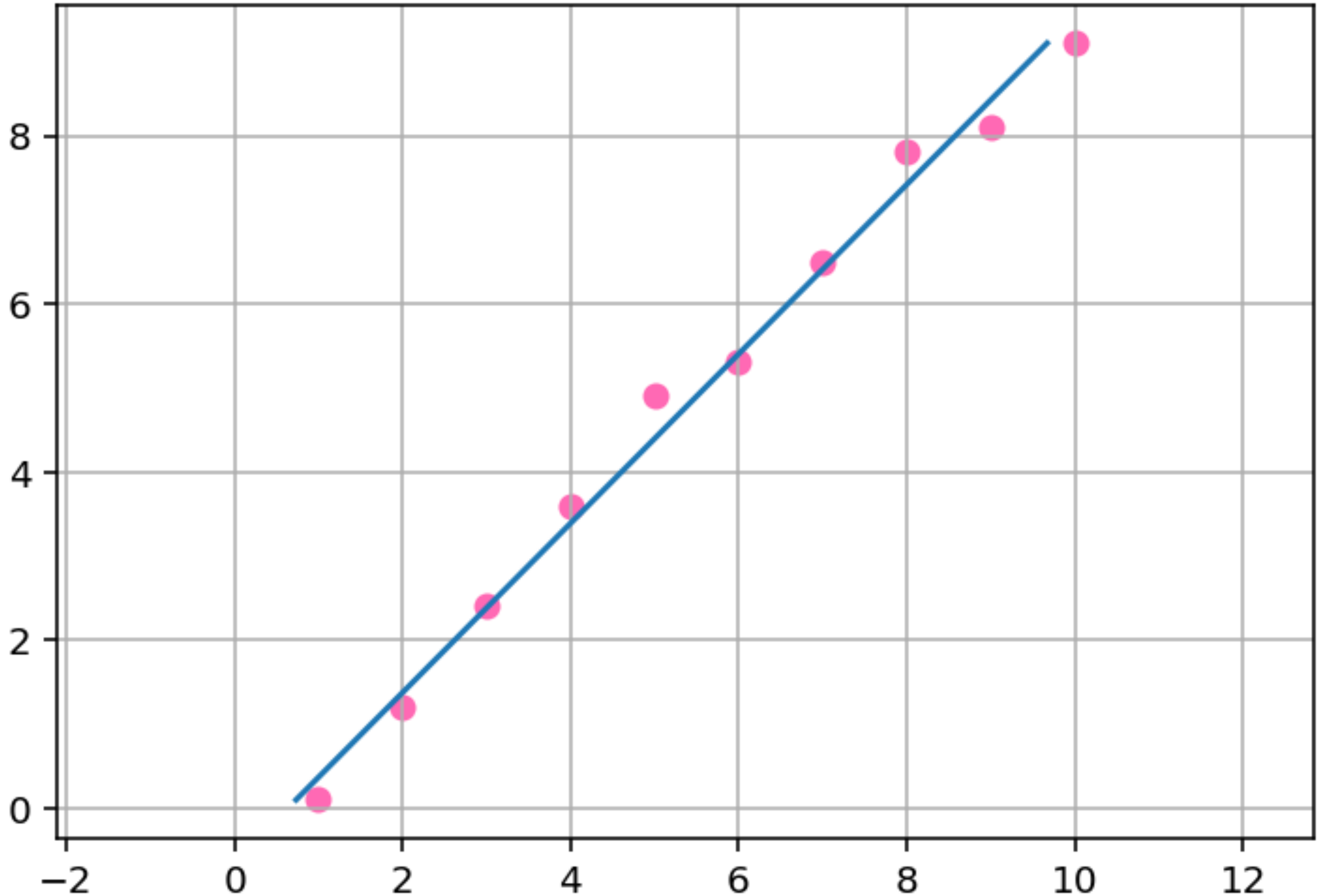
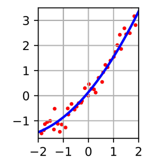


Constrained Least Squares

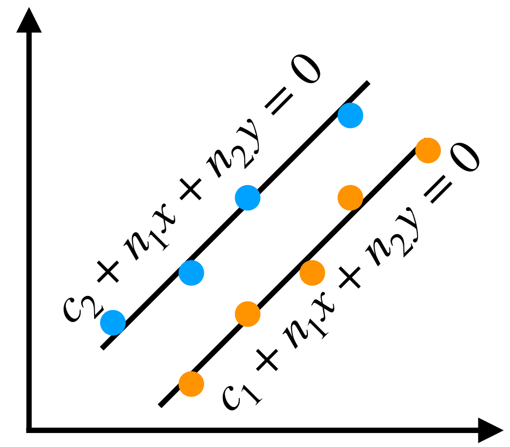
Px = [1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0]

Py = [0.1, 1.2, 2.4, 3.6, 4.9, 5.3, 6.5, 7.8, 8.1, 9.1]

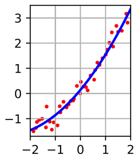




Constrained Least Squares



$$\begin{bmatrix} 1 & 0 & x_{p_1} & y_{p_1} \\ 1 & 0 & x_{p_2} & y_{p_2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & x_{p_n} & y_{p_n} \\ 0 & 1 & x_{q_1} & y_{q_1} \\ 0 & 1 & x_{q_2} & y_{q_2} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & x_{q_m} & y_{q_m} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{p+q} \end{bmatrix} \quad \text{and} \quad n_1^2 + n_2^2 = 1$$



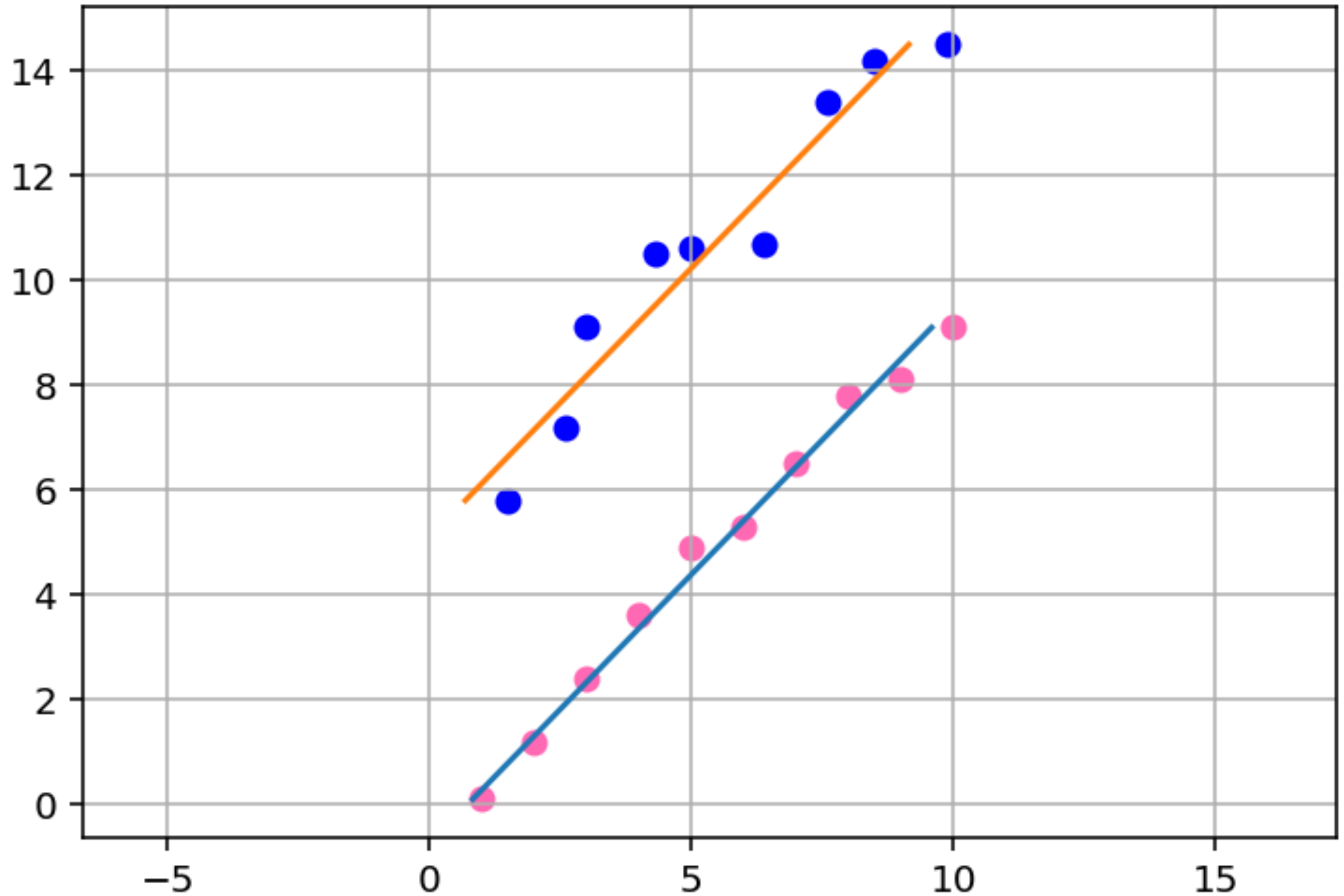
Constrained Least Squares

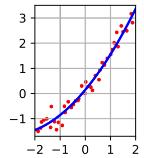
Px = [1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0]

Py = [0.1, 1.2, 2.4, 3.6, 4.9, 5.3, 6.5, 7.8, 8.1, 9.1]

Qx = [1.5, 2.6, 3.0, 4.3, 5.0, 6.4, 7.6, 8.5, 9.9]

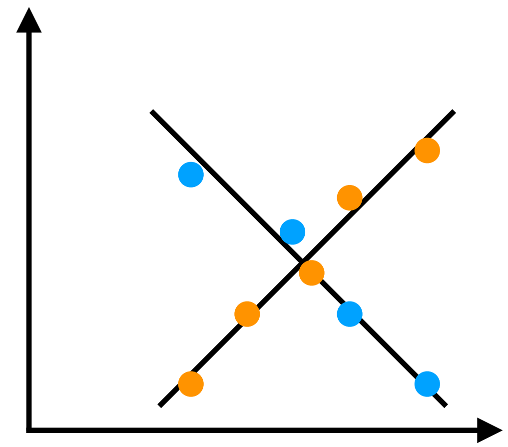
Qy = [5.8, 7.2, 9.1, 10.5, 10.6, 10.7, 13.4, 14.2, 14.5]



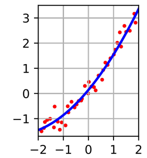


Constrained Least Squares

If **(n_1, n_2)** is the normal vector of the first line, then the second line must have the normal vector **($-n_2$, n_1)** in order to be orthogonal.



$$
\begin{bmatrix}
1 & 0 & x_{p_1} & y_{p_1} \\
1 & 0 & x_{p_2} & y_{p_2} \\
\vdots & \vdots & \vdots & \vdots \\
1 & 0 & x_{p_n} & y_{p_n} \\
0 & 1 & y_{q_1} & -x_{q_1} \\
0 & 1 & y_{q_2} & -x_{q_2} \\
\vdots & \vdots & \vdots & \vdots \\
0 & 1 & y_{q_m} & -x_{q_m}
\end{bmatrix}
\begin{bmatrix} c_1 \\ c_2 \\ n_1 \\ n_2 \end{bmatrix}
=
\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{p+q} \end{bmatrix}
\quad \text{and} \quad n_1^2 + n_2^2 = 1
$$



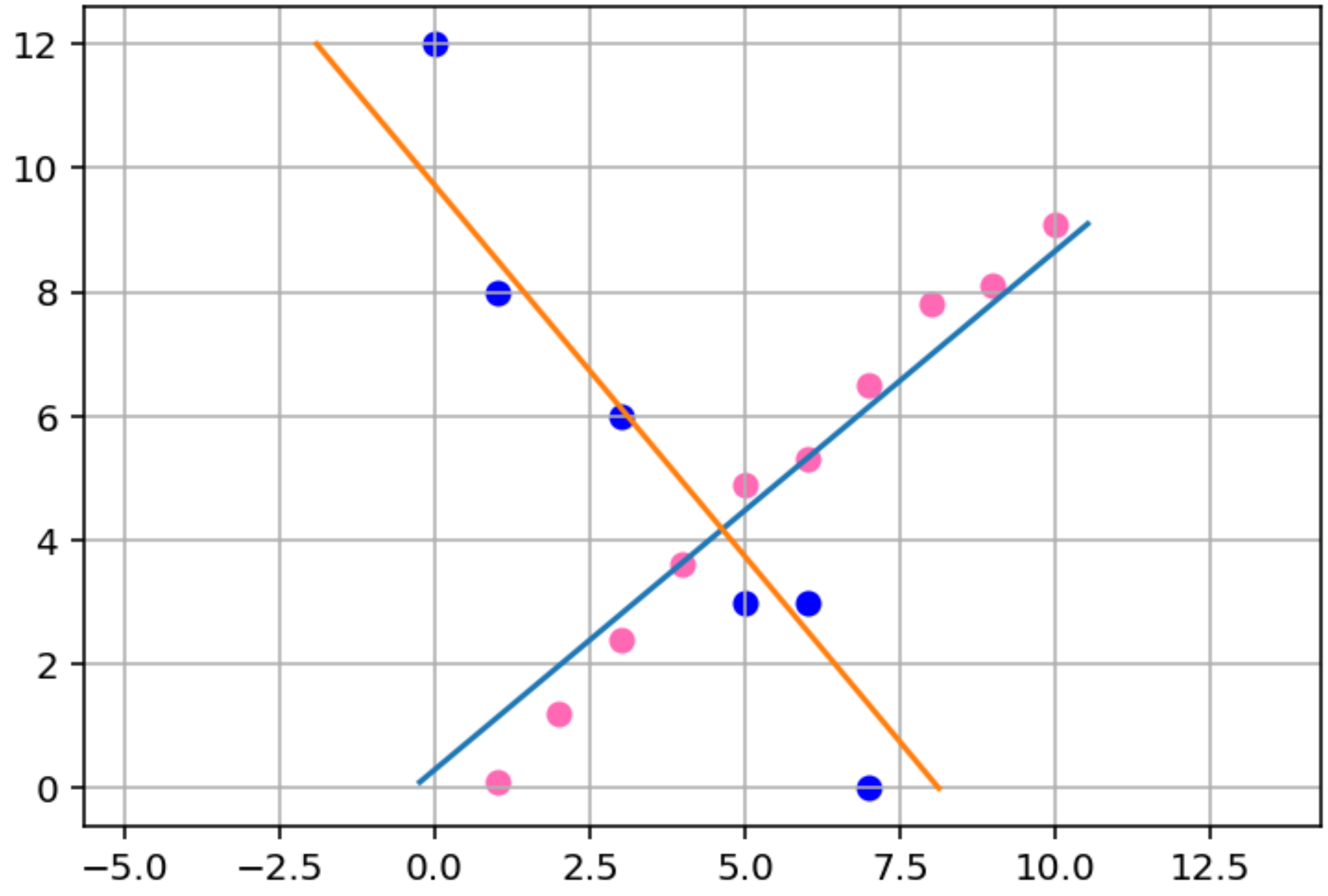
Constrained Least Squares

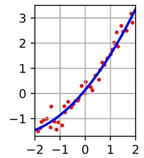
Px = [1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0]

Py = [0.1, 1.2, 2.4, 3.6, 4.9, 5.3, 6.5, 7.8, 8.1, 9.1]

Qx = [0, 1, 3, 5, 6, 7]

Qy = [12, 8, 6, 3, 3, 0]





Constrained Least Squares

Numerische Mathematik 7, 206—216 (1965)

Numerical Methods for Solving Linear Least Squares Problems*

By

G. Golub

Abstract. A common problem in a Computer Laboratory is that of finding linear least squares solutions. These problems arise in a variety of areas and in a variety of contexts. Linear least squares problems are particularly difficult to solve because they frequently involve large quantities of data, and they are ill-conditioned by their very nature. In this paper, we shall consider stable numerical methods for handling these problems. Our basic tool is a matrix decomposition based on orthogonal Householder transformations.