



# HAB718 Spor Biyomekaniğinde Hareket Analizi



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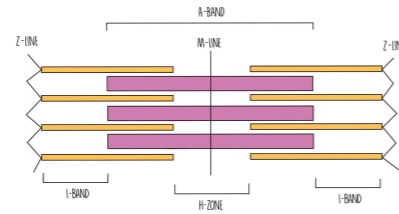
# HAB718 Spor Biyomekaniğinde Hareket Analizi

## #10

- Hill-Type Muscle Modeling

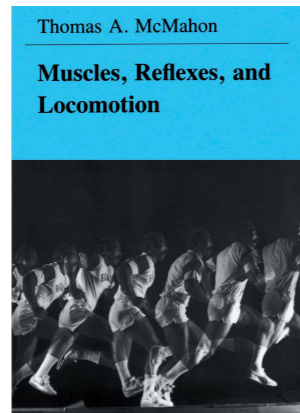


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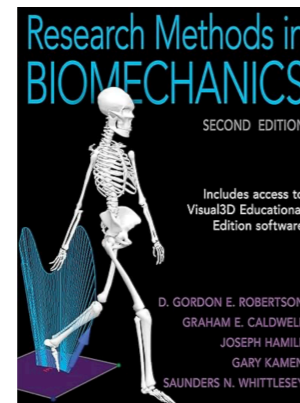
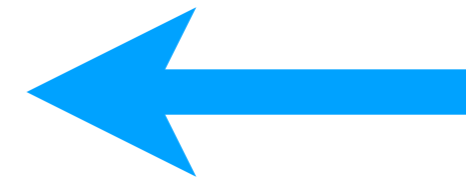
## Muscle Modelling

### Chapter 1



### Fundamental Muscle Mechanics

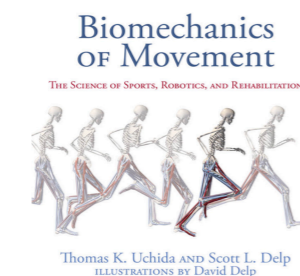
This book is about muscles and how they work in the bodies of vertebrate animals. Its special concern is locomotion on land. It is written for a reader who has some interest in the physical as well as the biological sciences, and who suspects that mechanics, thermodynamics, and engineering control theory might be useful, as well as biochemistry and histology, in understanding how muscles operate.



### Chapter 9

## Muscle Modeling

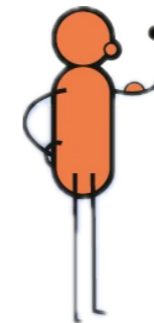
*Graham E. Caldwell*



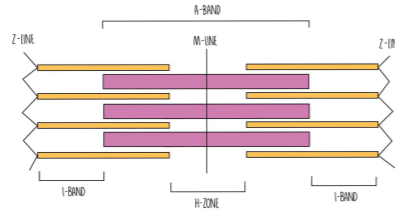
### 5

### Muscle Architecture and Dynamics

If you want to understand function, study structure.  
—Francis Crick



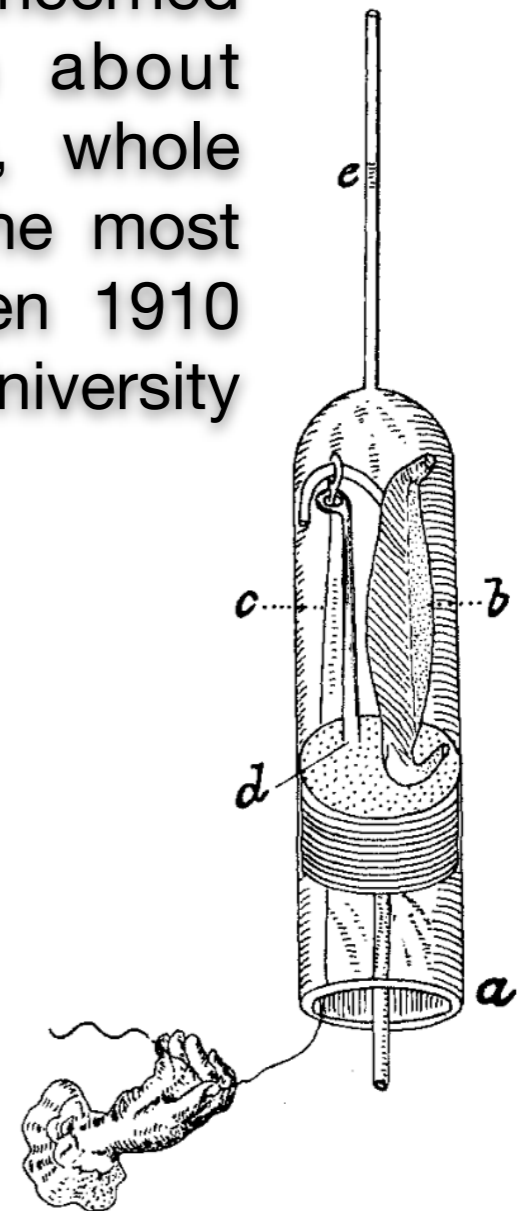
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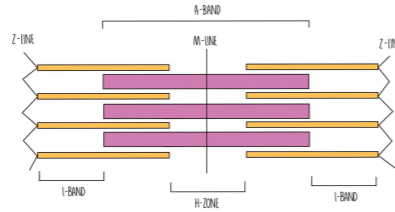
## Muscle Modelling

Swammerdam's experiment was actually one of the first recorded mechanical experiments on an isolated muscle preparation. The remainder of this chapter is concerned with what has been learned since then about fundamental muscle mechanics using single, whole muscles removed from the animal. Many of the most important experiments were performed between 1910 and 1950 by **A. V. Hill** and his collaborators at University College, London.

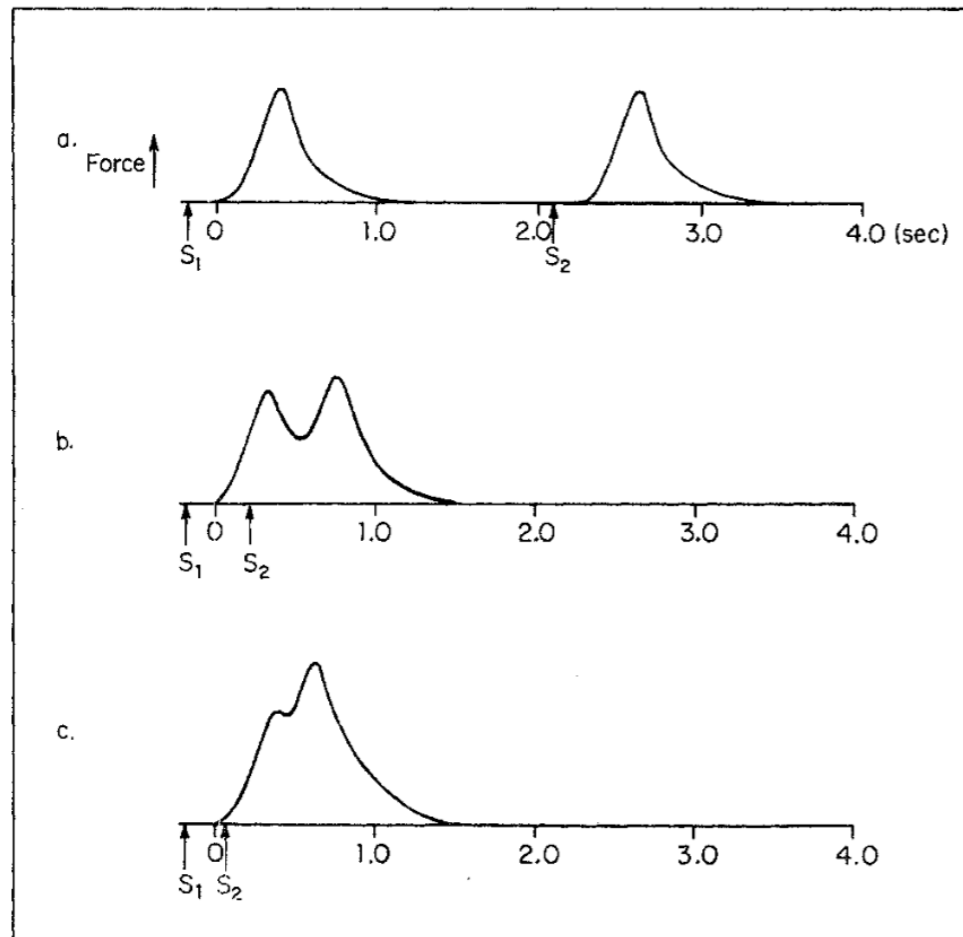
**Fig. 1.2.** The experiment of Jan Swammerdam, circa 1663, showing that a muscle does not increase in volume as it contracts. A frog's muscle (*b*) is placed in an air-filled tube closed at the bottom (*a*). When the fine wire (*c*) is pulled, the nerve is pinched against the support (*d*), causing the muscle to contract. The drop of water in the capillary tube (*e*) does not move up when the muscle contracts. From Needham (1971).



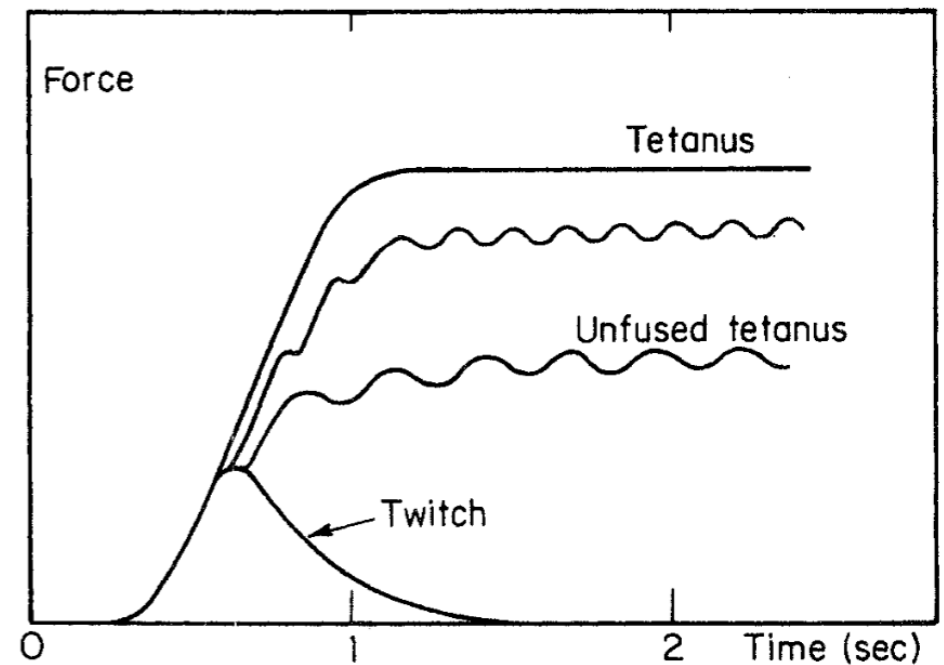
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## Muscle Modelling

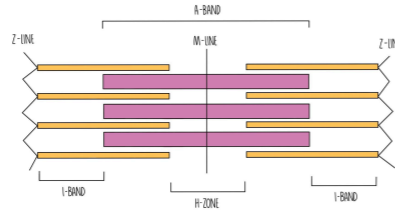


**Fig. 1.3.** Two-twitch experiments. (a) When two identical electrical stimuli,  $S_1$  and  $S_2$ , are given with a suitable time interval separating them, the two transient force events (twitches) are identical. (b and c) When the stimuli are moved closer together, the second twitch reaches a higher force maximum than the first.



**Fig. 1.4.** Twitch and tetanus. When a series of stimuli is given, muscle force rises to an uneven plateau (unfused tetanus) which has a ripple at the frequency of stimulation. As the frequency is increased, the plateau rises and becomes smoother, reaching a limit as the tetanus becomes fused.

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## Muscle Modelling

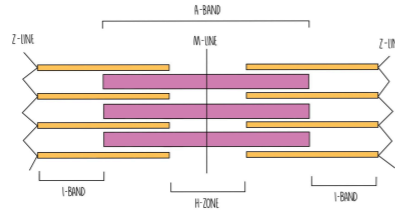
The tetanic fusion frequency is higher, about 50 or 60 shocks per second, in mammalian muscle at body temperature.

Most of what has been said so far was known to the Victorians. Helmholtz, as he put his ear to his own arm, correctly perceived that the muscle vibrates, and produces a sound whose pitch (about 30 Hz) is determined by the number of electric excitations sent to it in a second.

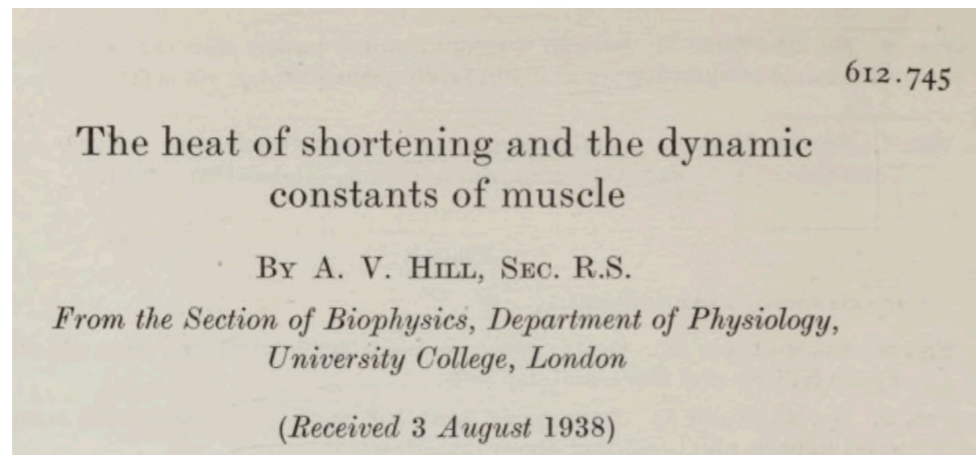


Hermann Von Helmholtz (1821-1894). German Physicist, Anatomist, And Physiologist.

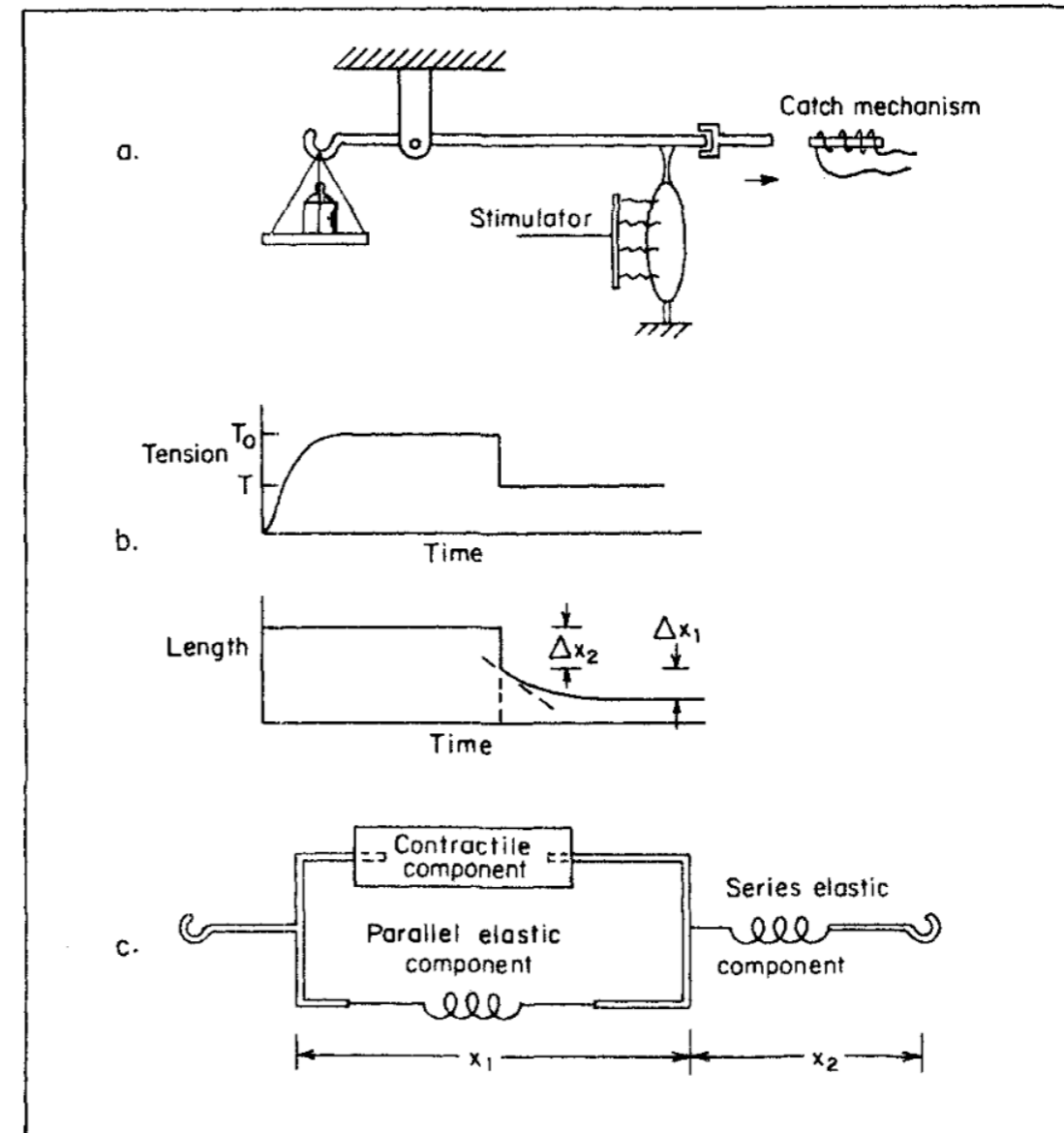
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## Muscle Modelling

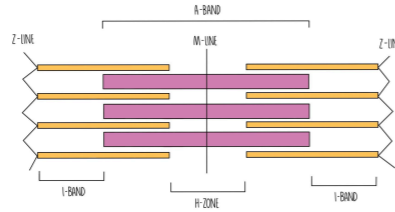


The hope was recently expressed (Hill 1937, p. 116) that with the development of a more accurate and rapid technique for muscle heat measurement, a much more consistent picture might emerge of the energy relations of muscles shortening (or lengthening) and doing positive (or negative) work. This hope has been realized, and some astonishingly simple and accurate relations have been found, relations, moreover, which (among other things) determine the effect of load on speed of shortening, **allow the form of the isometric contraction to be predicted**, and are the basis of the so-called “visco-elasticity” of skeletal muscle.



**Fig. 1.8.** (a) Quick-release apparatus. When the catch is withdrawn, the muscle is exposed to a constant force determined by the weight in the pan. (b) The muscle is stimulated tetanically. Upon release of the catch, the muscle shortens rapidly by an amount  $\Delta x_2$  which depends on the difference in force before and after release. (c) A conceptual model of muscle. For an alternative but equivalent model, see solved problem 2 at the end of the chapter.

# HAB718 Spor Biyomekaniğinde Hareket Analizi



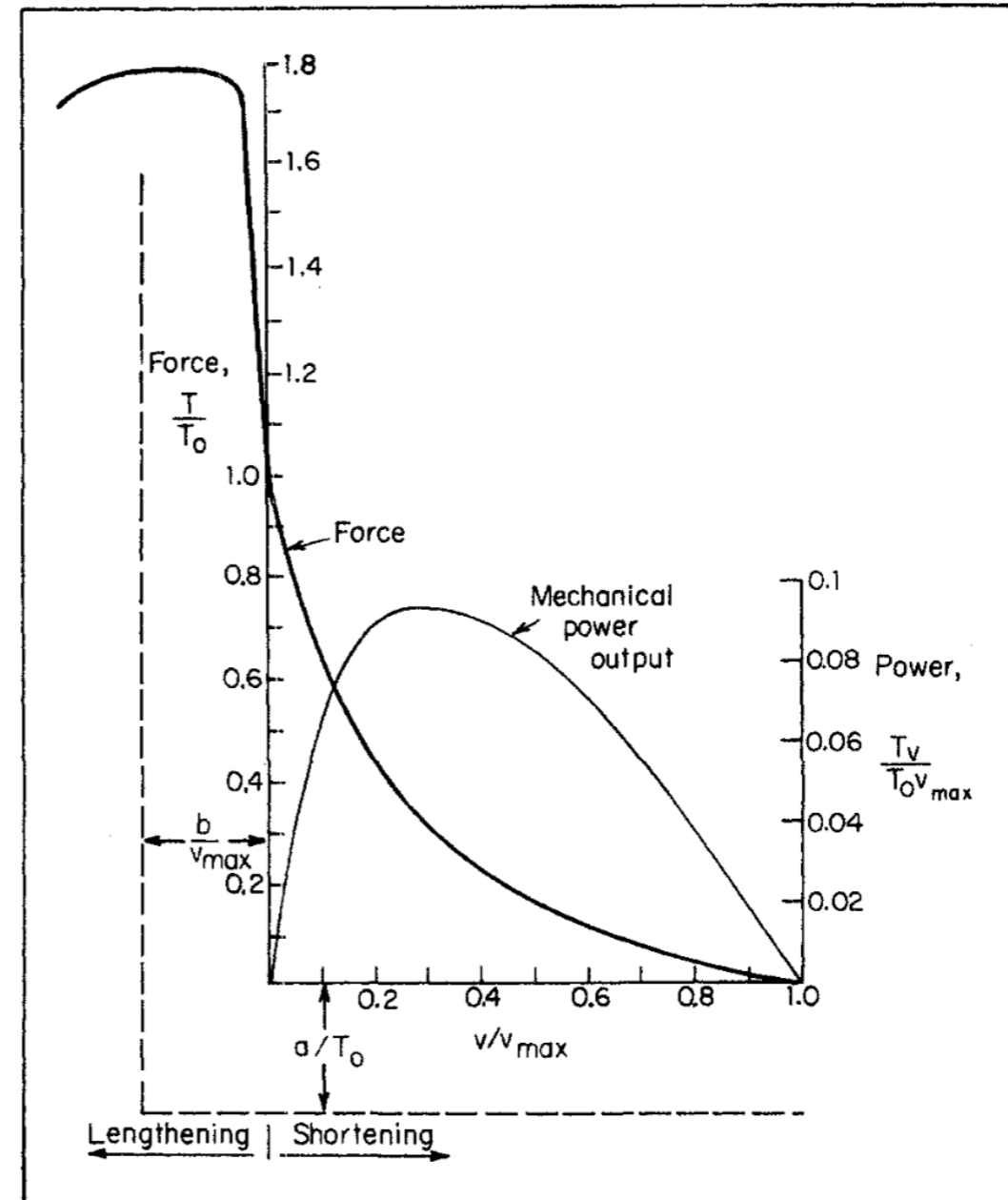
## Muscle Modelling

$$v' = (1 - T') / (1 + T'/k), \quad (1.4)$$

where  $v' = v/v_{\max}$ ,  $T' = T/T_0$ , and  $k = a/T_0 = b/v_{\max}$ . For most vertebrate muscles, the curve described by Hill's equation has a similar shape. In fact, for most muscles,  $k$  usually lies within the range  $0.15 < k < 0.25$ .

It is important to note at this point that mechanical power output available from a muscle,

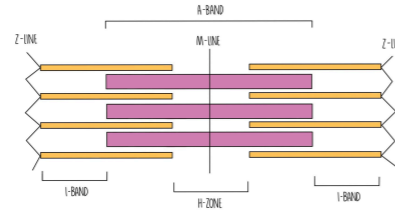
$$Power = Tv = \frac{v(bT_0 - av)}{v + b}, \quad (1.5)$$



**Fig. 1.10.** Hill's force-velocity curve. The shortening part of the curve was calculated from eq. (1.4) with  $k = 0.25$ . The asymptotes for Hill's hyperbola (broken lines) are parallel to the  $T/T_0$  and  $v/v_{\max}$  axes. Near zero shortening velocity, the lengthening part of the curve has a negative slope approximately six times steeper than the shortening part. The externally delivered power was calculated from the product of tension and shortening velocity.

# HAB718 Spor Biyomekaniğinde Hareket Analizi

## Muscle Modelling



### EXAMPLE 9.1

#### MUSCLE MODEL ALGORITHM

A Hill-type muscle model can be implemented in several ways. The most flexible arrangement is to write a software subroutine that contains the code for the muscle model in a general form. The model for any specific muscle is then implemented by writing a calling subroutine that contains the model parameters (e.g.,  $P_o$ , FL information,  $a$  and  $b$  dynamic constants, SEC elasticity) for that particular muscle. The calling subroutine passes the specific parameters to the general model subroutine so that the model output represents that individual muscle. This permits the general model subroutine to be used for more than one specific muscle, which is advantageous when we are using a musculoskeletal model in which several different muscles are represented and are required to produce force simultaneously. This scheme can use the muscle model algorithm and software language of the researcher's choice.

The Hill model iterative algorithm presented here (Baildon and Chapman 1983; Caldwell and Chapman 1989, 1991) is similar in concept (but not necessarily implementation) to others found in the literature (Bobbert et al. 1986a; Pandy et al. 1990; van den Bogert et al. 1998; Winters and Stark 1985; Zajac 1989). The algorithm has explicit equations representing the CC and SEC prop-

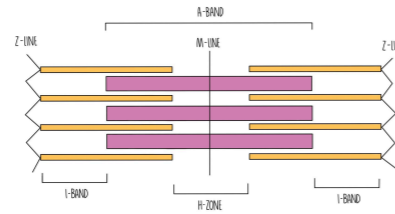
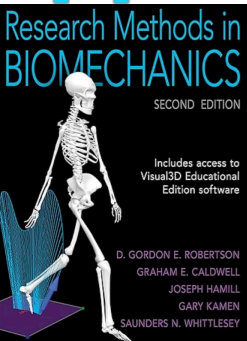
- For each time step  $\Delta t$ 
  - Need predicted muscle forces for  $N$  different muscles ( $i = 1, 2, \dots, i + 1, \dots, N$ )
    - For muscle  $i$ 
      - Get muscle parameters for CC and SEC
      - Get muscle length and stimulation inputs
      - Get CC activation and length from previous time step
      - Get SEC length and force from previous time step
      - Call muscle model subroutine
        - 1. SEC force =  $f$  (SEC length)
        - CC force = SEC force
        - 2. CC activation =  $f$  (stimulation)
        - 3. CC velocity =  $f$  (CC force, activation, length)
        - 4. CC length =  $\int$  CC velocity  $dt$
        - 5. SEC length = muscle length — CC length
      - Repeat for next muscle  $i + 1$
  - Repeat for each time step  $\Delta t$

▲ **Figure 9.14** Flowchart of muscle model algorithm.

erties and should give the user a sense of how the CC and SEC interact dynamically (figure 9.14). The description of the algorithm indicates exactly where functions representing the CC and SEC properties are needed, and table 9.1



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## Muscle Modelling

**Table 9.1** Model Equations and Parameters

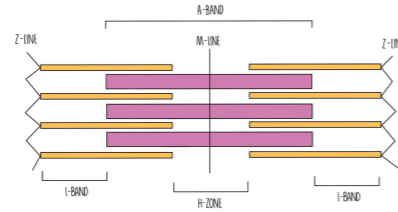
Model component	Relation	Equation	Parameters
CC	FV concentric	$(P + a)(v + b) = (P_o + a) b$	$P_o$ = maximal isometric force $a$ = Hill dynamic constant $b$ = Hill dynamic constant
	FV eccentric	$S = b(P_s - P_o)/(P_o + a)$ $P = P_s - S(P_s - P_o)/(S - v)$	$P_s$ = eccentric force saturation level (% of $P_o$ )
	FL (parabola)	$RF = [c0(RL - 100)^2] + 100$	$c0$ = parabola width coefficient
SEC	FΔL	$RF = 0.0258 [\exp (\text{stiff}) (RLS)] - 0.0258$	$\text{stiff}$ = nonlinear stiffness coefficient

CC = contractile component;  $P$  = force;  $RF$  = relative force;  $RL$  = relative CC length;  $RLS$  = relative SEC length; SEC = series elastic component;  $v$  = CC velocity.

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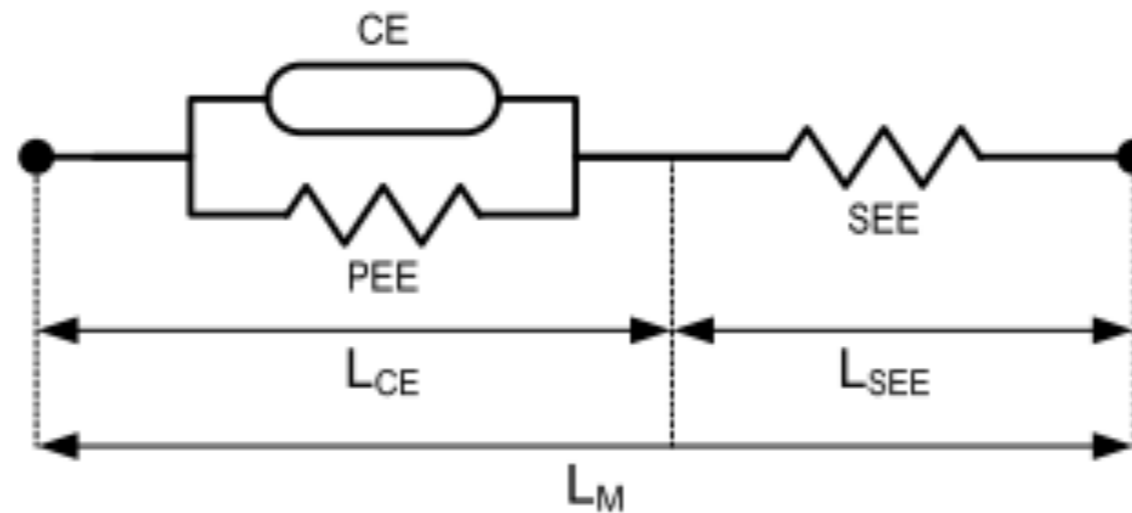


# HAB718 Spor Biyomekaniğinde Hareket Analizi



## Muscle Modelling

### Hill-type muscle model

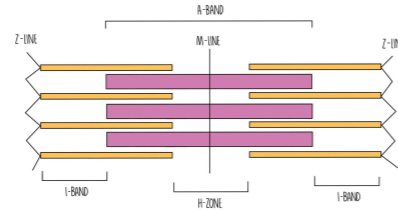


The model consists of three elements:

- A contractile element (**CE**) that represents the muscle fibers that contract based on the muscle activation state.
- A serial elastic element (**SEE**) that represents the tendons that connect the muscle to the bones.
- A parallel elastic element (**PEE**) that represents the passive elastic material surrounding muscle fibers.

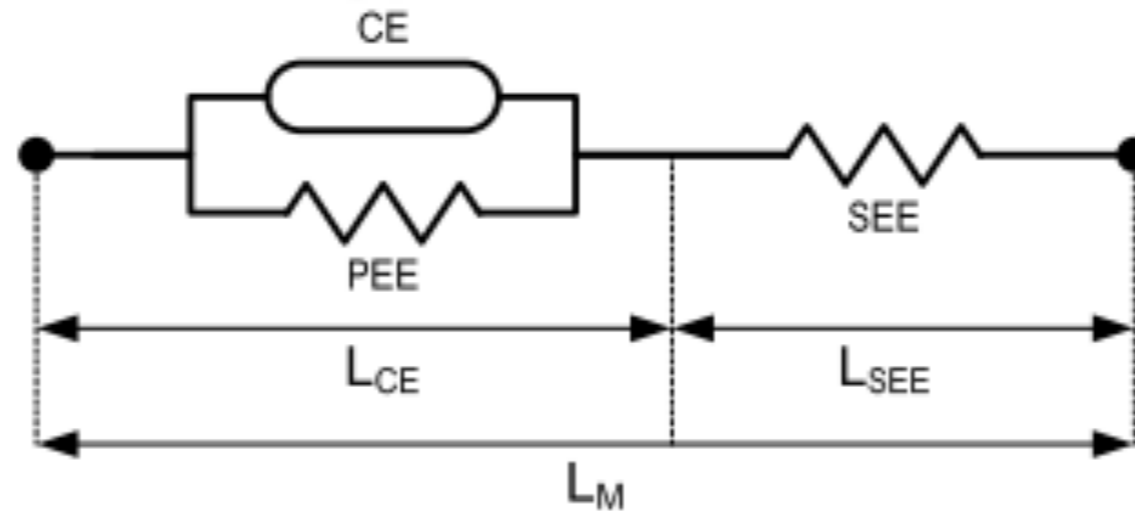


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## Muscle Modelling

### Hill-type muscle model



Forces:

$F_{max}$  : maximum isometric force. Optimized constant.

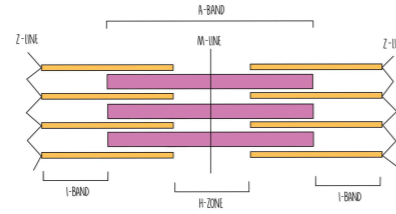
$F_{CE}$  : contractile element force.

$F_{SEE}$  : serial element force.

$F_{PEE}$  : parallel element force.

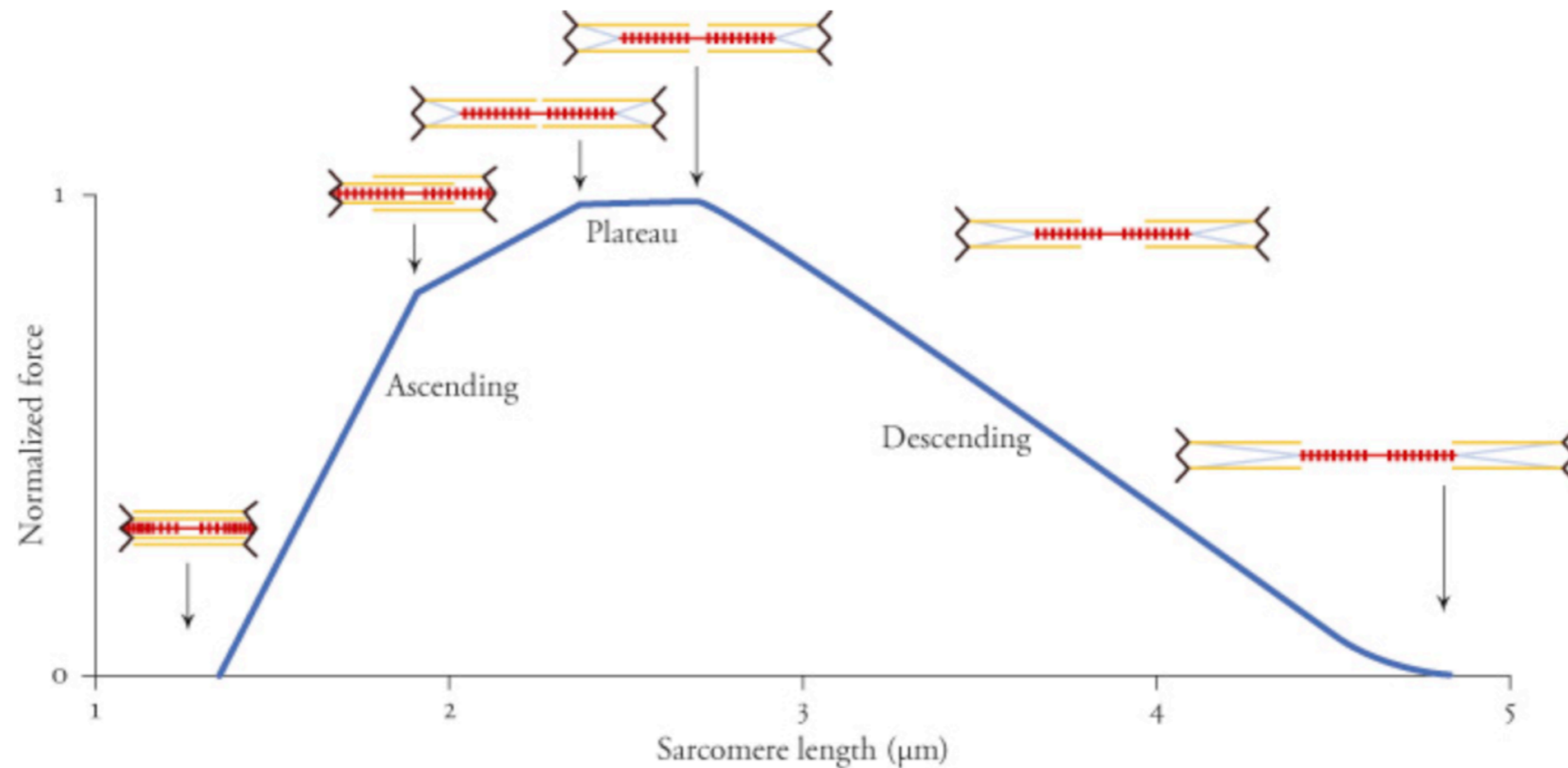
$$F_M = F_{CE} + F_{PEE} + F_{SEE}$$

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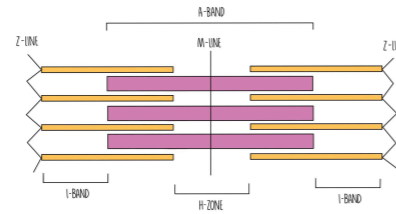
## Muscle Modelling

### Contractile Element



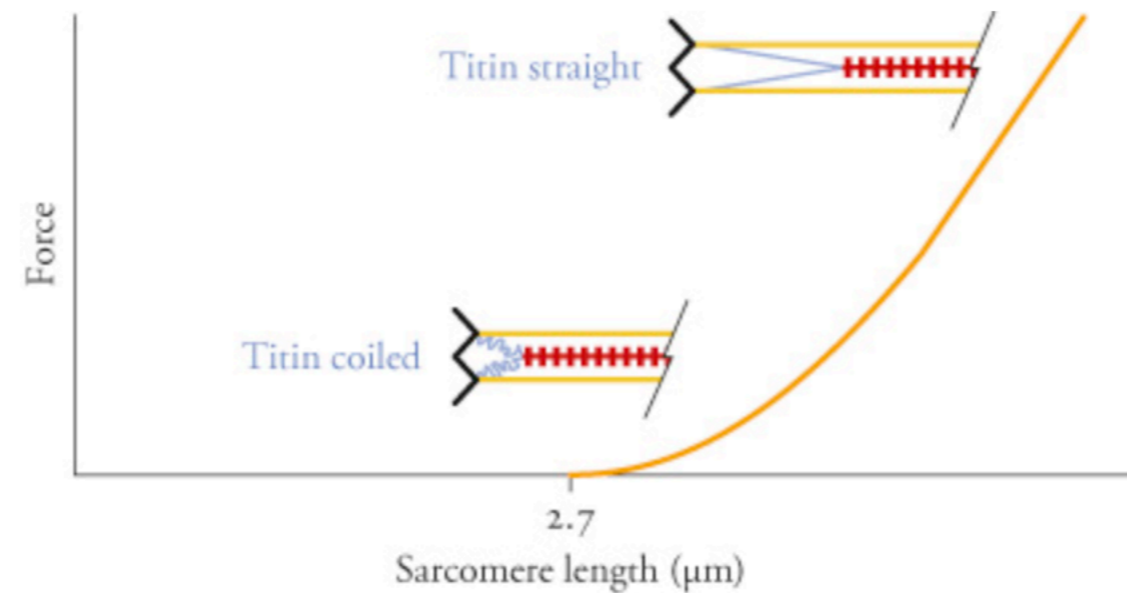
**Figure 4.6** The active force generated by a sarcomere is a function of its length. Force varies as the thick and thin filaments slide past each other. Force increases with length in the ascending limb of the curve, plateaus, and decreases with length in the descending limb. Force generation peaks when the maximum number of cross-bridges are formed. Adapted from Gordon et al. (1966).

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## Muscle Modelling

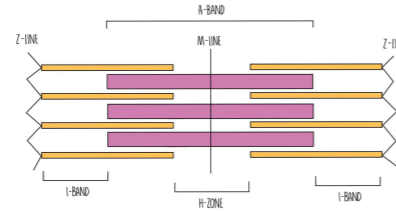
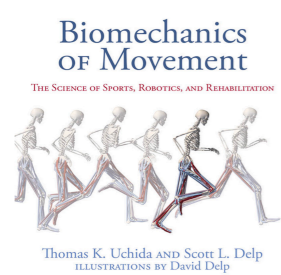
### Contractile Element



**Figure 4.7** Titin attaches the thick filaments to the Z-discs at either end of the sarcomere and develops force when stretched.

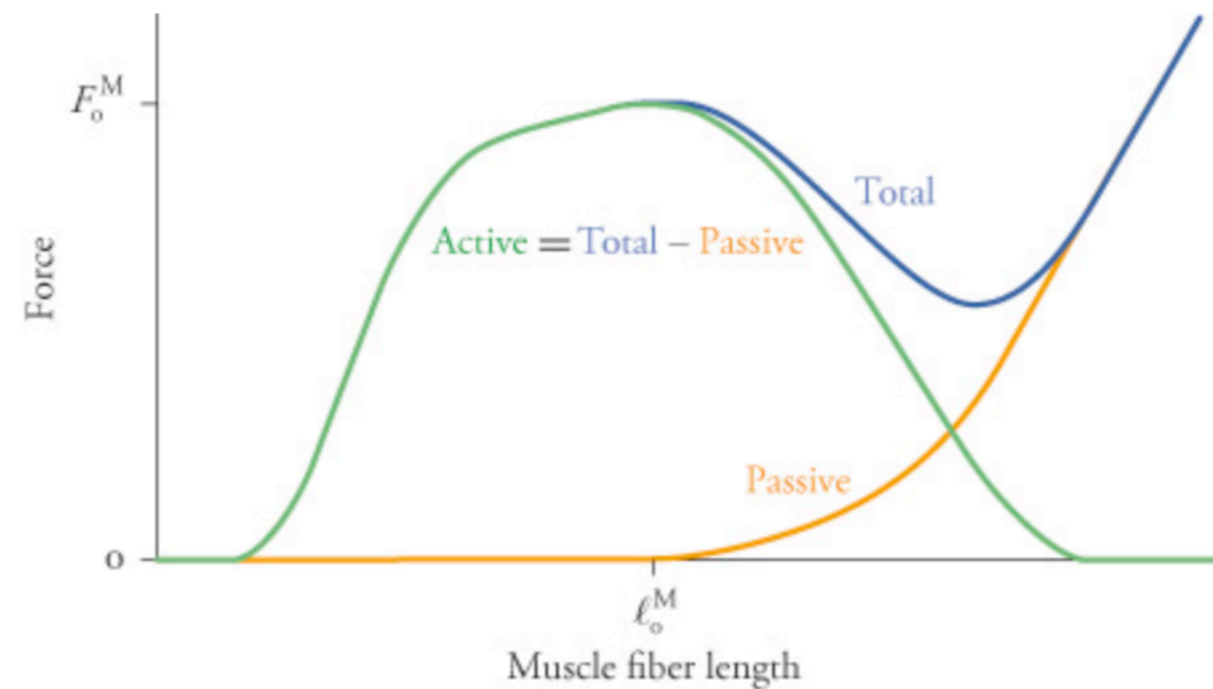


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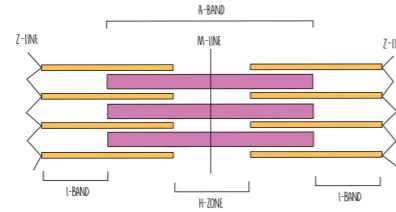
## Muscle Modelling

### Contractile Element



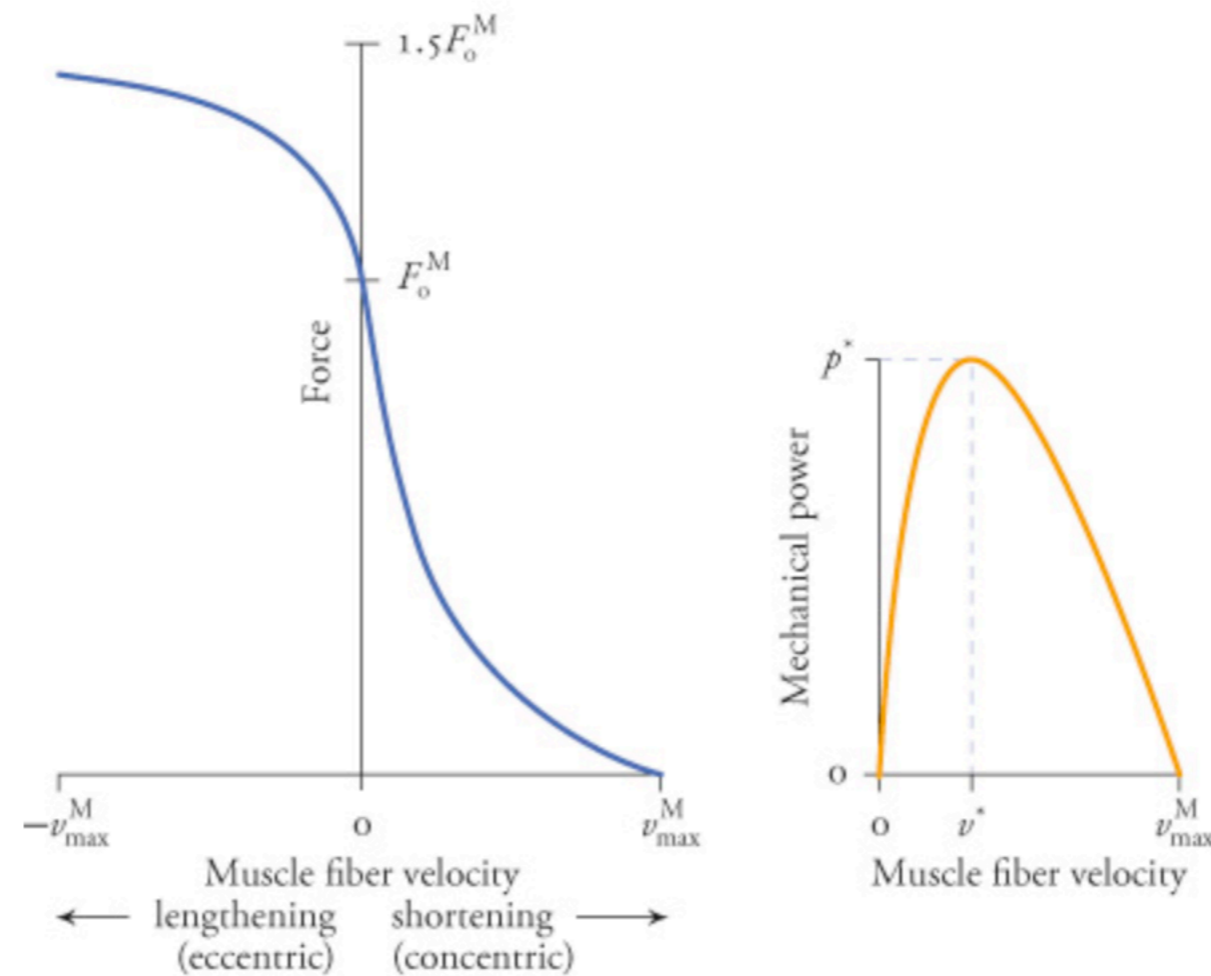
**Figure 4.8** The active force–length curve can be determined by subtracting measurements of the passive force from the total force over a range of fiber lengths. The peak active force,  $F_o^M$ , occurs at the optimal muscle fiber length,  $\ell_o^M$ .

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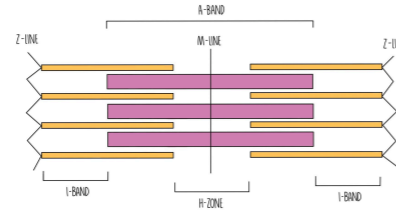
## Muscle Modelling

Force-Velocity relationship was first observed by A. V. Hill in 1938



**Figure 4.9** Muscle fiber force and power as functions of the fiber's velocity. Force-generating capacity increases when lengthening and decreases when shortening (left). Mechanical power, the product of force and velocity, reaches a maximum ( $p^*$ ) at about one-third of the maximum shortening velocity ( $v^*$ ; right).

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## Muscle Modelling

### Serial Elastic Element

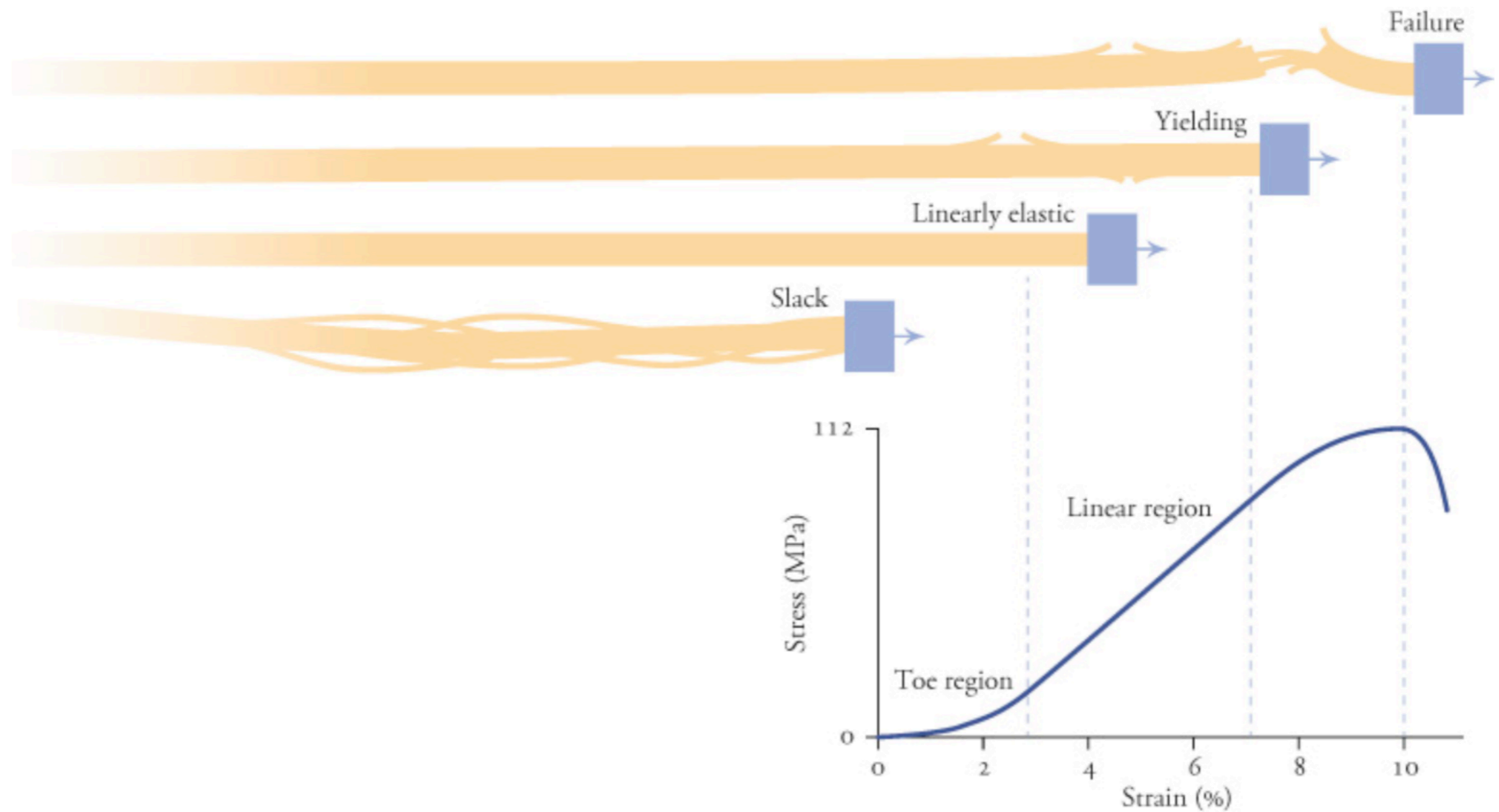
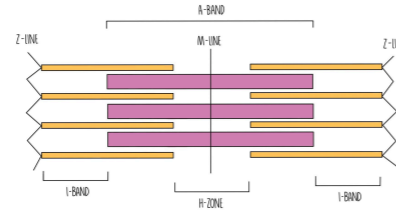
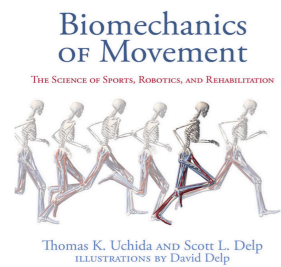


Figure 5.7 Tendon stress-strain relationship.



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## Muscle Modelling

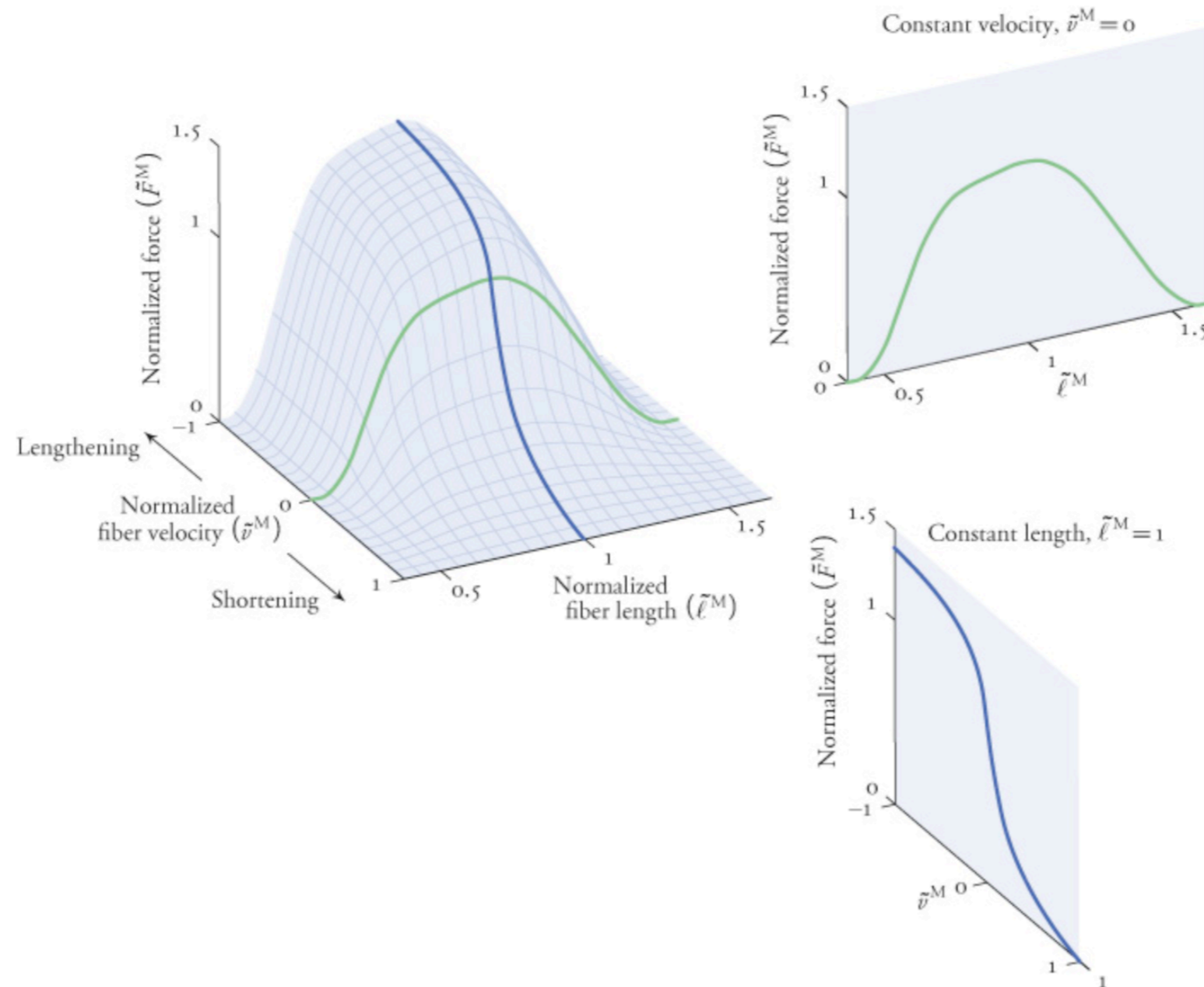
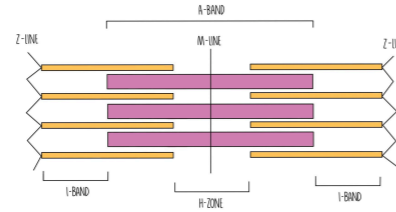
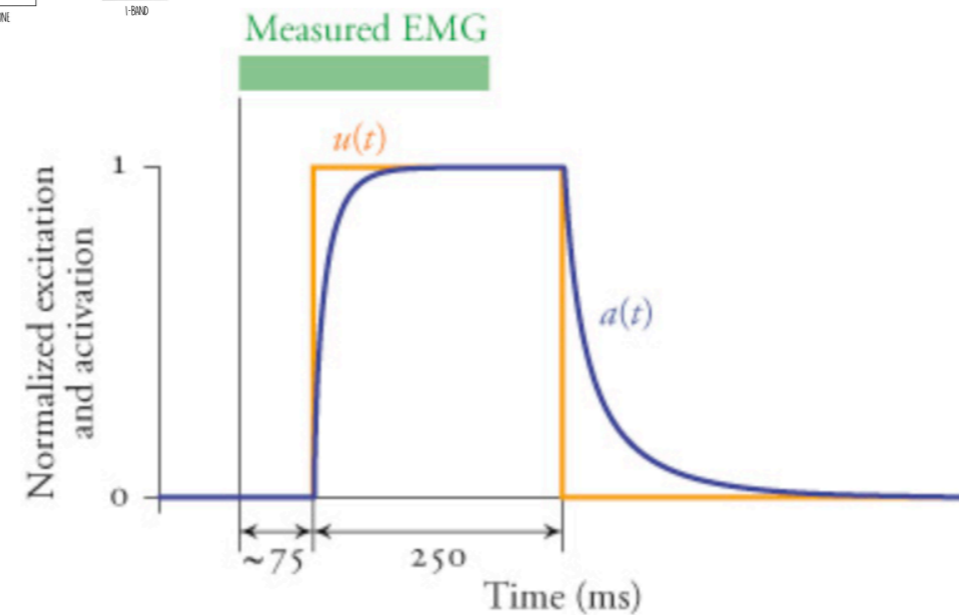


Figure 4.17 Muscle force-generating capacity varies with fiber length and velocity. Adapted from Lieber (2010).

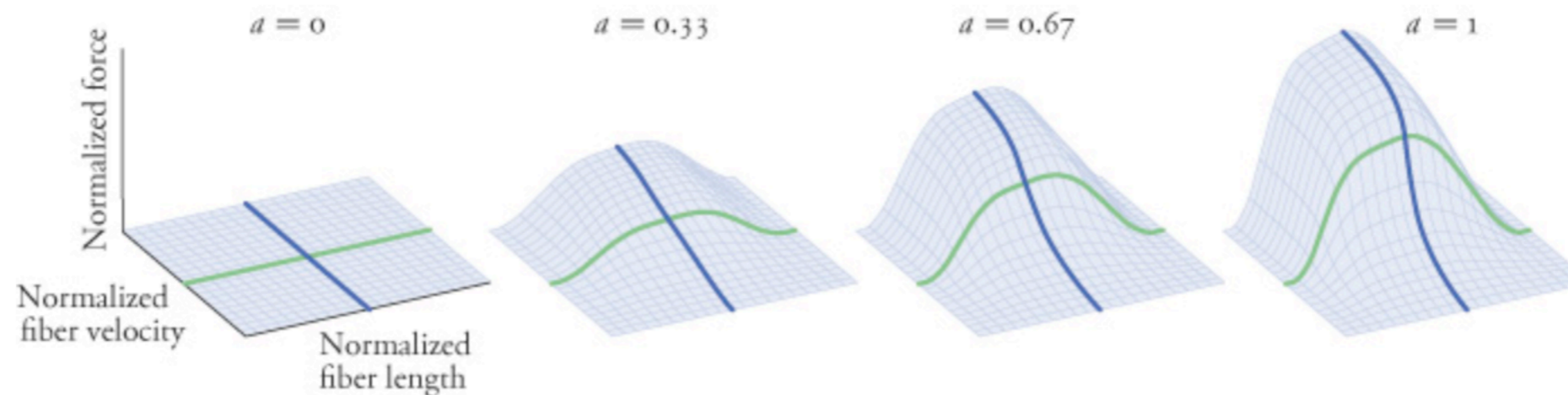
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## Muscle Modelling



**Figure 4.16** A computational model of activation dynamics relates excitation ( $u(t)$ ) to activation ( $a(t)$ ) using a first-order ordinary differential equation. Excitation often lags behind a measured EMG signal, and force production lags further still.



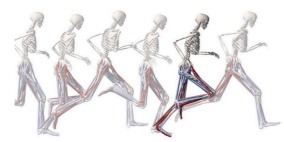
**Figure 4.18** The nervous system modulates muscle force through rate encoding and motor unit recruitment, collectively modeled by muscle activation ( $a$ ).



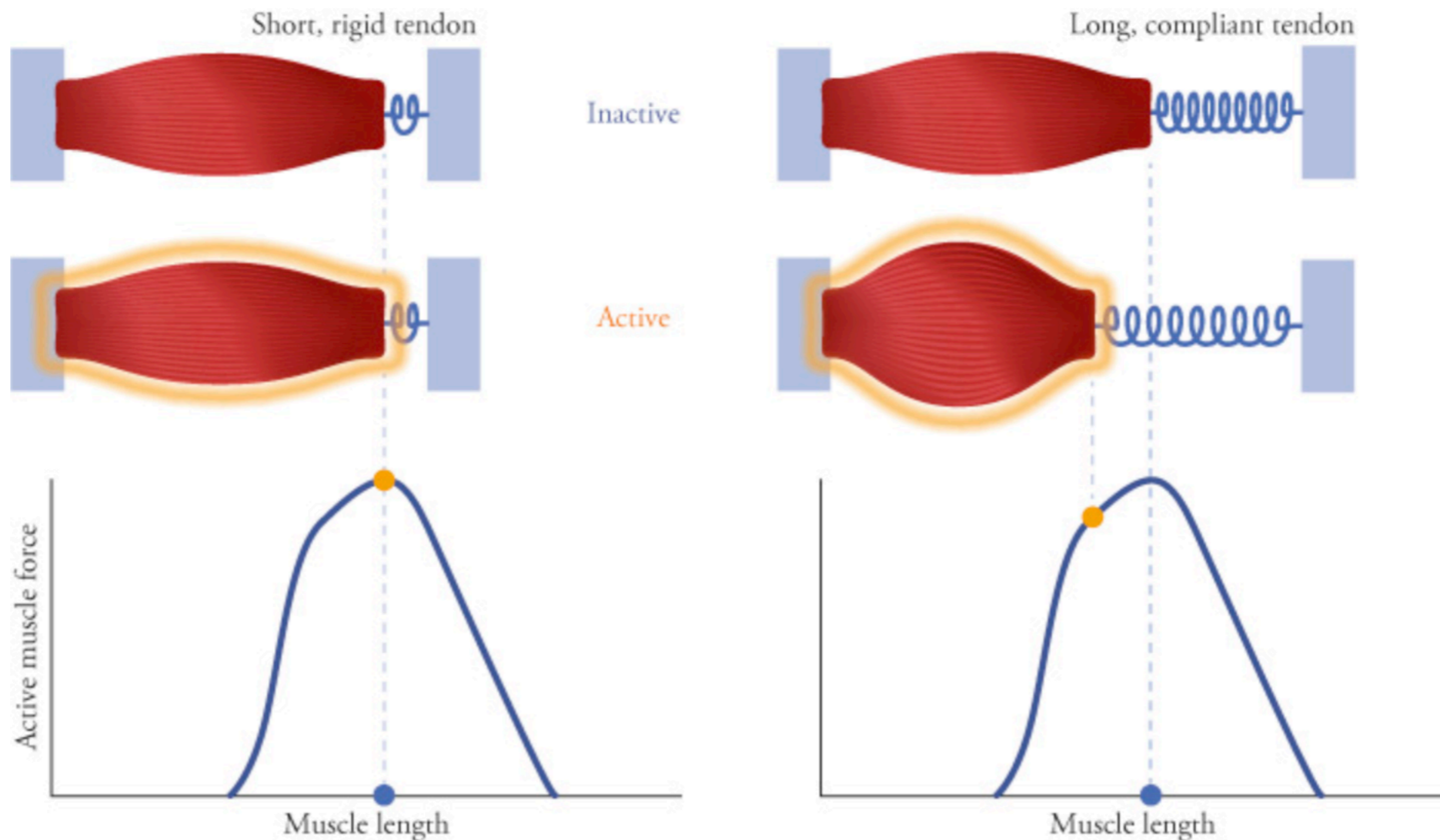
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Biomechanics  
OF Movement

THE SCIENCE OF SPORTS, ROBOTICS, AND REHABILITATION

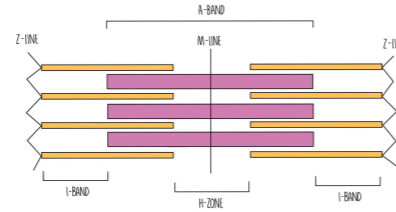


Thomas K. Uchida AND Scott L. Delp  
ILLUSTRATIONS BY David Delp

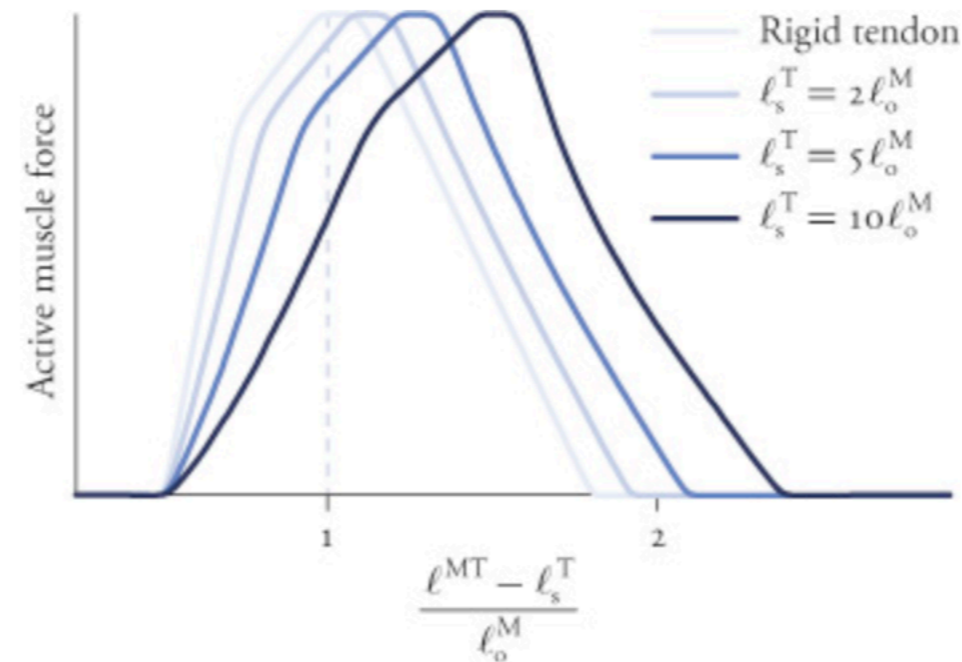


**Figure 5.8** Tendon compliance affects muscle force generation. In both cases shown, a parallel-fibered muscle is at its optimal length when the muscle is inactive (top). If the tendon is relatively short and rigid (left), there will be negligible change in the length of the muscle fibers when the muscle is activated. If the tendon is long and compliant (right), the tendon will stretch when the muscle is activated, thereby shortening the muscle fibers and reducing the generated force.

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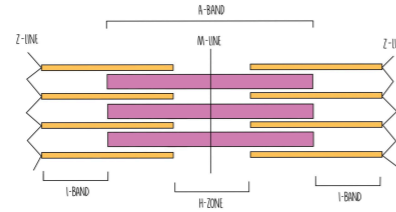


## Muscle Modelling



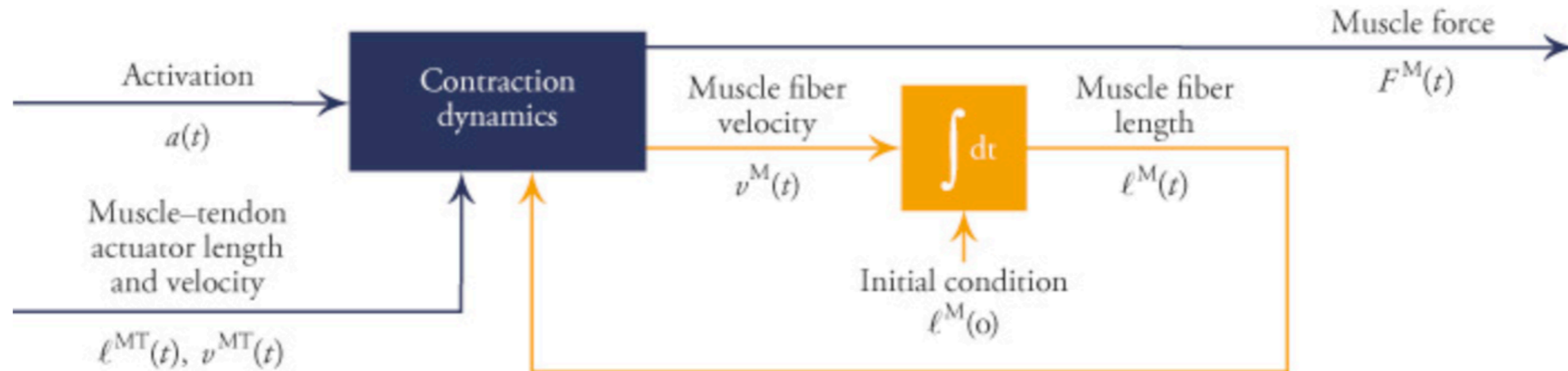
**Figure 5.9** Effects of tendon compliance on the active force–length curve. Increasing the slack length of the tendon relative to the optimal muscle fiber length (i.e., increasing tendon compliance) broadens the range of lengths over which a muscle–tendon actuator can generate active force.

# HAB718 Spor Biyomekaniğinde Hareket Analizi



## Muscle Modelling

### Computing Muscle Force with a Compliant Tendon



**Figure 5.12** Muscle fiber length ( $\ell^M(t)$ ) is integrated forward in time to compute contraction dynamics with a compliant tendon.